AN

ARITHMETIC

FOR

PREPARATORY SCHOOLS,

HIGH SCHOOLS, AND ACADEMIES.

BY

CHARLES A. HOBBS, A.M.,

MASTER OF MATHEMATICS IN THE BELMONT SCHOOL, BELMONT, MASS.

NEW YORK:

A. LOVELL & CO.

1896.

Hillebrand

PREFACE.

This book is designed particularly for pupils in preparatory schools, and it is likewise adapted for the use of all pupils who desire a thorough knowledge of Arithmetic.

The four fundamental operations of Arithmetic should be learned by a thorough drill in the elementary schools. This cannot be too strongly insisted upon, as accuracy and readiness of work in all parts of Arithmetic are dependent on these fundamental operations. After such a drill the pupil is ready for rapid advancement. Since this fundamental work belongs to the elementary schools, it has been omitted in this book, and the space has been given to examples for more advanced pupils.

Special attention has been paid to the selection of examples, over a thousand of which have been taken from entrance papers given at various universities and colleges. The 275 miscellaneous examples at the end of the book are all taken from such entrance papers and from entrance papers given at the United States Military and Naval Academies at West Point and Annapolis. An abundance and variety of examples, sufficient to render the pupil master of the subject, will be found in all parts of the book.

In the selection of illustrative examples, great care has been taken to present those which will make clear to the pupil all the difficulties he is liable to meet. The solu-

tions are given in full in order that the principles involved may be clearly understood with but little aid from the teacher.

The adaptability of a text-book to school purposes can be determined only by actual use in the school-room. This treatise has already stood this test, since nearly every part of it has been used by the author in classes, whose members have without exception passed successfully their college entrance examinations in Arithmetic.

No attempt has been made to introduce novel methods, but in all cases methods are given which experience has shown to be well adapted to the needs of the pupil. In the arrangement of subjects, no important departure has been made. The Metric System, a thorough knowledge of which is required by all first-class universities and colleges, is given directly after Compound Numbers, and thereafter the two systems are used side by side, thus giving the pupil a thoroughly practical as well as theoretical knowledge of the system.

It is expected that the teacher will use his judgment with regard to omissions. In the endeavor to make the book complete, certain subjects have been included which are not necessary to a good knowledge of Arithmetic; a careful study of college entrance papers shows that these subjects are insisted on by some colleges.

The author desires to express his thanks to the many persons who have aided him by valuable suggestions, and also to the many institutions which have responded so promptly and often to requests for entrance papers.

CHARLES A. HOBBS.

BELMONT, July 1st, 1889.

CONTENTS.

ARITHMETIC.

CHAPTER I.

INTRODUCTION.

1. Before studying this book the student should be perfectly familiar with the four fundamental operations of Arithmetic — Addition, Subtraction, Multiplication, and Division. In order to ensure accuracy it is always advisable to test each step of work.

In Addition add the column downwards and then upwards, and if the results are alike, they may be considered correct.

In Subtraction the best test is to add the subtrahend and the remainder, and if the work is correct, the sum is the same as the minuend.

In Multiplication and Division the easiest way to test the work is simply to repeat each step; however, if further tests are desired, the following can be used: in Multiplication divide the product by the multiplier, and if the work is correct, the result is the same as the multiplicand; in Division multiply the quotient by the divisor and add the remainder, if any, and if the work is correct, the result is the same as the dividend.

2. The fundamental operations can also be tested by the method known as casting out the nines. The **excess of**

nines is a term used to denote the remainder arising from dividing a number by 9.

$$10 = 9 + 1, \qquad 1000 = 9 \times 111 + 1,$$
$$100 = 9 \times 11 + 1, \qquad 10000 = 9 \times 1111 + 1, \text{ etc.}$$

We thus see that a unit of any order equals one more than 9 multiplied by some number. From this it follows that any number of units of any order equals that number of units added to 9 multiplied by some number. For example, $50 = 9 \times 5 + 5$, $600 = 9 \times 66 + 6$, $7000 = 9 \times 777 + 7$, etc. Therefore every number consists of a certain number of nines increased by the sum of its digits. For example, 7654 equals a number of nines increased by $7 + 6 + 5 + 4$; this excess equals $9 \times 2 + 4$, so that the excess of nines is 4. This process can be applied to any number; hence *the excess of nines in any number equals the excess of nines in the sum of its digits.*

Addition.

$$\begin{array}{rrl} & 82765 \ldots & 1 \\ & 4912 \ldots & 7 \\ & 25754 \ldots & 5 \\ & \underline{6732} \ldots & \underline{0} \\ 4 \ldots & 120163 & 13 \ldots 4 \end{array}$$

The excess of nines in the first number is 1, in the second number 7, in the third number 5, and in the fourth number 0. The sum is 13, and the excess of nines is 4. The sum of the numbers is 120163, in which the excess of nines is 4. The excess is the same in each case; therefore the work may be considered correct.

Subtraction.

$$\begin{array}{rl} 89643 \ldots\ldots\ldots & 3 \\ \underline{3598} \ldots & 7 \\ 86045 \ldots & \underline{5} \\ & 12 \ldots 3 \end{array}$$

The minuend equals the sum of the subtrahend and remainder. The excess of nines in the minuend is 3. The excess of nines in the subtrahend is 7, and in the remainder 5. The excess in the sum of these two excesses is 3, the same as in the minuend; therefore the work may be considered correct.

MULTIPLICATION.

$$\begin{array}{rl} 857\ldots & 2 \\ 62\ldots & \underline{8} \\ \overline{1714} & \overline{16}\ldots 7 \\ 5142 & \\ \hline 53134\ldots & 7 \end{array}$$

The excess of nines in the multiplicand is 2; in the multiplier 8. The product is 16, and the excess of nines is 7. The product of the numbers is 53134, in which the excess of nines is 7. The excess is the same in each case; therefore the work may be considered correct.

DIVISION.

$$\begin{array}{r} 563)87614(155 \\ \underline{563} \\ 3131 \\ \underline{2815} \\ 3164 \\ \underline{2815} \\ 349 \end{array} \qquad \begin{array}{l} 87614\ldots\ldots\ldots 8 \\ \quad 563\ldots 5 \\ \quad 155\ldots 2 \\ \quad 349\ldots 7 \\ 5\times 2+7=17\ldots . 8 \end{array}$$

The dividend equals the product of the divisor and quotient plus the remainder. The excess of nines in the dividend is 8. The excess of nines in the divisor is 5, in the quotient 2, and in the remainder 7. The product of 5 and 2 plus 7 equals 17, in which the excess is 8, the same as in the dividend; therefore the work may be considered correct.

3. The **excess of elevens** is a term used to denote the remainder arising from dividing a number by 11.

$$10 = 11 - 1, \qquad 1000 = 11 \times 91 - 1,$$
$$100 = 11 \times 9 + 1, \qquad 10000 = 11 \times 909 + 1, \text{ etc.}$$

We thus see that a unit in an odd place equals one more than 11 multiplied by some number, and a unit in an even place equals one less than 11 multiplied by some number. From this it follows that any number of units in an odd place equals that number of units added to 11 multiplied by some number, and any number of units in an even place equals that number of units subtracted from 11 multiplied by some number. Therefore every number consists of a

certain number of elevens increased by the sum of the digits in the odd places, decreased by the sum of the digits in the even places. For example, 45316 equals a number of elevens increased by $4 + 3 + 6$, decreased by $5 + 1$, and the excess of elevens is $13 - 6$, or 7. 18394 equals a number of elevens increased by $1 + 3 + 4$, decreased by $8 + 9$; 17 cannot be subtracted from 8, so one of the elevens is added to 8, and the excess of elevens is $11 + 8 - 17$, or 2. This process can be applied to any number; hence *the excess of elevens in any number can be found by subtracting the sum of the digits in the even places from the sum of the digits in the odd places (increased, if necessary, by* 11 *or some number of* 11*'s)*.

The processes of testing the fundamental operations by casting out the elevens are similar to those by casting out the nines.

4. There are a few principles of Multiplication and Division which should be borne in mind.

(1) Multiplying either multiplicand or multiplier by a number multiplies the product by the same number.

(2) Dividing either multiplicand or multiplier by a number divides the product by the same number.

(3) Multiplying the dividend or dividing the divisor by a number multiplies the quotient by the same number.

(4) Dividing the dividend or multiplying the divisor by a number divides the quotient by the same number.

(5) Multiplying or dividing both dividend and divisor by the same number does not change the quotient.

5. The student should always take great care in his use of signs. The signs + and − always denote the points of separation in a problem, and the parts between these signs

are called **terms**. Each term should be simplified by itself before the operations of addition and subtraction are performed. In the expression $15+12\div3-5\times2$ there are three terms, 15, $12\div3$, and 5×2; hence the value is $15+4-10=9$.

The **parenthesis**, ( ), is used to denote that the expression contained therein should be used as a whole. The expression should be simplified at first, and its value substituted. For example, $(18+7-5)\div(10-6)+3\times8=20\div4+3\times8$ $=5+24=29$.

NOTE. Brackets, [ ], braces, { }, and the vinculum, $\overline{\quad}$, may be used with the same meaning as a parenthesis.

EXAMPLES.

Find the value of

1. $28\times4+32\div8-16$.
2. $10+6\times3-24\div6+12$.
3. $56\div7+12\times3-52\div2$.
4. $22-6\times3+2\times5+50\div25$.
5. $28\times6\div14+9\times8\div12+42\div7\times3$.
6. $99\times8+51\times10-7\times104+26$.
7. $99\times(8+51)\times10-(7\times104+26)$.
8. $(99\times8)+(51\times10-7)\times104+26$.
9. $(99\times8+51)\times10-7\times(104+26)$.
10. $(99\times8)+(51\times10)-(7\times104)+26$.
11. $99\times8+51\times(10-7)\times104+26$.
12. $99\times(8+51)\times(10-7)\times(104+26)$.
13. $(105\div21+80\div5)\times(81+36\div9)$.
14. $(3146+279-2141)\div(370-263)+91\times3$.

15. $(2142 - 1729) \times (3666 - 2514) \div (354 \div 6)$.

16. $327 \times 6 \div 109 + 52 \times 5 - (42 + 8 \times 4)$.

17. $(46 - 8) \times 11 + 17 \times 15 + (83 \times 4 - 327) \times 10 - 39 \times 14$.

18. $864 \div 12 - 124 \div (775 \div 25) + 54 \div (61 - 34)$.

19. $949 \div 13 - (119 \div 7 + 1176 \div 21) + 3648 \div 32 - (306 \div 51 - 5 + 672 \div 6)$.

20. From $126 + (16 + 4) \times 2$ take $(48 \div 2) + 34 \times 6 \div (17 - 5)$.

CHAPTER II.

DECIMAL FRACTIONS.

6. When a number is used to express whole units, it is called an **integer,** or **integral number.**

As in the decimal system of notation a unit of any order has one tenth the value of a unit of the next order to the left, we can continue our notation toward the right beyond units' place. To do this, we place a **decimal point (.)** immediately after the number in units' place, and the next place to the right is called **tenths**; a digit in this place has one tenth the value of the same digit in units' place. For example, .3 is read *three tenths.* The places are continued to the right indefinitely, and the names are given in the table annexed.

Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.	Etc.
.0	0	0	0	0	0	0	0	0	

The places at the right of the decimal point are called **decimal places,** and the numbers thus written are called **decimal fractions,** or **decimals**. They are read like whole numbers, adding the name of the right-hand place. For example, .67 is read *sixty-seven hundredths.* It is to be noticed that the number consists of six tenths and seven hundredths, but as six tenths is the same as sixty hundredths, the entire decimal is sixty-seven hundredths. .5432 is read *five thousand four hundred thirty-two ten-thousandths.*

When whole numbers and decimal fractions are written together, the word "and" should be used to connect the

two parts. For example, 362.459 is read *three hundred sixty-two and four hundred fifty-nine thousandths.*

NOTE. Zeros may be annexed or omitted at the right of a decimal fraction without altering the value, for either with or without these zeros the significant digits are in the same decimal places.

When a decimal is written without an integer, a zero may be put in units' place, or the place may be left vacant.

EXERCISES IN NUMERATION.

Read the following numbers:

1. .6.
2. .572.
3. .05072.
4. .090067.
5. .31402.
6. .00000041.
7. 27.8.
8. 2.78.
9. .6008.
10. 6000.0008.
11. 41.002.
12. 750.0081.
13. 75000.81.
14. 3.14159.
15. 28.0097.

EXERCISES IN NOTATION.

Write the following in figures:

1. Nine tenths.
2. Eighty-two hundredths.
3. Eighty-two ten-thousandths.
4. Three hundred sixty-one thousandths.
5. Three hundred sixty-one millionths.
6. Nine thousand two hundred nine hundred-thousandths.
7. Thirty thousand six hundred-thousandths.
8. Thirty thousand and six hundred-thousandths.
9. Sixteen and three hundred forty-one thousandths.
10. One hundred fifty-five millionths.

11. One hundred and fifty-five millionths.

12. Six and five ten-millionths.

13. Sixty-five ten-millionths.

14. Forty-five and fifty-eight hundredths.

15. Three hundred twenty-nine thousandths.

16. Three hundred and twenty-nine thousandths.

Addition of Decimals.

7. Units of the same order should be written in the same column. This can easily be accomplished by making the decimal points fall under each other. The process of adding is precisely the same as in whole numbers.

I. Find the sum of 18.47, 159.363, 70.00451, and 0.926.

$$\begin{array}{r} 18.47 \\ 159.363 \\ 70.00451 \\ 0.926 \\ \hline 248.76351 \end{array}$$

We begin at the right and add as in whole numbers. The decimal point comes under the decimal points of the problem.

EXAMPLES.

1. Add together 12.613, 0.00175, 257.8425, and 0.001345.

2. Add together 17.429, 0.0173, 1156.8, and 0.0001723.

3. Add together 0.20765, 0.00631, 6758.13247, and 5.973.

4. Add together 3107.8192, 0.0624, 0.00414, and 47.2875.

5. Add together 371.87007, 0.00731, 5768.45321, and 0.0093.

6. Add together 11.431, 0.00101, 243.342, 400, and 1.3734.

7. Add together 16.41215, 9.736, 0.00304, 188.24, and 29.03069.

8. Add together 0.61692, 243.734, 901, 68.45213, and 8.386.

9. Add ten thousand and one millionth, four hundred-thousandths, ninety-six hundredths, and forty-seven million sixty thousand and eight billionths.

10. Find the sum of the following numbers: fifty-seven and three thousandths, three hundred and sixty-four hundred-thousandths, forty-seven thousand and eight thousand seven hundred-thousandths, eighty-seven millionths, and four hundred and twenty-seven ten-thousandths.

Subtraction of Decimals.

8. The process is the same as in whole numbers, taking care to have the decimal points under each other.

I. From 934.2963 subtract 47.794.

$$\begin{array}{r} 934.2963 \\ 47.794 \\ \hline 886.5023 \end{array}$$

In the subtrahend there is no digit in the place of ten-thousandths, so we annex a zero mentally, and this zero subtracted from the 3 of the minuend leaves 3.

II. From 35.2 subtract 24.543.

$$\begin{array}{l} 35.2 \\ 24.543 \\ \hline 10.657 \end{array}$$

In the minuend there are no digits in the places of hundredths and thousandths, so we annex two zeros mentally, and then subtract as in whole numbers.

EXAMPLES.

1. Subtract 284.7654 from 321.07659.
2. Subtract 17.2398 from 27.06.
3. Subtract 29.9189 from 240.775.
4. Subtract 84.736568 from 100.3064231.
5. Subtract 49.934 from 500.39.
6. Subtract 70.2574 from 365.71.

7. Subtract 876.351 from 1000.01.

8. Subtract 185.939131 from 186.847.

9. Find the difference between 0.0000005 and 0.00005.

10. From ten take six millionths.

11. From two hundred and six thousandths take two hundred six thousandths.

12. What is the value of thirty-six million minus thirty-six millionths?

Multiplication and Division by 10, 100, 1000, etc.

9. Since a unit of any order is ten times as large as a unit of the next order to the right, when we move a digit one place to the left, we multiply it by 10. This is the same as moving the decimal point one place to the right. For example, $70 \times 10 = 700$; $85.43 \times 10 = 854.3$. If the decimal point be moved two places to the right, the number is multiplied by 100; to multiply by any number of 10's, the decimal point is moved as many places to the right as there are zeros in the multiplier. For example, $0.95 \times 100 = 95$; $2.5 \times 1000 = 2500$; $0.00043 \times 10000 = 4.3$.

For the same reason, when we move a digit one place to the right, we divide it by 10. This is the same as moving the decimal point one place to the left. For example, $26 \div 10 = 2.6$; $0.49 \div 10 = 0.049$. To divide by any number of 10's, the decimal point is moved as many places to the left as there are zeros in the divisor. For example, $195 \div 100 = 1.95$; $4.53 \div 10000 = 0.000453$.

In like manner, to multiply by 0.1, 0.01, 0.001, etc., move the decimal point as many places to the left as there are decimal places in the multiplier. To divide by 0.1, 0.01, 0.001, etc., move the decimal point as many places to the right as there are decimal places in the divisor.

EXAMPLES.

Find the value of

1. 8.7×10.
2. 0.0069×10.
3. 95.6×100.
4. 0.0453×100.
5. 4.069×1000.
6. 0.00094×10000.
7. $9.2 \div 10$.
8. $7.49 \div 100$.
9. $0.036 \div 100$.
10. $854.3 \div 1000$.
11. $1.00182 \div 1000$.
12. $76.541 \div 10000$.
13. 4.7×0.1.
14. 8.76×0.01.
15. 0.0469×0.01.
16. 0.037×0.001.
17. 4.62×0.001.
18. 573.7×0.00001.
19. $10 \div 0.1$.
20. $53.4 \div 0.01$.
21. $97.42 \div 0.001$.
22. $0.48 \div 0.001$.
23. $0.1 \div 0.0001$.
24. $7.32 \div 0.00001$.

Multiplication of Decimals.

10. I. Multiply 4.92 by 0.3.

$$\begin{array}{r} 4.92 \\ 0.3 \\ \hline 1.476 \end{array}$$

If units of any order be multiplied by an integer, the product consists of units of the same order. If 4.92 be multiplied by 3, the product is 14.76. But the multiplier is 0.3, a number one tenth as large; hence the product is one tenth as large, and the decimal point must be moved one place to the left, giving 1.476 for the answer.

II. Multiply 0.718 by 0.028.

$$\begin{array}{r} 0.718 \\ 0.028 \\ \hline 5744 \\ 1436 \\ \hline 0.020104 \end{array}$$

0.718 multiplied by 28 would give 20.104. But the multiplier is only one thousandth of 28; hence the decimal point must be moved three places to the left, giving 0.020104 for the answer.

From these two examples we see that we *multiply as in whole numbers, pointing off as many decimal places in the product as there are in both multiplicand and multiplier.*

EXAMPLES.

1. Multiply 6.4 by 1.5.
2. Multiply 0.64 by 0.15.
3. Multiply 0.09 by 0.0016.
4. Multiply 0.427 by 345.
5. Multiply 76000 by 1.05.
6. Multiply 0.076 by 0.0105.
7. Multiply 37900000 by 2.005.
8. Multiply 0.0379 by 0.2005.
9. Multiply 34.27 by 60000.
10. Multiply 200.043 by 2.021.
11. Multiply 0.785 by 0.0191.
12. Multiply 2.708 by 0.007005.
13. Multiply 947.36 by 0.00423.
14. Multiply 2.708 by 70050000.
15. Multiply 8.764 by 40.015.
16. Multiply 25.3784 by 12.567.
17. Multiply 0.0400268 by 0.260075.
18. Multiply 3 hundredths by 300 thousandths.
19. Multiply five thousand and three ten-thousandths by five thousand three ten-thousandths.
20. Multiply twelve thousand five hundred and six hundred seventy-five millionths by four thousand sixteen ten-thousandths.

21. Multiply six hundred twenty-five ten-millionths by three hundred and eight thousandths.

Contracted Multiplication of Decimals.

11. In many examples in multiplication of decimals, only a certain number of accurate decimal places are required in the product. All extra work involving figures beyond the required degree of accuracy can be avoided by the use of the following method:

Invert the order of the figures of the multiplier, and place them so that the tenths' figure may be under that order of decimals to which it is proposed to limit the product. Multiply the multiplicand by each figure of the multiplier, beginning at the figure immediately above it, and taking in the number carried from the right hand. Place the first figure of each partial product in the same column, and add the partial products, rejecting the sum of the right-hand column, after carrying the nearest ten.

I. Multiply 29.637842 by 85.916, the result to be correct to three decimal places.

```
29.637842
   61958
--------
23710274
 1481892
  266740
    2964
    1778
--------
2546.365
```

In order to ensure accuracy to three decimal places, the partial products should be accurate to four decimal places. Hence four decimal places must be multiplied by the units' figure, and 5 is placed under the figure in the fourth decimal place; this brings the figure in tenths' place under the order of decimals to which the product is to be limited. As the multiplier is now arranged, each partial product obtained by beginning to multiply at the figure directly above contains four decimal places; hence the first figures of the partial products are to be placed in the same column.

Begin at the right to multiply. $8 \times 4 = 32$; however, 2 must be carried from the right, for $8 \times 2 = 16$, which is nearer 20 than 10; hence we set down 4 and carry 3 to the next place. The process is then continued as in ordinary multiplication.

EXAMPLES.

1. Multiply 3.7185625 by 2.2134125, the result to be correct to two decimal places.

2. Multiply 8.170663 by 1461.203, the result to be correct to three decimal places.

3. Multiply 78.5126 by 37.8759, the result to be correct to four decimal places.

4. Multiply 375.76843 by 3.14159, the result to be correct to four decimal places.

5. Multiply 13.50629 by 0.36472, the result to be correct to five decimal places.

6. Multiply 5.7203716 by 2.71728, the result to be correct to five decimal places.

7. Multiply 87.896397 by 3.5298875, the result to be correct to five decimal places.

8. Multiply 0.86858896 by 1.0986123, the result to be correct to six decimal places.

9. Multiply 0.69314718 by 0.43429448, the result to be correct to seven decimal places.

10. Multiply 3.1415926 by itself, the result to be correct to seven decimal places.

Division of Decimals.

12. Since units of any order multiplied by a whole number yield units of the same order in the product, units of any order divided by units of the same order yield a whole number as the quotient; in other words, *with equal decimal places in dividend and divisor the quotient is a whole number,*

I. Divide 38.4 by 6.

$$\begin{array}{r} 6\overline{)38.4} \\ 6.4 \end{array}$$

Since the divisor is a whole number, the quotient is a whole number as far as the dividend is a whole number. Hence the decimal point in the quotient is directly below the decimal point of the dividend.

II. Divide 0.58961 by 0.07.

$$\begin{array}{r} 0.07\overline{)0.58{}^\times961} \\ 8.423 \end{array}$$

Mark off by a star as many decimal places in the dividend as there are in the divisor. Then the quotient is a whole number as far as this star, and the decimal point is directly below the star.

III. Divide 3.12 by 8000.

$$\begin{array}{r} 8\cancel{0}\cancel{0}\cancel{0}\overline{){}^\times003.12} \\ 0.00039 \end{array}$$

If the decimal point be moved three places to the left in both dividend and divisor, the quotient is not changed, and the problem becomes .00312 divided by 8, which equals .00039. If, however, we cross off the zeros in the divisor, and place a star three places to the left of the decimal point in the dividend, the effect is the same.

In Long Division the quotient can be written above the dividend, and then the decimal point is directly above the star.

IV. Divide 82.32 by 2.1.

$$\begin{array}{r} 39.2 \\ 2.1\overline{)82.3{}^\times2} \\ 63 \\ \hline 193 \\ 189 \\ \hline 42 \\ 42 \\ \hline \end{array}$$

Since there is one decimal place in the divisor, the star comes after the 3, and in the quotient the decimal point is directly above the star.

V. Divide 0.03969 by 4900.

$$\begin{array}{r} 0.0000081 \\ 49\cancel{0}\cancel{0}\overline{){}^\times00.03969} \\ 392 \\ \hline 49 \\ 49 \\ \hline \end{array}$$

Since there are two zeros at the end of the divisor, the star should be placed two places to the left of the decimal point. Then divide by 49, and the decimal point in the quotient is directly above the star.

VI. Divide 403920 by 0.00108.

```
          374000000.
0.00108)403920.00000×
         324
          799
          756
           432
           432
```

In order to make decimal places even, zeros must be annexed to the dividend. Then the division is as before, and the decimal point in the quotient is directly above the star.

VII. Divide 0.05 by 4.3 to five decimal places.

```
      0.01162 +
4.3)0.0×50000
        43
         70
         43
         270
         258
          120
           86
           34
```

In this example the divisor is not contained an exact number of times in the dividend. In all such cases it is customary to carry the division to a certain number of decimal places, and then stop. The answer can be written 0.01162 +, the + sign denoting that there is a remainder. If, however, the + sign is omitted, the answer should be written 0.01163, which would be nearer correct than 0.01162, because the next figure would be greater than 5.

From these examples we see that the method for Division of Decimals is as follows: *If necessary, annex zeros to the dividend in order to make the decimal places equal in dividend and divisor. Then mark off by a star as many decimal places in the dividend as there are in the divisor. Divide as in whole numbers, and in the quotient place the decimal point directly below or above the star.*

When the divisor is a whole number ending in zeros, *cross off the zeros, and place the star in the dividend as many places to the left of the decimal point as there are zeros in the divisor.*

NOTE. In problems where the divisor is not contained an exact number of times in the dividend, five decimal places in the quotient is ordinarily far enough to carry the division. *If the next digit is to be less than 5, keep the last digit as it comes in the division; if the next digit is to be 5, or more, increase the last digit by one.*

EXAMPLES.

1. Divide 769.428 by 200.
2. Divide 76.9428 by 0.0002.
3. Divide 0.000064 by 0.008.
4. Divide 9.00081 by 900.
5. Divide 0.000144 by 120000.
6. Divide 0.01625 by 0.000025.
7. Divide 0.000744 by 0.62.
8. Divide 67.56785 by 0.035.
9. Divide 0.09 by 0.0016.
10. Divide 287.1 by 3300.
11. Divide 0.0002548 by 0.0364.
12. Divide 0.00064708 by 6.72.
13. Divide 0.00309824 by 0.376.
14. Divide 2926.5 by 0.3902.
15. Divide 29.265 by 390.2.
16. Divide 1.096641 by 1521.
17. Divide 0.0018891 by 3.75.
18. Divide 190.914 by 270800.
19. Divide 10.85 by 0.0775.
20. Divide 3336.894963 by 72530.
21. Divide 0.00091471 by 9.43.
22. Divide 189695.4 by 2.708.
23. Divide 76.125 by 463000.
24. Divide 8.21 by 0.41.
25. Divide 0.821 by 410.

26. Divide 0.314 by 1.785.

27. Divide 0.10724 by 0.003125.

28. Divide 2.838913 by 708.4.

29. Divide 0.011825369 by 5.884.

30. Divide 695.57270875 by 52.35775.

31. Divide four millionths by four million.

32. Divide 300 thousandths by 3 hundred-thousandths.

33. Divide sixteen thousandths by forty-five hundred.

34. Divide eighty-four and eighty-four hundredths by forty-eight thousandths.

35. Divide fifty millionths by six hundred twenty-five ten-thousandths.

36. Divide two thousand five hundred one and four tenths by four thousand one hundred twenty-five ten-millionths.

37. Divide 59285 ten-millionths by 835 hundred-thousandths.

38. Divide four thousand three hundred twenty-two and four thousand five hundred seventy-three ten-thousandths by eight thousand and nine thousandths.

Contracted Division of Decimals.

13. When examples in division of decimals are required to be accurate only to a certain number of decimal places, the work can be shortened by the use of the following method:

Determine by inspection the position of the decimal point in the quotient, and the number of significant figures in the quotient can at once be determined. Write the divisor so

as to contain two more figures than the quotient. Cut off the right-hand figure of the divisor, and then divide; in multiplying the divisor by the figure of the quotient, the product must be increased by the number carried from the right hand. Instead of bringing down each time a figure at the right of the remainder, cut off the right-hand figure of the divisor, and proceed as before.

I. Divide 7.97647964 by 3.7876476, the result to be correct to four decimal places.

$$
\begin{array}{r|l}
 & \quad\;\; 2.1059 \\
3.7\cancel{8}\cancel{7}\cancel{6}\cancel{4}\cancel{8} & 7.976479{\times}64 \\
 & 757530 \\
\hline
 & \;\;40118 \\
 & \;\;37876 \\
\hline
 & \;\;\;\;2242 \\
 & \;\;\;\;1894 \\
\hline
 & \;\;\;\;\;\;348 \\
 & \;\;\;\;\;\;340 \\
\hline
\end{array}
$$

3 is contained in seven twice; hence the quotient contains one integral place, and the entire number of figures in the quotient is five. Write the first seven figures of the divisor, changing 7 to 8 on account of the 6 dropped. Before dividing cut off the 8 in the divisor. Then in multiplying by 2, 2 must be carried from the right hand, for $2 \times 8 = 16$, which is nearer 20 than 10. The first remainder is increased by 1 on account of the 9 in the dividend. Then cut off the 4 in the divisor and proceed as before.

NOTE. If necessary, zeros can be annexed to the divisor to make the required number of figures; zeros occurring before the first significant figure are not to be counted.

EXAMPLES.

1. Divide 3698.779375 by 375.625, the result to be correct to three decimal places.

2. Divide 0.046 by 0.00762089, the result to be correct to four decimal places.

3. Divide 0.32165 by 0.003516, the result to be correct to four decimal places.

4. Divide 0.765439 by 359.21, the result to be correct to five decimal places.

5. Divide 0.22165 by 0.0035216, the result to be correct to five decimal places.

6. Divide 6.38572164 by 0.0752681, the result to be correct to five decimal places.

7. Divide 29.48495554 by 378.6725, the result to be correct to six decimal places.

8. Divide 100.016 by 3.056, the result to be correct to six decimal places.

9. Divide 0.765439 by 359.21, the result to be correct to seven decimal places.

10. Divide 2.71828128 by 3.1415926, the result to be correct to seven decimal places.

MISCELLANEOUS EXAMPLES.

1. Simplify $8.763 - 4.12 + 78.326 + 1.1126 - 68.0816$.

2. Simplify $198.63 + 21.3711 - 100.416 - 45.79 + 8.3$.

3. Simplify
$$8.72 \times 5.4 + 196 \times 0.004 - 6.25 \times 4.8 - 0.06 \times 21.7.$$

4. Simplify
$$3.71 \times 8 + 2.64 \div 160 + 7.55 \times 0.07 + 0.071 \times 25.$$

5. Simplify
$$84 \times 1.13 - (66 - 1.2 \times 2.4) + 100 \times (4 \times 0.018 + 0.189).$$

6. Simplify
$$94.5 \div 250 + 16 \div (4.5 \div 0.225) + 87.25 \div (1.6 - 0.35).$$

7. Simplify $(15 - 10 \times 0.3) \times 6.192 \div (7 \times 5.4 - 35.048)$.

8. The difference between two numbers is 94.32, and the smaller is 147.631; find the larger.

9. Divide the sum of four thousandths and four millionths by their difference.

10. Divide 876.196 by 2.12. If the decimal point were moved in the dividend two places to the left, and in the divisor one place to the right, how many times greater or less would the quotient be?

11. Multiply forty-eight ten-thousandths by two and one thousandth, and divide the result by one million.

12. Divide 375 by 0.75 and 0.75 by 375, and find the sum and difference of the quotients.

13. The product of three numbers is 5.76; one of them is 0.024, and another is 0.06; find the third.

14. The product of three numbers equals 70.04597; two of them equal 3.91 and 3.0005 respectively; find the third.

15. What number divided by 28.15 will give 1.216 as the quotient and 1.5195 as the remainder?

16. The dividend is 7423.973, the quotient is 12.13, and the remainder is 0.413; what is the divisor?

17. Find the number of rods of fence necessary to enclose a field, the sides of which are respectively 42.78 rods, 51.3 rods, 27 rods, and 37.22 rods.

18. Two men walk respectively 26.7 miles and 22.94 miles per day; how much further does the first walk than the second?

19. If the year is considered as 365.25 days instead of 365.242264 days, how great will be the error in 1880 years?

20. From a tank containing 1200 gallons, 22.75 barrels of 31.5 gallons each were pumped out; how many gallons remained?

21. How many barrels, each containing 44.5 gallons, can be filled from 16554 gallons of oil?

22. 4 cords of wood are worth as much as 13.4 bushels of rye; how much rye can be obtained for 15 cords of wood?

23. A merchant bought 972 bushels of wheat; how many bins, each containing 16.25 bushels, will be filled, and how much remains?

24. Two wheels of a carriage are respectively 13.5 feet and 11.75 feet in circumference; how much oftener does one turn than the other in going 4000 feet?

UNITED STATES MONEY.

14. The money in use in the United States is expressed in a decimal system, of which the unit is a dollar. The symbol for dollars ($) is placed before the number used to represent them. One dollar equals 100 cents (cts.), and cents are written as a decimal fraction of a dollar. For example, $6.43 means six dollars and forty-three cents. There are three other denominations sometimes mentioned, which are not in common use, — ten dollars equal one eagle, ten cents equal one dime, and one tenth of a cent is a mill. All operations in United States Money (sometimes called Federal Money) are performed as in decimal fractions.

NOTE 1. In general when the final result in a problem contains mills, if less than 5 they are rejected, if 5 or more they are called another cent. If, however, the final result is the value of one article where a number are considered, any fraction of a cent should be retained.

NOTE 2. In business transactions C is often used for hundred, and M for thousand, when the price is by the hundred or by the thousand.

EXAMPLES.

1. A farmer sold 24 cows for $32.25 apiece; how much did he receive?

2. A drover sold 42 hogs for $246.75; how much apiece did he receive?

3. A merchant paid \$52 for a lot of cloth at 8 cts. a yard; how many yards did he buy?

4. A merchant's receipts for a week were as follows: Monday, \$102.79; Tuesday, \$72.73; Wednesday, \$150.65; Thursday, \$127.70; Friday, \$205; Saturday, \$278.92. Find the amount of his receipts for the entire week.

5. A clerk has a yearly salary of \$1000; he pays \$312 for board, \$157.50 for clothing, and \$372.25 for all other expenses. How much does he save in a year?

6. Bought three loads of wood, containing respectively 2.15 cords, 1.98 cords, and 1.625 cords; find the cost at \$3.15 a cord.

7. Find a man's daily wages when he was paid \$29.70 for 22 days' work.

8. At \$12.375 a ton, how many tons of hay can be bought for \$2326.50?

9. Bought 3 pounds of tea at 72 cts. a pound, 8 pounds of coffee at 28 cts. a pound, and 15 pounds of rice at 6 cts. a pound; find the amount of the bill.

10. If \$31.75 be paid for 5 barrels of flour, what would 28 barrels cost at the same rate?

11. If 0.62 of a ton of hay be worth \$11.47, find the value of 8.75 tons.

12. A farmer sold in one month 62 pounds of butter at 28 cts. a pound, 45 dozen eggs at 18 cts. a dozen, and 27 chickens at 55 cts. apiece; find the amount of his receipts.

13. A lady bought 12 yards of crash at 14 cts. a yard, and 8 yards of cotton cloth at 18 cts. a yard, and gave a \$5 bill in payment; how much change should she receive?

14. If the price of gas be \$1.75 per M, find the amount of a man's bill when 12240 cubic feet have been consumed.

15. Sold 7250 cigars at $4.20 per C; find the amount received.

16. Paid $10.44 for 1440 bricks; what was the price per M?

17. A pedler sells beets, six in a bunch, at 10 cts. a bunch, and gains 1 ct. on each bunch; find the cost per C.

18. How many tons of coal at $4.75 a ton must be given in exchange for 19 barrels of flour at $6.25 a barrel?

19. How many dozen eggs at 18 cts. a dozen must be given in exchange for 28 pounds of sugar at 11 cts. a pound and 8 pounds of coffee at 29 cts. a pound?

20. A merchant bought a load of grain for $50, and by retailing it at $1.20 a bushel, he gained $22; how many bushels were there in the load?

21. To send a telegram from New York to Boston costs 25 cts. for 10 words and 2 cts. for each additional word; find the cost of a telegram containing 28 words.

22. A grocer bought 16 barrels of sugar, each containing 232 pounds, for $335, and sold it at 10 cts. a pound; how much was his gain?

23. Bought a roll of carpet, containing 82 yards, for $45, and sold it for 75 cts. a yard; find the amount of profit.

24. Bought a horse for $125, a carriage for $140, and a harness for $18; kept them a month at an expense of $17.25, and then sold the team for $300. Did I gain or lose, and how much?

25. A merchant bought 150 barrels of apples for $300; he sold seven tenths of them at $2.25 a barrel, and the remainder at $1.875 a barrel. Did he gain or lose, and how much?

CHAPTER III.

FACTORS.

15. A number which can be contained in another without a remainder is called a **divisor** or **factor** of that number. When a number has no factor except itself and one, it is called a **prime number**; when it has other factors besides itself and one, it is called a **composite number**. When two numbers have no common factor except one, they are said to be **prime to each other.**

Numbers of which 2 is a factor are called **even numbers**; all others are **odd numbers.**

When a number is applied to some particular object or objects, it is called a **concrete number**; when not applied to any object, it is called an **abstract number**. For example, 4 and 7 are abstract numbers, but 4 boys and 7 books are concrete numbers.

An **exponent**, or **index**, is a small figure placed at the upper right-hand corner of a number to show how many times it is used as a factor. For example, $5^4 = 5 \times 5 \times 5 \times 5$.

16. For determining at sight whether certain numbers are contained in a given number, the following tests can be used:

(1) A number is divisible by 2 if its right-hand figure is zero or an even digit.

(2) A number is divisible by 3 if the sum of its digits is divisible by 3. For example, in 741, $7 + 4 + 1 = 12$, which is divisible by 3; hence 741 is divisible by 3.

(3) A number is divisible by 4 if the two right-hand figures are zeros, or if the number expressed by them is divisible by 4.

(4) A number is divisible by 5 if its right-hand figure is 0 or 5.

(5) A number is divisible by 6 if it is an even number, and at the same time is divisible by 3.

(6) A number is divisible by 8 if its three right-hand figures are zeros, or if the number expressed by them is divisible by 8.

(7) A number is divisible by 9 if the sum of its digits is divisible by 9.

(8) A number is divisible by 10 if its right-hand figure is 0.

(9) A number is divisible by 11 if the sums of the alternate digits are the same, or if the difference between these sums can be divided by 11. For example, in 7458, $7+5=4+8$; hence 7458 is divisible by 11. In 19382, $1+3+2=6$, $9+8=17$, and the difference between 6 and 17 is 11; hence 19382 is divisible by 11.

17. To find whether a number is prime or composite, divide by the prime numbers in succession until one of them is contained in the number, or else the quotient is less than the divisor. In the former case the number is composite; in the latter case the number is prime. For example, take 491. Dividing in succession by 2, 3, 5, 7, 11, 13, 17, and 19, none of them are contained in 491, and in every case the quotient is greater than the divisor. Divide by 23, and the quotient is 21 with a remainder; hence 491 is a prime number. If the number were composite, both divisor and quotient would be factors. We have seen that 491 has no factor less than 23, and as the divisors grow greater, the quotients grow less and will be less than 23. Then since

there is no factor less than 23, in no case will the quotient be a whole number, and there are no prime factors for the number.

Factoring.

18. The prime numbers which multiplied together produce a given composite number are called the **prime factors** of that number.

I. Find the prime factors of 182.

$$\begin{array}{r} 2\overline{)182} \\ 7\overline{)91} \\ 13 \end{array}$$

$182 = 2 \times 7 \times 13.$

Divide by 2, the least number that is a factor of 182. Then divide the quotient by 7, and we find that the three prime factors of 182 are 2, 7, and 13. For convenience of work it is best always to divide by the least possible factor.

II. Find the prime factors of 3465.

$$\begin{array}{r} 3\overline{)3465} \\ 3\overline{)1155} \\ 5\overline{)385} \\ 7\overline{)77} \\ 11 \end{array}$$

$3465 = 3^2 \times 5 \times 7 \times 11.$

When the same factor occurs more than once, it is best to write that factor with an exponent.

EXAMPLES.

Find the prime factors of the following numbers:

1. 176.	7. 792.	13. 4800.
2. 210.	8. 1221.	14. 6902.
3. 360.	9. 1836.	15. 8364.
4. 384.	10. 1872.	16. 10917.
5. 432.	11. 2310.	17. 37125.
6. 480.	12. 2346.	18. 179487.

19. Which of the numbers 5, 9, 13, 18, 21, and 25 are prime numbers? Which of them are prime to the number 10?

20. Select the prime numbers between 50 and 100.

21. Make a list of all the prime numbers below 40, and use it to prove that 541 is prime.

22. Which of the numbers 293, 371, 385, 440, 524, 617, and 713 are prime?

23. Of what number are 2, 3, 5, 7, 11, and 13 the prime factors?

24. How many of the different divisors of 150 are prime, and how many are composite?

25. Find all the prime factors common to 1001 and 616.

Greatest Common Divisor.

19. A **common divisor** of two or more numbers is a number that will be contained exactly in each of them.

The **greatest common divisor** of two or more numbers is the greatest number that will be contained exactly in each of them. For example, 3 and 4 are common divisors of 24 and 36, but 12 is the greatest common divisor.

For convenience G.C.D. is used to represent the greatest common divisor.

Greatest common measure (G.C.M.) and **highest common factor** (H.C.F.) are expressions which have the same meaning as greatest common divisor.

I. Find the G.C.D. of 56, 84, and 140.

$$56 = 2^3 \times 7.$$
$$84 = 2^2 \times 3 \times 7.$$
$$140 = 2^2 \times 5 \times 7.$$
$$\text{G.C.D.} = 2^2 \times 7 = 28.$$

The G.C.D. is the product of all the common prime factors. 2 and 7 are the common factors, but 2 occurs twice in each number, and hence will occur twice in the G.C.D. The G.C.D. is thus seen to be $2^2 \times 7$, which equals 28.

To find the G.C.D. of two or more numbers, *resolve the numbers into their prime factors, and find the product of the common factors, taking each factor the least number of times it occurs in any number.*

The following arrangement of work may be used:

$$\begin{array}{r|rrr} 2 & 56 & 84 & 140 \\ \hline 2 & 28 & 42 & 70 \\ \hline 7 & 14 & 21 & 35 \\ \hline & 2 & 3 & 5 \end{array}$$

G.C.D. $= 2^2 \times 7 = 28$.

Arrange the numbers in a line, and divide by all the prime numbers that will be contained in all the numbers. The divisors are the common prime factors, and the product of the divisors is the G.C.D.

EXAMPLES.

Find the G.C.D. of

1. 48 and 120.
2. 84, 126, and 140.
3. 48, 26, 72, and 24.
4. 6, 8, 20, 36, and 48.
5. 45, 75, 90, 135, 150, and 180.
6. 66, 78, 102, and 114.
7. 66, 308, and 506.
8. 119 and 231.
9. 168, 192, and 216.
10. 120, 228, and 720.
11. 144 and 780.
12. 105, 231, and 1001.
13. 156, 234, and 260.
14. 189, 243, and 297.
15. 240, 560, and 616.
16. 252, 315, 420, and 504.
17. 256, 480, and 1296.
18. 432 and 1872.
19. 936 and 2925.
20. 720, 336, and 1736.
21. 927, 342, and 861.
22. 252, 588, 924, and 1092.
23. 4815, 4905, and 5085.
24. 1209, 1885, 2457, 2691, and 2717.
25. Find all the common divisors of 225, 2025, and 8100.
26. What is the length of the longest boards that will exactly fit three floors 42, 63, and 105 feet long respectively?

27. A man has three farms of 56, 72, and 88 acres respectively, and wishes to fence them into the largest possible fields, having each the same number of acres. How many acres could he put in each?

28. How many gallons are there in the largest vessel that will exactly measure the contents of three hogsheads, containing respectively 143, 104, and 156 gallons?

29. Find the length of the longest pole that will exactly measure the sides of a field, which are respectively 72, 126, 162, and 90 feet.

30. Three military companies consisting respectively of 36, 42, and 54 men are divided into squads, each containing the same number; find the largest number of men each squad may contain, and the number of squads in each company.

20. When the numbers cannot be factored easily, a different method is employed for finding the G.C.D.

I. Find the G.C.D. of 161 and 368.

```
161)368(2
    322
    ---
     46)161(3
        138
        ---
         23)46(2
            46
            --
```

Divide 368 by 161. If 161 were contained exactly, it would be the G.C.D. However, there is a remainder 46. Divide 161 by 46, and there is a remainder 23. Divide 46 by 23, and there is no remainder. 23, the last divisor, is the G.C.D.

Since 23 is a divisor of 46, it is a divisor of 138, which equals 46×3, and is then a divisor of 161, which equals $138 + 23$. Since 23 is a divisor of 161, it is a divisor of 322, which equals 161×2, and is then a divisor of 368, which equals $322 + 23 \times 2$. Hence 23 is a common divisor of 161 and 368.

Furthermore, 161 and 368 are each a certain number of times the G.C.D. $161 \times 2 = 322$, which is a certain number of times the G.C.D. $368 - 322 = 46$, which must be a certain number of times the G.C.D., because if a number is a divisor of two other numbers, it is a divisor of their difference. $46 \times 3 = 138$, which is a certain number of times the

G.C.D. Then 161 — 138, which equals 23, is a certain number of times the G.C.D., and the G.C.D. cannot be greater than 23. But it has been proved that 23 is a common divisor; hence it is the G.C.D.

The method may be stated as follows: *Divide the greater number by the less, and the divisor by the remainder, and so on till there is no remainder, each time dividing the last divisor by the last remainder. The last divisor is the G.C.D.*

When there are more than two numbers, *find the G.C.D. of any two of them, then of that divisor and a third number, and so on till all the numbers have been used. The last G.C.D. is the one required.*

EXAMPLES.

Find the G.C.D. of

1. 187 and 153.
2. 323 and 374.
3. 434, 539, and 616.
4. 1235 and 1495.
5. 1181 and 2741.
6. 1417 and 1469.
7. 630, 840, and 2772.
8. 13212 and 1841.
9. 3432 and 4760.
10. 4939 and 3143.
11. 3696 and 1440.
12. 9249 and 10920.
13. 2618, 39039, and 1771.
14. 43700 and 9430.
15. 13860 and 38500.
16. 17640 and 18375.
17. 4994, 12485, and 16117.
18. 36864, and 20736.
19. 156, 585, 442, and 1287.
20. 1274, 2002, 2366, 7007, and 13013.

Least Common Multiple.

21. A **multiple** of a number is any number in which it is contained exactly.

A **common multiple** of two or more numbers is a number that will exactly contain each of them.

The **least common multiple** of two or more numbers is the least number that will exactly contain each of them. For example, 48 and 72 are common multiples of 6 and 8, but 24 is the least common multiple.

For convenience L.C.M. is used to represent the least common multiple.

I. Find the L.C.M. of 36, 42, and 88.

$36 = 2^2 \times 3^2.$
$42 = 2 \times 3 \times 7.$
$88 = 2^3 \times 11.$

L.C.M. $= 2^3 \times 3^2 \times 7 \times 11 = 5544.$

The L.C.M. must consist of all the different factors that are in the numbers, and each factor must be present as many times as it is in any one number. The L.C.M. is thus seen to be $2^3 \times 3^2 \times 7 \times 11$, which equals 5544.

To find the L.C.M. of two or more numbers, *resolve the numbers into their prime factors, and find the product of all the different factors, taking each factor the greatest number of times it occurs in any number.*

If the numbers are prime to each other, their product is their L.C.M.

The following arrangement of work may be used:

$$
\begin{array}{r|rrr}
2 & 36 & 42 & 88 \\ \hline
2 & 18 & 21 & 44 \\ \hline
3 & 9 & 21 & 22 \\ \hline
 & 3 & 7 & 22
\end{array}
$$

L.C.M. $= 2^2 \times 3^2 \times 7 \times 22 = 5544.$

Arrange the numbers in a line and divide by all the prime numbers that will be contained in any two of the numbers, bringing down in the line below the quotients and the numbers that cannot be divided, and so on until the numbers in the line are prime to each other. The product of the divisors and the numbers in the last line is the L.C.M.

EXAMPLES.

Find the L.C.M. of

1. 15, 18, and 35.
2. 20, 24, and 36.
3. 36, 48, and 72.
4. 10, 14, 15, 21, 30, and 42.

5. 48, 98, 21, and 27.
6. 18, 32, 48, and 52.
7. 48, 26, 72, and 24.
8. 21, 36, 50, and 64.
9. 14, 36, 108, and 144.
10. 72, 80, 84, and 96.
11. 91, 52, 39, 28, and 21.
12. 3, 91, 78, 182, and 231.
13. 108, 217, 54, and 31.
14. 84, 126, and 140.
15. 156, 234, and 260.
16. 105, 476, and 306.
17. 144 and 780.
18. 240, 560, and 616.
19. 740, 333, and 296.
20. 945 and 1485.
21. 936 and 2925.
22. 504, 924, and 2184.
23. 1209, 1885, 2457, 2691, and 2717.

24. Find the L.C.M. of the nine digits.

25. Find the L.C.M. of the even numbers from 10 to 20 inclusive.

26. What is the shortest length that can be measured by either of three measures, which are respectively 9, 15, and 24 inches long?

27. Find the contents of the smallest cistern that can be exactly measured by either one of three casks containing respectively 18, 25, and 30 gallons.

28. What is the width of the narrowest walk that can be paved with blocks each 12 inches long and 15 inches wide, allowing the blocks to run either lengthwise or across the walk?

29. What is the smallest sum of money that can be made up either of 2-cent, of 3-cent, of 5-cent, of 10-cent, or of 25-cent pieces?

30. Four boys start together to run around a square; the first can run around in 12 minutes, the second in 15 minutes, the third in 16 minutes, and the fourth in 18 minutes; how long will it be before they all meet at the starting-point?

22. When the numbers cannot be factored easily, find the G.C.D. of two or more of the numbers, and use it as a divisor in the first line, and then proceed as before.

I. Find the L.C.M. of 368, 483, and 532.

```
368)483(1          23)368(16     23)483(21     23)532(23
    368                23            46            46
    ---                ---           --            --
    115)368(3          138           23            72
        345            138           23            69
        ---            ---           --            --
         23)115(5                                   3
            115
            ---
```

23)	368	483	532
2)	16	21	532
2)	8	21	266
7)	4	21	133
	4	3	19

$$\text{L.C.M.} = 23 \times 2^2 \times 7 \times 4 \times 3 \times 19$$
$$= 146832.$$

The G.C.D. of 368 and 483 is found to be 23. Then divide all the numbers by 23. It is contained in the first two, but not in the third. However, the numbers are now so much simplified that the method as previously given can be applied.

When there are but two numbers, the L.C.M. can be found by dividing one number by the G.C.D. and multiplying the quotient by the other number.

EXAMPLES.

Find the L.C.M. of

1. 187 and 153.
2. 391 and 493.
3. 209, 247, and 253.
4. 187, 539, and 847.
5. 630, 840, and 2772.
6. 1417 and 1469.
7. 1011, 1685, and 2359.
8. 1517 and 1763.
9. 1217, 1422, and 1611.
10. 3150 and 2310.
11. 9249 and 10920.
12. 4939 and 3143.
13. 2618, 39039, and 1771.
14. 43700 and 9430.
15. 13860 and 38500.
16. 2520, 2772, and 30888.
17. 17640 and 18375.
18. 340200, 583200, and 2268000.

Cancellation.

23. Division may be indicated by writing the dividend above a line and the divisor below the same line. For example, $\frac{42}{6}$ means 42 divided by 6. This method of indicating division is commonly used when there are several factors in either dividend or divisor. Such an example in division can be simplified by striking out, or cancelling, like factors in dividend and divisor. This does not affect the result, because when both dividend and divisor are divided by the same number the quotient remains the same.

I. Find the value of $\frac{7 \times 5 \times 30 \times 12}{20 \times 7 \times 6}$.

$$\frac{\cancel{7} \times \cancel{5} \times \overset{5}{\cancel{30}} \times \overset{3}{\cancel{12}}}{\underset{4}{\cancel{20}} \times \cancel{7} \times \cancel{6}} = 15.$$

Cancel 7 and 7. Then cancel 5 in the dividend and 20 in the divisor, writing 4 below the 20, as 20 divided by 5 equals 4. Then cancel 4 and 12, writing 3 above the 12; cancel 6 and 30, writing 5 above the 30. The result is 5×3, which equals 15.

II. How many pounds of sugar worth 9 cents a pound must be given in exchange for 18 dozen of eggs worth 16 cents a dozen?

$$\frac{\overset{2}{\cancel{18}} \times 16}{\cancel{9}} = 32 \text{ pounds.}$$

The value of the eggs is 18×16 cents, and as many pounds of sugar can be obtained as 9 is contained times in 18×16. Simplify by cancellation.

EXAMPLES.

Find the value of

1. $\frac{5 \times 8 \times 3 \times 16}{8 \times 15 \times 4}$.

2. $\frac{11 \times 33 \times 21 \times 13}{13 \times 7 \times 11 \times 3}$.

3. $\frac{9 \times 25 \times 64}{5 \times 18 \times 32}$.

4. $\frac{20 \times 56 \times 12}{21 \times 10 \times 8}$.

5. $\dfrac{792}{11 \times 4 \times 9}$.

6. $\dfrac{108 \times 132}{99 \times 144}$.

7. $\dfrac{54 \times 84 \times 99}{9 \times 22 \times 63}$.

8. $\dfrac{57 \times 119 \times 16}{17 \times 12 \times 19}$.

9. How many yards of cloth worth 22 cents a yard must be given in exchange for 11 bushels of potatoes worth 60 cents a bushel?

10. If 50 oranges cost 75 cents, find the cost of 30 oranges.

11. How many barrels of flour can be bought for $247, when 8 barrels cost $52?

12. If 16 men can dig a ditch in 21 days, how long will it take 24 men?

13. If the work of 11 men equals the work of 17 boys, how many men's work will equal the work of 68 boys?

14. Allowing 17 bushels of wheat to make 4 barrels of flour, how many bushels will be necessary to make 68 barrels?

15. When 15 barrels of pork, each containing 200 pounds, are worth $250, find the value of 60 pounds.

16. How many dresses, each containing 16 yards, can be made from 20 pieces of cloth, 52 yards in each piece?

17. If 54 men can build a wall in 35 days, working 10 hours a day, how many men will be necessary to build it in 15 days, working 9 hours a day?

18. A gardener sells 75 crates of berries, 24 boxes in a crate, at 8 cents a box, and receives in return 12 rolls of matting, 40 yards in a roll; find the price of the matting a yard.

CHAPTER IV.

COMMON FRACTIONS.

24. A unit may be divided into any number of equal parts, and any number of these parts may be taken together. For example, $\frac{5}{7}$ means that the unit is divided into seven equal parts, of which five are taken; it is read *five sevenths.* As has been stated in § 23, $\frac{5}{7}$ also means five divided by seven, but these two meanings are really the same, because the quotient arising from dividing five by seven is five sevenths.

An expression used to denote one or more of the equal parts of a unit is called a **fraction.** When it is represented by two numbers, one written above the other with a dividing line between, it is called a **common fraction,** or **vulgar fraction.** When it is represented by figures at the right of the decimal point, as shown in Chapter II., it is called a **decimal fraction.**

In common fractions the number below the line, which shows into how many equal parts the unit is divided, is called the **denominator.** The number above the line, which shows how many of the equal parts are taken, is called the **numerator.** The two numbers are called the **terms** of the fraction.

A **proper fraction** is one whose numerator is less than its denominator; as $\frac{3}{8}$.

An **improper fraction** is one whose numerator is equal to or greater than its denominator; as $\frac{6}{6}$, $\frac{19}{8}$. When the numerator is greater than the denominator, more than one unit must be divided into equal parts; for example, $\frac{19}{8}$ means

that three or more units have each been divided into eight equal parts, and nineteen of these parts are taken.

A **mixed number** is an integer and a fraction expressed together; as $6\frac{7}{15}$, which is read *six and seven fifteenths.*

A **compound fraction** is a fraction of a fraction; as $\frac{2}{3}$ of $\frac{5}{11}$.

A **complex fraction** is one which has a fraction in one or both of its terms; as $\frac{6}{\frac{7}{8}}$, $\frac{\frac{3}{4}}{5\frac{5}{9}}$.

The **reciprocal** of a number is the quotient arising from dividing 1 by that number. For example, the reciprocal of 7 is $\frac{1}{7}$.

25. Since a fraction is an expression of division, the last three principles of § 4 may be restated for fractions as follows:

(1) Multiplying the numerator or dividing the denominator by a number multiplies the fraction by the same number.

(2) Dividing the numerator or multiplying the denominator by a number divides the fraction by the same number.

(3) Multiplying or dividing both numerator and denominator by the same number does not change the value of the fraction.

Reduction of Fractions to Lowest Terms.

26. A fraction is in its lowest terms when the numerator and denominator are prime to each other.

1. Reduce $\frac{135}{315}$ to its lowest terms.

$\frac{135}{315} = \frac{27}{63} = \frac{3}{7}$. Since dividing both numerator and denominator by the same number does not change the value of the fraction, we can divide both terms by 5 and thus obtain $\frac{27}{63}$ for a value of the fraction in lower terms. Then divide both terms by 9, and we obtain $\frac{3}{7}$. Since 3 and 7 are prime to each other, the fraction is in its lowest terms.

To reduce a fraction to its lowest terms, *divide numerator and denominator successively by their common factors.* Since the product of all the common factors is the G.C.D., in all cases where the common factors are not easily seen, *divide numerator and denominator by their G.C.D.*

EXAMPLES.

Reduce the following fractions to their lowest terms:

1. $\frac{42}{66}$.
2. $\frac{48}{108}$.
3. $\frac{39}{91}$.
4. $\frac{22}{132}$.
5. $\frac{99}{495}$.
6. $\frac{288}{432}$.
7. $\frac{280}{616}$.
8. $\frac{625}{875}$.
9. $\frac{392}{560}$.
10. $\frac{208}{494}$.
11. $\frac{253}{483}$.
12. $\frac{323}{714}$.
13. $\frac{697}{1558}$.
14. $\frac{945}{1485}$.
15. $\frac{4343}{5757}$.
16. $\frac{3941}{6193}$.
17. $\frac{5063}{9047}$.
18. $\frac{16200}{24840}$.
19. $\frac{17640}{29400}$.
20. $\frac{23820}{39700}$.
21. $\frac{184800}{1180410}$.

22. Ascertain whether the fraction $\frac{2458}{61937}$ is in its lowest terms or not, and explain the process you employ.

Reduction of Improper Fractions to Whole or Mixed Numbers.

27. I. Reduce $\frac{132}{12}$ to a whole number.

$\frac{132}{12} = 11.$ $\frac{132}{12}$ is the same as $132 \div 12$, which equals 11.

II. Reduce $\frac{141}{12}$ to a mixed number.

$\frac{141}{12} = 11\frac{9}{12} = 11\frac{3}{4}.$ $\frac{141}{12}$ is the same as $141 \div 12$, which equals $11\frac{9}{12}$, and this reduces to $11\frac{3}{4}$.

EXAMPLES.

Reduce to whole or mixed numbers

1. $\frac{81}{7}$.
2. $\frac{99}{12}$.
3. $\frac{105}{13}$.
4. $\frac{285}{15}$.
5. $\frac{255}{13}$.
6. $\frac{500}{21}$.

7. $\frac{589}{19}$.
10. $\frac{841}{66}$.
13. $\frac{1365}{91}$.
8. $\frac{729}{27}$.
11. $\frac{1024}{83}$.
14. $\frac{1800}{247}$.
9. $\frac{775}{30}$.
12. $\frac{1203}{88}$.
15. $\frac{3463}{572}$.

Reduction of Whole or Mixed Numbers to Improper Fractions.

28. I. Reduce 8 to sevenths.

$8 = \frac{56}{7}$. Since there are 7 sevenths in 1, in 8 there are 8 times 7 sevenths, which equals $\frac{56}{7}$.

A whole number may be written as a fraction with 1 for the denominator. For example, $13 = \frac{13}{1}$.

II. Reduce $12\frac{7}{9}$ to an improper fraction.

$12\frac{7}{9} = \frac{108+7}{9} = \frac{115}{9}$. $12 = \frac{108}{9}$; adding $\frac{7}{9}$ to this, the result is $\frac{115}{9}$.

EXAMPLES.

Reduce to improper fractions

1. $3\frac{5}{8}$.
4. $12\frac{6}{17}$.
7. $9\frac{21}{29}$.
2. $7\frac{8}{11}$.
5. $16\frac{3}{20}$.
8. $12\frac{13}{30}$.
3. $10\frac{10}{13}$.
6. $21\frac{7}{15}$.
9. $15\frac{19}{41}$.
10. Reduce 9 to eighths.
11. Reduce 12 to elevenths.
12. Reduce 19 to thirteenths.
13. Reduce 25 to fifteenths.
14. Reduce 42 to twenty-fourths.

Least Common Denominator.

29. When several fractions have the same denominator, they are said to have a **common denominator**. It is always possible to reduce two or more fractions to equivalent frac-

tions having a common denominator; but the most useful common denominator is the least common multiple of the denominators, which is known as the **least common denominator**. For convenience L.C.D. is used to represent the least common denominator.

I. Reduce $\frac{5}{6}$, $\frac{7}{9}$, and $\frac{11}{12}$ to equivalent fractions having the L.C.D.

$$\frac{5}{6}=\frac{5\times 6}{36}=\frac{30}{36}.$$
$$\frac{7}{9}=\frac{7\times 4}{36}=\frac{28}{36}.$$
$$\frac{11}{12}=\frac{11\times 3}{36}=\frac{33}{36}.$$

We find the L.C.M. of 6, 9, and 12 to be 36. 6 must be multiplied by 6 to obtain 36; hence 5 must be multiplied by 6 in order to keep the fraction of the same value, because multiplying both numerator and denominator by the same number does not change the value of the fraction. A similar process is applied to the other two fractions.

To reduce fractions to equivalent fractions having the L.C.D., *in each fraction multiply both terms by the quotient arising from dividing the L.C.D. by the denominator.*

NOTE. Fractions should always be in their lowest terms before finding the L.C.D.

EXAMPLES.

Reduce to equivalent fractions having the L.C.D.

1. $\frac{1}{3}$, $\frac{5}{6}$, and $\frac{5}{8}$.
2. $\frac{2}{5}$, $\frac{5}{9}$, and $\frac{7}{12}$.
3. $\frac{16}{36}$, $\frac{9}{12}$, and $\frac{12}{16}$.
4. $\frac{3}{4}$, $\frac{5}{14}$, and $\frac{13}{18}$.
5. $\frac{3}{5}$, $\frac{4}{9}$, and $\frac{8}{13}$.
6. $\frac{8}{12}$, $\frac{10}{16}$, and $\frac{8}{15}$.
7. $\frac{5}{6}$, $\frac{3}{13}$, and $\frac{7}{17}$.
8. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$.
9. $\frac{1}{2}$, $\frac{5}{6}$, $\frac{7}{12}$, and $\frac{11}{18}$.
10. $\frac{3}{8}$, $\frac{5}{16}$, $\frac{7}{24}$, and $\frac{3}{48}$.
11. $\frac{9}{15}$, $\frac{5}{18}$, $\frac{14}{21}$, and $\frac{18}{27}$.
12. $\frac{7}{13}$, $\frac{11}{15}$, $\frac{2}{11}$, and $\frac{6}{21}$.
13. $\frac{7}{10}$, $\frac{11}{12}$, $\frac{4}{15}$, $\frac{6}{25}$, and $\frac{1}{60}$.
14. $\frac{8}{13}$, $\frac{15}{39}$, $\frac{22}{33}$, $\frac{25}{52}$, and $\frac{52}{65}$.
15. $\frac{5}{14}$, $\frac{27}{35}$, $\frac{3}{68}$, $\frac{99}{112}$, $\frac{100}{119}$, and $\frac{63}{170}$.
16. $\frac{18}{55}$, $\frac{19}{60}$, $\frac{20}{77}$, $\frac{21}{132}$, $\frac{22}{165}$, and $\frac{23}{288}$.

Addition of Fractions.

30. I. Find the sum of $\frac{2}{17}$, $\frac{5}{17}$, and $\frac{8}{17}$.

$\frac{2}{17}+\frac{5}{17}+\frac{8}{17}=\frac{2+5+8}{17}=\frac{15}{17}$. Quantities to be added together must be of the same kind. Since the denominators are alike, we have merely to add the numerators.

II. Find the sum of $\frac{3}{8}$, $\frac{5}{6}$, and $\frac{7}{12}$.

$$\frac{3}{8}+\frac{5}{6}+\frac{7}{12}=\frac{9+20+14}{24}=\frac{43}{24}=1\frac{19}{24}.$$

When the denominators are unlike, the fractions must first be reduced to equivalent fractions having the L.C.D., and then added.

III. Find the sum of $2\frac{1}{3}$, $3\frac{2}{9}$, $6\frac{11}{12}$, and $\frac{5}{18}$.

$$2\frac{1}{3}+3\frac{2}{9}+6\frac{11}{12}+\frac{5}{18}=11\frac{12+8+33+10}{36}=11\frac{63}{36}=12\frac{27}{36}=12\frac{3}{4}.$$

The sum of the whole numbers is 11, and the sum of the fractions is $\frac{63}{36}$; the two sums taken together equal $12\frac{27}{36}$, which reduced to its lowest terms becomes $12\frac{3}{4}$.

To add fractions, *reduce the fractions to equivalent fractions having the L.C.D., and write the sum of the numerators over the L.C.D.* When there are mixed numbers, add the whole numbers and fractions separately, and combine the results. Improper fractions should be reduced to mixed numbers before adding.

EXAMPLES.

Find the sum of

1. $\frac{1}{3}$, $\frac{3}{8}$, and $\frac{1}{12}$.
2. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.
3. $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{4}{15}$.
4. $\frac{4}{25}$, $\frac{6}{30}$, $\frac{8}{12}$, and $\frac{11}{15}$.
5. $\frac{3}{4}$, $\frac{7}{8}$, $\frac{8}{9}$, and $\frac{11}{12}$.
6. $\frac{2}{9}$, $\frac{3}{8}$, $\frac{1}{4}$, and $\frac{7}{16}$.
7. $\frac{4}{7}$, $\frac{5}{9}$, $\frac{13}{18}$, and $\frac{19}{21}$.
8. $\frac{5}{8}$, $\frac{8}{11}$, $\frac{11}{16}$, and $\frac{13}{22}$.
9. $\frac{7}{10}$, $\frac{11}{12}$, $\frac{4}{15}$, $\frac{6}{25}$, and $\frac{1}{60}$.
10. $\frac{1}{2}$, $\frac{4}{48}$, $\frac{25}{100}$, and $\frac{7}{42}$.
11. $\frac{5}{6}$, $\frac{4}{9}$, $2\frac{3}{15}$, and $3\frac{3}{20}$.
12. $5\frac{2}{9}$, $3\frac{7}{18}$, $2\frac{7}{36}$, $\frac{11}{12}$, and $2\frac{1}{3}$.

13. $\frac{17}{25}$, $4\frac{1}{3}$, $\frac{13}{75}$, and $1\frac{2}{15}$.
14. $5\frac{3}{7}$, $\frac{11}{4}$, $\frac{3}{8}$, and $\frac{13}{56}$.
15. $2\frac{5}{6}$, $\frac{11}{14}$, $3\frac{1}{3}$, and $\frac{5}{7}$.
16. $\frac{1}{9}$, $\frac{1}{15}$, $\frac{1}{18}$, and $4\frac{1}{6}$.
17. $5\frac{3}{7}$, $\frac{14}{15}$, $\frac{19}{42}$, and $3\frac{1}{3}$.
18. $\frac{2}{3}$, $\frac{14}{25}$, $11\frac{4}{15}$, and $\frac{17}{30}$.
19. $5\frac{11}{13}$, $2\frac{3}{4}$, $\frac{9}{26}$, and $\frac{15}{52}$.
20. $5\frac{11}{18}$, $\frac{7}{8}$, $3\frac{4}{27}$, and $1\frac{1}{16}$.
21. $\frac{14}{15}$, $\frac{1}{17}$, and $1\frac{104}{105}$.
22. $4\frac{11}{70}$, $7\frac{9}{35}$, $1\frac{1}{7}$, $\frac{13}{14}$, and $1\frac{17}{21}$.
23. $\frac{4}{9}$, $\frac{7}{13}$, $1\frac{35}{36}$, and $\frac{15}{91}$.
24. $\frac{11}{17}$, $\frac{10}{221}$, and $2\frac{7}{13}$.
25. $12\frac{1}{12}$, $13\frac{5}{16}$, $17\frac{2}{9}$, and $\frac{19}{144}$.
26. $18\frac{9}{21}$, $12\frac{17}{35}$, $104\frac{16}{105}$, and $29\frac{51}{210}$.

Subtraction of Fractions.

31. I. Subtract $\frac{4}{9}$ from $\frac{7}{12}$.

$$\frac{7}{12}-\frac{4}{9}=\frac{21-16}{36}=\frac{5}{36}.$$

The fractions must first be reduced to equivalent fractions having the L.C.D., and then subtracted.

II. Subtract $4\frac{5}{21}$ from $7\frac{9}{14}$.

$$7\frac{9}{14}-4\frac{5}{21}=3\frac{27-10}{42}=3\frac{17}{42}.$$

The difference between the whole numbers is 3, and the difference between the fractions is $\frac{17}{42}$; these two differences taken together equal $3\frac{17}{42}$.

III. Subtract $5\frac{3}{4}$ from $11\frac{2}{7}$.

$$11\frac{2}{7}-5\frac{3}{4}=6\frac{8-21}{28}=5\frac{36-21}{28}=5\frac{15}{28}.$$

We cannot subtract $\frac{21}{28}$ from $\frac{8}{28}$, so we take 1 from 6, which makes $\frac{28}{28}$ to be added to $\frac{8}{28}$; the example thus becomes $5\frac{36-21}{28}$, which equals $5\frac{15}{28}$.

To subtract fractions, *reduce the fractions to equivalent fractions having the L.C.D., and write the difference between the numerators over the L.C.D.* When there are mixed numbers, subtract the whole numbers and fractions separately, and combine the results. Improper fractions should be reduced to mixed numbers before subtracting.

EXAMPLES.

Find the difference between

1. $\frac{5}{6}$ and $\frac{3}{8}$.
2. $\frac{9}{10}$ and $\frac{4}{5}$.
3. $\frac{7}{8}$ and $\frac{6}{7}$.
4. $\frac{11}{12}$ and $\frac{3}{16}$.
5. $\frac{5}{12}$ and $\frac{2}{15}$.
6. $\frac{7}{13}$ and $\frac{8}{17}$.
7. $\frac{8}{21}$ and $\frac{4}{25}$.
8. $\frac{11}{30}$ and $\frac{5}{42}$.
9. $\frac{13}{132}$ and $\frac{15}{156}$.
10. $\frac{17}{1763}$ and $\frac{12}{1517}$.
11. $18\frac{2}{3}$ and $15\frac{1}{4}$.
12. $3\frac{3}{13}$ and $2\frac{9}{10}$.
13. $9\frac{1}{13}$ and $8\frac{4}{7}$.
14. $4\frac{3}{8}$ and $3\frac{5}{7}$.
15. $15\frac{2}{33}$ and $12\frac{3}{11}$.
16. $36\frac{9}{10}$ and $\frac{4}{8}$.
17. $7\frac{2}{13}$ and $6\frac{5}{11}$.
18. $20\frac{1}{20}$ and $19\frac{1}{19}$.
19. $4\frac{3}{20}$ and $2\frac{12}{17}$.
20. $19\frac{4}{57}$ and $9\frac{9}{19}$.
21. $20\frac{3}{25}$ and $15\frac{15}{16}$.
22. $10\frac{1}{143}$ and $9\frac{3}{13}$.

Addition and Subtraction of Fractions Combined.

32. I. Simplify $8\frac{7}{24} - 2\frac{7}{8} - 3\frac{1}{4} + 6\frac{3}{16} - 5\frac{2}{3}$.

$$8\frac{7}{24} + 6\frac{3}{16} = 14\frac{14+9}{48} = 14\frac{23}{48}.$$
$$2\frac{7}{8} + 3\frac{1}{4} + 5\frac{2}{3} = 10\frac{21+6+16}{24} = 10\frac{43}{24} = 11\frac{19}{24}.$$
$$14\frac{23}{48} - 11\frac{19}{24} = 3\frac{23-38}{48} = 2\frac{71-38}{48} = 2\frac{33}{48} = 2\frac{11}{16}.$$

The three terms preceded by the minus sign are to be subtracted from the sum of the remaining terms. Subtracting the sum of these terms gives the same result as if they were subtracted separately.

EXAMPLES.

Simplify

1. $9\frac{3}{4} - 6\frac{5}{24} + 2\frac{5}{8} - \frac{2}{3} + 4\frac{7}{18}$.
2. $7\frac{1}{2} - 3\frac{8}{33} - \frac{2}{11} - \frac{1}{3} + 5\frac{1}{6}$.
3. $28\frac{3}{28} - 7\frac{3}{7} + 16\frac{3}{16} + 4\frac{3}{4} - 14\frac{3}{14}$.
4. $12\frac{1}{8} + 3\frac{61}{120} - 1\frac{2}{9} + 2\frac{11}{36} - 16\frac{43}{60}$.

5. $15 - 2\frac{17}{50} - 4\frac{9}{10} - 6\frac{3}{8} - \frac{3}{40}$.

6. $7\frac{7}{8} - 6\frac{6}{7} + 5\frac{5}{6} - 4\frac{4}{5} + 3\frac{3}{4} - 2\frac{2}{3} + 1\frac{1}{2}$.

7. $40\frac{23}{36} - 3\frac{7}{24} - 5\frac{5}{16} - 14\frac{9}{64} - 8 + 12\frac{107}{144} - 16\frac{89}{192}$.

8. $13\frac{4}{9} - 2\frac{5}{18} - 6\frac{9}{16} + 3 - 1\frac{5}{12} + 8\frac{7}{8} - \frac{23}{72} - 10\frac{19}{48}$.

9. Add together $\frac{2}{9}$, $\frac{16}{21}$, and $\frac{4}{13}$, and from their sum subtract $\frac{6}{15}$.

10. $\frac{4}{13}$ of a pole is in the mud, $\frac{5}{21}$ of it is in the water, and the rest of it is in the air; what part of it is in the air?

11. In a school $\frac{1}{4}$ of the scholars are Germans, $\frac{2}{9}$ Irish, $\frac{1}{12}$ English, $\frac{1}{18}$ Swedes, and the remainder Americans; what part of the scholars are Americans?

12. A man performs a journey of $84\frac{7}{8}$ miles in five days. He travels $12\frac{8}{9}$ miles the first day, $18\frac{1}{3}$ miles the second day, $16\frac{3}{4}$ miles the third day, and $20\frac{5}{6}$ miles the fourth day; how far does he travel the fifth day?

13. A merchant bought two pieces of cloth containing $45\frac{1}{2}$ yards and $53\frac{7}{8}$ yards respectively. He sold $12\frac{3}{4}$ yards, $15\frac{5}{6}$ yards, $18\frac{1}{9}$ yards, and $14\frac{5}{12}$ yards; how many yards were there remaining?

14. A painter receives \$15 for painting a room. He expends \$$6\frac{1}{4}$ for labor, \$$4\frac{9}{10}$ for paint, and \$$2\frac{6}{25}$ for varnish; find the amount he gained.

Multiplication of Fractions.

33. I. Multiply $\frac{2}{7}$ by 3.

$\frac{2}{7} \times 3 = \frac{6}{7}$. Since multiplying the numerator by a number multiplies the fraction by the same number, we multiply the numerator by 3, and obtain $\frac{6}{7}$ as the result.

II. Multiply $\frac{5}{7} \times \frac{4}{9}$.

$\frac{5}{7} \times \frac{4}{9} = \frac{20}{63}$. If we multiply the numerators together, we obtain 5×4. Since dividing either multiplicand or multiplier by a number divides the product by the same number, if we divide one by 7 and the other by 9, we divide the product by 7×9. 5 divided by 7 equals $\frac{5}{7}$, and 4 divided by 9 equals $\frac{4}{9}$; hence the product of $\frac{5}{7}$ and $\frac{4}{9}$ is 5×4 divided by 7×9, which may be written $\frac{5 \times 4}{7 \times 9}$, and is equal to $\frac{20}{63}$.

III. Multiply $\frac{5}{9}$ by $\frac{12}{35}$.

$$\frac{\cancel{5}}{\underset{3}{\cancel{9}}} \times \frac{\overset{4}{\cancel{12}}}{\underset{7}{\cancel{35}}} = \frac{4}{21}.$$

$\frac{5}{9} \times \frac{12}{35} = \frac{5 \times 12}{9 \times 35}$. The principle of cancellation can then be applied, but it gives the same result to apply the cancellation at once to the example.

IV. Multiply together $\frac{8}{9}$, $\frac{16}{35}$, and $5\frac{1}{4}$.

$$\frac{8}{9} \times \frac{16}{35} \times 5\tfrac{1}{4} = \frac{8}{\underset{3}{\cancel{9}}} \times \frac{\overset{4}{\cancel{16}}}{\underset{5}{\cancel{35}}} \times \frac{\overset{3}{\cancel{21}}}{\cancel{4}} = \frac{32}{15} = 2\tfrac{2}{15}.$$

Reduce $5\frac{1}{4}$ to an improper fraction, and proceed as in the preceding example.

To find the product of a whole number and a fraction, *write the product of the whole number and the numerator over the denominator.*

To find the product of several fractions, *write the product of the numerators over the product of the denominators, first cancelling the factors common to a numerator and denominator.* Mixed numbers should be reduced to improper fractions before multiplying, and a whole number should be treated as a numerator.

Compound fractions are simplified by multiplying together the simple fractions. For example, $\frac{2}{3}$ of $\frac{6}{7} = \frac{2}{3} \times \frac{6}{7}$.

EXAMPLES.

Find the product of

1. $\frac{2}{11} \times 4$.
2. $48 \times \frac{5}{16}$.
3. $\frac{4}{7}$ of $\frac{5}{8}$.
4. $\frac{5}{6}$ of $\frac{3}{20}$.
5. $\frac{1}{8} \times 3\frac{1}{3}$.
6. $6\frac{3}{10} \times \frac{25}{27}$.
7. $1\frac{1}{8} \times 1\frac{1}{2}$.
8. $\frac{10}{11} \times 6\frac{3}{5}$.
9. $5\frac{1}{3} \times 1\frac{1}{8}$.

10. $8\frac{3}{11} \times 2\frac{3}{26}$.
11. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.
12. $\frac{5}{16} \times \frac{8}{11} \times \frac{33}{40}$.
13. $1\frac{7}{10} \times \frac{21}{34}$ of $7\frac{1}{7}$.
14. $\frac{2}{3} \times \frac{5}{7} \times 1\frac{3}{4} \times 1\frac{1}{8}$.
15. $10\frac{1}{2} \times 2\frac{5}{6} \times \frac{4}{7} \times 3\frac{3}{20}$.
16. $\frac{14}{17} \times 3\frac{6}{7} \times 3\frac{7}{9} \times \frac{11}{12}$.
17. $2\frac{3}{4} \times 4\frac{4}{5} \times 5\frac{5}{6} \times \frac{1}{7}$.
18. $2\frac{1}{8} \times 1\frac{1}{84} \times 1\frac{4}{17} \times \frac{8}{85}$.
19. $\frac{7}{18}$ of $3\frac{1}{12} \times 1\frac{7}{11} \times \frac{3}{74}$.
20. $5\frac{6}{17} \times \frac{85}{144}$ of $\frac{36}{49}$ of $1\frac{1}{13}$.
21. $\frac{12}{35} \times 2\frac{5}{9} \times \frac{3}{106} \times 1\frac{7}{46}$.
22. $3\frac{5}{6} \times \frac{17}{39} \times 2\frac{11}{12} \times \frac{50}{51}$ of $\frac{14}{15}$.
23. $1\frac{4}{11} \times 2\frac{13}{15} \times 2\frac{25}{43} \times 1\frac{3}{17} \times \frac{51}{55}$.
24. $1\frac{5}{11} \times 17\frac{7}{8} \times 1\frac{15}{47} \times 1\frac{1}{93} \times 2\frac{5}{39}$.
25. $\frac{2}{3} \times \frac{25}{28} \times \frac{7}{10} \times 7\frac{1}{5} \times 16\frac{1}{60}$.

34. In finding the product of a mixed number and a whole number, or of two mixed numbers, we can use another method, which is particularly useful when the integral parts of the mixed numbers are large numbers.

I. Multiply 23 by $6\frac{7}{11}$.

$$\begin{array}{r} 23 \\ \underline{6\frac{7}{11}} \\ 11)\underline{161} \\ 14\frac{7}{11} \\ \underline{138} \\ 152\frac{7}{11} \end{array}$$

Multiply 23 by 7 and divide the result by 11, which is the same as multiplying by $\frac{7}{11}$; this gives $14\frac{7}{11}$. Then multiply 23 by 6, and add 138 thus obtained to $14\frac{7}{11}$. The entire product is $152\frac{7}{11}$.

II. Multiply $18\frac{2}{5}$ by $8\frac{1}{4}$.

$$\begin{array}{r} 18\frac{2}{5} \\ \underline{8\frac{1}{4}} \\ \frac{1}{10} \\ 4\frac{1}{2} \\ 3\frac{1}{5} \\ \underline{144} \\ 151\frac{4}{5} \end{array}$$

$\frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$; $18 \times \frac{1}{4} = 4\frac{1}{2}$; $\frac{2}{5} \times 8 = 3\frac{1}{5}$; $18 \times 8 = 144$. The sum of these partial products is $151\frac{4}{5}$.

EXAMPLES.

Find the product of

1. $41 \times 3\frac{7}{9}$.
2. $23 \times 6\frac{3}{4}$.
3. $25 \times 6\frac{4}{7}$.
4. $32 \times 10\frac{5}{16}$.
5. $29 \times 21\frac{8}{15}$.
6. $18 \times 11\frac{5}{13}$.

7. $8\frac{1}{3} \times 6\frac{1}{4}$.

8. $8\frac{2}{3} \times 5\frac{5}{9}$.

9. $22\frac{1}{2} \times 8\frac{3}{4}$.

10. $11\frac{9}{16} \times 8\frac{4}{7}$.

11. $14\frac{2}{7} \times 8\frac{2}{5}$.

12. $12\frac{1}{12} \times 4\frac{4}{11}$.

13. $18\frac{4}{5} \times 12\frac{2}{3}$.

14. $25\frac{6}{7} \times 16\frac{3}{10}$.

15. $31\frac{13}{22} \times 14\frac{2}{7}$.

16. $66\frac{2}{3} \times 37\frac{1}{2}$.

17. $112\frac{1}{2} \times 31\frac{1}{4}$.

18. $168\frac{3}{4} \times 83\frac{1}{3}$.

Division of Fractions.

35. I. Divide $\frac{5}{9}$ by $\frac{6}{7}$.

$\frac{5}{9} \div \frac{6}{7} = \frac{5}{9} \times \frac{7}{6} = \frac{35}{54}$.

$\frac{5}{9}$ divided by 1 equals $\frac{5}{9}$. If we divide the divisor by 7, we multiply the quotient by 7; hence $\frac{5}{9}$ divided by $\frac{1}{7}$ equals $\frac{5 \times 7}{9}$. If we now multiply the divisor by 6, we divide the quotient by 6; hence $\frac{5}{9}$ divided by $\frac{6}{7}$ equals $\frac{5 \times 7}{9 \times 6}$. This is the same as $\frac{5}{9} \times \frac{7}{6}$, which equals $\frac{35}{54}$.

To divide a fraction by a fraction, *invert the divisor and proceed as in multiplication.*

II. Divide $\frac{3}{10}$ of $2\frac{1}{6}$ by $\frac{7}{12}$ of $5\frac{1}{5}$.

$$\frac{3}{\underset{2}{\cancel{10}}} \times \frac{\cancel{13}}{\cancel{6}} \times \frac{\overset{2}{\cancel{12}}}{7} \times \frac{\cancel{5}}{\underset{2}{\cancel{26}}} = \frac{3}{14}.$$

When the divisor contains more than one factor, each factor should be inverted.

EXAMPLES.

Find the quotient of

1. $\frac{2}{3} \div \frac{5}{6}$.

2. $18 \div \frac{3}{5}$.

3. $\frac{9}{13} \div 15$.

4. $\frac{15}{22} \div \frac{25}{32}$.

5. $5\frac{4}{7} \div \frac{13}{14}$.

6. $\frac{16}{27} \div 2\frac{1}{3}$.

7. $5\frac{6}{11} \div 2\frac{1}{2}$.

8. $\frac{161}{6} \div 42$.

9. $1\frac{1}{2} \div 1\frac{1}{8}$.

10. $4\frac{4}{9} \div 6\frac{2}{3}$.

11. $100 \div 4\frac{7}{8}$.

12. $4\frac{10}{11} \div 10\frac{4}{11}$.

13. $\frac{87}{186} \div \frac{5}{16}$.

14. $7\frac{11}{12} \div 1\frac{19}{24}$.

15. $\frac{17}{96} \div 6\frac{3}{8}$.

16. $10\frac{1}{8} \div 1\frac{11}{16}$.

17. $10\frac{4}{5} \div 13$.

18. $31\frac{7}{9} \div 1\frac{62}{81}$.

19. $3\frac{8}{17} \div 2\frac{16}{21}$.

20. $3\frac{14}{15} \div 7\frac{24}{25}$.

21. $\frac{7}{257} \div \frac{3}{241}$.

22. $10\frac{18}{23} \div 1\frac{9}{115}$.

23. $5\frac{2}{7} \div \frac{74}{343}$.

24. $7\frac{5}{29} \div 3\frac{141}{145}$.

25. $\frac{3}{5}$ of $\frac{7}{9} \div \frac{4}{3}$ of $\frac{9}{10}$.

26. $\frac{7}{8}$ of $\frac{16}{19} \div \frac{4}{11}$ of $\frac{33}{29}$.

27. $\frac{4}{9}$ of $91 \div \frac{10}{27}$ of 637.

28. $\frac{2}{3}$ of $7\frac{3}{4} \div \frac{4}{5}$ of $11\frac{4}{11}$.

29. $\frac{6}{17}$ of $2\frac{1}{9} \div \frac{8}{51}$ of $5\frac{15}{16}$.

30. $\frac{1}{5}$ of $\frac{6}{7}$ of $2\frac{1}{2} \div \frac{3}{10}$ of $1\frac{3}{7}$.

31. $\frac{7}{8}$ of $\frac{6}{11} \div \frac{8}{15}$ of $\frac{7}{32}$ of $4\frac{1}{11}$.

32. $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{6}{25} \div \frac{2}{5}$ of $\frac{1}{2}$ of $\frac{5}{8}$.

36. When the divisor is either a whole or a mixed number, a different method may often be used to advantage.

I. Divide $29\frac{6}{7}$ by 4.

$$\begin{array}{r} 4\overline{)29\frac{6}{7}} \\ 7\frac{13}{28} \end{array}$$

29 divided by 4 equals 7 with a remainder of 1. $1\frac{6}{7}$ divided by 4 equals $\frac{13}{7} \times \frac{1}{4}$, which equals $\frac{13}{28}$.

II. Divide $52\frac{5}{8}$ by $3\frac{2}{3}$.

$$\begin{array}{l} 3\frac{2}{3})\ 52\frac{5}{8} \\ \underline{3 \qquad 3} \\ 11\overline{)157\frac{7}{8}} \\ \qquad 14\frac{31}{88} \end{array}$$

Since multiplying both dividend and divisor by the same number does not affect the quotient, we can multiply both dividend and divisor by 3, the denominator of the divisor, and then proceed as in the preceding example.

EXAMPLES.

Find the quotient of

1. $22\frac{1}{2} \div 3$.
2. $19\frac{2}{3} \div 5$.
3. $28\frac{9}{11} \div 7$.
4. $33\frac{1}{3} \div 12$.
5. $44\frac{4}{9} \div 15$.
6. $87\frac{1}{7} \div 21$.
7. $156\frac{1}{4} \div 25$.
8. $128\frac{4}{7} \div 30$.
9. $22 \div 6\frac{2}{3}$.
10. $21 \div 5\frac{3}{5}$.
11. $64 \div 9\frac{2}{11}$.
12. $13\frac{4}{5} \div 2\frac{1}{3}$.
13. $48\frac{1}{6} \div 5\frac{1}{2}$.
14. $56\frac{2}{3} \div 8\frac{1}{3}$.
15. $104\frac{1}{11} \div 8\frac{6}{7}$.
16. $90\frac{6}{7} \div 11\frac{1}{9}$.
17. $115\frac{5}{8} \div 21\frac{2}{5}$.
18. $402\frac{1}{2} \div 30\frac{1}{4}$.

Short Methods of Multiplication and Division.

37. Any exact fractional part of a number is called an **aliquot part** of that number. For example, 2, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are aliquot parts of 10.

To multiply by aliquot parts of 10, 100, 1000, etc., *multiply by 10, 100, 1000, etc., as the case may require, and then find the required part.* For example, since $16\frac{2}{3} = \frac{1}{6}$ of 100, $24 \times 16\frac{2}{3} = \frac{1}{6}$ of $2400 = 400$.

To divide by aliquot parts of 10, 100, 1000, etc., *divide by 10, 100, 1000, etc., as the case may require, and then multiply by the denominator of the fraction expressing the aliquot part.* For example, since $12\frac{1}{2} = \frac{1}{8}$ of 100, $225 \div 12\frac{1}{2} = 2.25 \times 8 = 18$.

To multiply by a number a little less than 10, 100, 1000, etc., *multiply by 10, 100, 1000, etc., and from the product subtract the product of the multiplicand by the difference between the multiplier and 10, 100, 1000, etc., as the case may require.* For example, $184 \times 99 = 18400 - 184 = 18216$; $184 \times 98 = 18400 - 368 = 18032$.

EXAMPLES.

1. Multiply 423 by 5.
2. Multiply 2918 by $2\frac{1}{2}$.
3. Multiply 57162 by $3\frac{1}{3}$.
4. Multiply 3143 by 25.
5. Multiply 4890 by $16\frac{2}{3}$.
6. Multiply 12792 by $33\frac{1}{3}$.
7. Multiply 804320 by $12\frac{1}{2}$.
8. Multiply 84322 by 250.
9. Multiply 7614 by 125.
10. Multiply 5436 by $166\frac{2}{3}$.
11. Divide 7165 by 5.
12. Divide 8775 by $2\frac{1}{2}$.
13. Divide $876\frac{2}{3}$ by $3\frac{1}{3}$.
14. Divide 11150 by 25.
15. Divide 2700 by $16\frac{2}{3}$.
16. Divide 42125 by $12\frac{1}{2}$.
17. Divide 1172 by $33\frac{1}{3}$.
18. Divide 87320 by 250.
19. Divide $1183\frac{1}{3}$ by $166\frac{2}{3}$.
20. Divide 33625 by 125.
21. Multiply 64 by 9.
22. Multiply 82 by 99.
23. Multiply 127 by 999.
24. Multiply 7342 by 9999.
25. Multiply 138 by 98.
26. Multiply 72 by 997.

Complex Fractions.

38. I. Reduce $\dfrac{5\frac{5}{12}}{6\frac{1}{2}}$ to a simple fraction.

$$\frac{5\frac{5}{12}}{6\frac{1}{2}} = \frac{\overset{5}{\cancel{65}}}{\underset{6}{\cancel{12}}} \times \frac{\cancel{2}}{\cancel{13}} = \frac{5}{6}.$$

A complex fraction indicates that the numerator is to be divided by the denominator. Hence we perform the division by inverting the divisor and proceeding as in multiplication.

II. Reduce $\dfrac{\frac{3}{4}}{5}$ to a simple fraction.

$$\frac{\frac{3}{4}}{5} = \frac{3}{20}.$$

In many cases the simplest solution is to multiply both numerator and denominator by their least common denominator. In this example we multiply both terms by 4.

III. Reduce $\dfrac{3\frac{1}{8} - 2\frac{1}{4}}{2\frac{5}{6} + 1\frac{2}{3}}$ to a simple fraction.

$$3\tfrac{1}{8} - 2\tfrac{1}{4} = 1\tfrac{1-2}{8} = \tfrac{9-2}{8} = \tfrac{7}{8}.$$

$$2\tfrac{5}{6} + 1\tfrac{2}{3} = 3\tfrac{5+4}{6} = 3\tfrac{9}{6} = 4\tfrac{3}{6} = 4\tfrac{1}{2}.$$

$$\frac{\frac{7}{8}}{4\frac{1}{2}} = \frac{7}{\underset{4}{\cancel{8}}} \times \frac{\cancel{2}}{9} = \frac{7}{36}.$$

The numerator and denominator must each be simplified, and then we can proceed as in the preceding examples.

EXAMPLES.

Reduce the following complex fractions to simple fractions:

1. $\dfrac{2\frac{1}{2}}{5\frac{2}{3}}$.
2. $\dfrac{10}{16\frac{2}{3}}$.
3. $\dfrac{9\frac{3}{7}}{2\frac{3}{4}}$.
4. $\dfrac{6\frac{1}{4}}{33\frac{1}{3}}$.
5. $\dfrac{9\frac{7}{9}}{3\frac{5}{7}}$.
6. $\dfrac{18\frac{2}{7}}{\frac{2}{3} \text{ of } \frac{3}{5} \text{ of } \frac{5}{2}}$.
7. $\dfrac{\frac{1}{2} \text{ of } \frac{5}{7} \text{ of } 7\frac{3}{8}}{19\frac{6}{25}}$.
8. $\dfrac{2\frac{1}{3} + 5\frac{1}{2}}{\frac{7}{6}}$.
9. $\dfrac{\frac{4}{7} \text{ of } \frac{3}{8}}{5\frac{51}{56} - 5\frac{1}{4}}$.
10. $\dfrac{4\frac{1}{7} - 2\frac{1}{4}}{6\frac{1}{2} - 2\frac{1}{7}}$.
11. $\dfrac{2\frac{3}{4} + 2\frac{7}{8}}{4\frac{3}{4} - 3\frac{1}{7}}$.
12. $\dfrac{3\frac{1}{3} - 2\frac{1}{5}}{9\frac{2}{7} + 5\frac{1}{4}}$.

13. $\dfrac{3\frac{2}{5} - 2\frac{1}{2}}{9\frac{1}{2} + 3\frac{1}{2}}.$

14. $\dfrac{3\frac{1}{4} + 1\frac{1}{2} \div \frac{2}{3}}{6\frac{1}{2} - \frac{3}{4} \times \frac{4}{3}}.$

15. $\dfrac{2\frac{5}{8} \div \dfrac{4}{\frac{5}{10}} \times 2}{2 - \dfrac{\frac{1}{2}}{4} \div 5}.$

16. $\dfrac{4\frac{1}{2} + 2\frac{1}{3} \div \frac{2}{3}}{6\frac{1}{2} - 1\frac{2}{3} \times \frac{3}{2}}.$

17. $\dfrac{17\frac{5}{12} - 9\frac{3}{4} + 4\frac{5}{7}}{\frac{5}{9} \times 9\frac{2}{7}}.$

18. $\dfrac{(4\frac{1}{2} + 7\frac{1}{2}) \div 3\frac{1}{2}}{\frac{1}{7} \times 2\frac{2}{7} \times 5\frac{1}{4}}.$

19. $\dfrac{4\frac{2}{11}}{5\frac{7}{8} + \dfrac{3\frac{1}{2}}{2\frac{2}{3}}}.$

20. $\dfrac{1\frac{3}{4} \div 3\frac{2}{11}}{\dfrac{45}{1\frac{1}{2}} - \dfrac{149}{20}}.$

21. $\dfrac{\frac{2}{9} + \frac{3}{8} + \frac{1}{4} - \frac{7}{12}}{3\frac{1}{3} - 2\frac{1}{2}}.$

22. $\dfrac{(3\frac{1}{2} - 2\frac{1}{4}) \div 1\frac{2}{3}}{1\frac{2}{3} + 2\frac{1}{2}}.$

23. $\dfrac{\frac{3}{4} \text{ of } \frac{2}{9} \div \frac{6}{5} \text{ of } \frac{5}{18}}{7\frac{1}{2} \div 1\frac{1}{9} \times 5\frac{2}{9}}.$

24. $\dfrac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{\dfrac{1}{2\frac{1}{2}} + \dfrac{1}{3\frac{1}{2}} + \dfrac{1}{4\frac{1}{2}}}.$

39. When in a series of fractions we have only the signs of multiplication and division, one operation is sufficient to obtain the result.

I. Simplify $\dfrac{\frac{3}{8} \text{ of } \frac{16}{17}}{5\frac{5}{6} - 2\frac{11}{24}} \div \dfrac{1\frac{7}{9}}{12\frac{3}{4}}.$

$$5\tfrac{5}{6} - 2\tfrac{11}{24} = 3\tfrac{20-11}{24} = 3\tfrac{9}{24} = 3\tfrac{3}{8}.$$

$$\frac{\frac{3}{8} \text{ of } \frac{16}{17}}{3\frac{3}{8}} \div \frac{1\frac{7}{9}}{12\frac{3}{4}} = \frac{\frac{3}{8} \text{ of } \frac{16}{17}}{3\frac{3}{8}} \times \frac{12\frac{3}{4}}{1\frac{7}{9}}$$

$$= \frac{3}{\cancel{8}} \times \frac{\cancel{16}}{\cancel{17}} \times \frac{8}{\underset{3}{\cancel{27}}} \times \frac{\overset{3}{\cancel{51}}}{4} \times \frac{\cancel{9}}{\cancel{16}} = \frac{3}{4}.$$

We must first perform the subtraction in the denominator of the first fraction. We can then invert the second fraction and obtain the result by one process of cancellation.

EXAMPLES.

1. Find in its simplest form the value of $\dfrac{11\frac{2}{3}}{12\frac{3}{5}} \div \dfrac{5}{9}$.

2. Multiply $\dfrac{4\frac{1}{3}}{8\frac{2}{3}}$ by $\dfrac{4\frac{1}{5}}{17\frac{1}{2}}$.

3. Multiply $\dfrac{2\frac{5}{7}}{4\frac{1}{8}}$ by $\frac{8}{13}$ of $2\frac{1}{4}$.

4. Multiply $\frac{3}{4}$ of $\dfrac{8\frac{4}{5}}{6\frac{2}{5}}$ by $\frac{4}{9}$ of $\dfrac{\frac{2}{7}}{16}$.

5. Multiply $\frac{2}{9}$ of $\frac{13}{9}$ of $4\frac{1}{5}$ by $\dfrac{36\frac{3}{4}}{2\frac{3}{75} \div 1\frac{3}{4}}$.

6. What is the product of $\frac{3}{5}$ of $\frac{7}{11}$ of 15 and $\frac{14}{15}$ of $11\frac{5}{8}$?

7. Reduce to its simplest form $\frac{2}{15}$ of $\frac{28}{3}$ of $3\frac{1}{8} \div \dfrac{24\frac{1}{2}}{1\frac{8}{55} \times 1\frac{1}{2}}$.

8. Divide $\dfrac{\frac{2}{3}}{\frac{5}{6}}$ by $\dfrac{7\frac{1}{2}}{3\frac{1}{3}}$.

9. Divide $\dfrac{3\frac{1}{4}}{6\frac{1}{2}} \times 72\frac{1}{2}$ by $\frac{2}{3}$ of $\frac{3}{5}$ of $9\frac{3}{8}$.

10. Divide $\dfrac{\frac{7}{8}}{\frac{3}{11}}$ of $12\frac{1}{2}$ by $\dfrac{\frac{1}{3}}{7\frac{1}{2}}$ of $8\frac{3}{4}$.

11. Divide $\frac{9}{10}$ of $\frac{7}{75}$ of $8\frac{1}{3}$ by $\dfrac{\frac{2}{5} \text{ of } 2\frac{7}{24}}{18\frac{1}{3}}$.

12. Divide $\frac{2}{19}$ of $\frac{48}{7}$ of $7\frac{1}{8}$ by $\dfrac{14\frac{1}{2}}{2\frac{17}{35} \times 1\frac{1}{2}}$.

13. Divide 10 times $\left(\dfrac{7}{9} \text{ of } \dfrac{1\frac{2}{3}}{12\frac{1}{2}} \text{ of } 9\frac{9}{10}\right)$ by $\dfrac{2\frac{3}{4}}{7\frac{1}{2}}$.

14. Reduce $\dfrac{7\frac{2}{3}}{3\frac{1}{4}} \times \dfrac{2\frac{1}{2}}{4\frac{3}{5}} \div \dfrac{\frac{5}{3}}{2\frac{1}{6}}$ to its simplest form.

15. Reduce $\dfrac{8\frac{1}{2} \times \frac{7}{13}}{\frac{1}{12} \text{ of } \dfrac{3\frac{1}{2}}{\frac{4}{5}}} \div \dfrac{\frac{1}{3}}{\frac{3}{4} \text{ of } \frac{7}{11}}$ to its simplest form.

16. $\dfrac{27}{37\frac{4}{5}} \times \dfrac{87\frac{2}{9}}{98\frac{1}{8}} \times \dfrac{\frac{7}{8}}{2\frac{1}{3}} \times \dfrac{81\frac{5}{11}}{128}$ equals what?

17. Reduce to its simplest form $\frac{3}{7}$ of $\dfrac{4\frac{5}{9}}{12\frac{1}{3}}$ of $\dfrac{3\frac{4}{11}}{11\frac{5}{7}} \div 1\frac{1}{11}$.

18. Divide $\frac{12}{25}$ of $\frac{3}{70}$ of $13\frac{3}{4}$ by $\dfrac{\frac{11}{20}}{2\frac{1}{2} - \frac{5}{9}}$.

19. Divide $\dfrac{3\frac{1}{3}}{7\frac{1}{2}}$ by $\dfrac{\frac{1}{2} + \frac{1}{6}}{\frac{1}{2} - \frac{1}{3}}$.

20. Divide $\dfrac{3\frac{8}{9} + \frac{6}{5} + \frac{1}{12}}{\frac{2}{3} \text{ of } 5\frac{7}{8}} \times \dfrac{1}{\frac{4}{5}}$ by $\dfrac{133}{141}$.

21. Divide $\dfrac{5}{9} \times \dfrac{3\frac{2}{3}}{\frac{5}{6} \times 1\frac{2}{9}}$ by $\dfrac{7\frac{2}{3} - 4\frac{1}{2}}{\frac{1}{14} \times 3\frac{5}{6}} \times \dfrac{7\frac{2}{3}}{31\frac{8}{9}}$.

22. Simplify $(\frac{1}{3} + \frac{4}{7}) \times \dfrac{20\frac{1}{4}}{3\frac{6}{7} + 2\frac{1}{4}}$.

23. Simplify $2\frac{1}{4} \times \dfrac{10\frac{3}{4} - 4\frac{11}{12}}{6\frac{3}{16} + 7\frac{2}{3}} \times \dfrac{3\frac{5}{11}}{1\frac{2}{5} \times 9\frac{1}{11}}$.

24. Simplify $5\frac{1}{3}$ of $\dfrac{1}{1\frac{1}{3} + 2\frac{1}{4}} \div \dfrac{4\frac{1}{5} + 5\frac{1}{4}}{4\frac{1}{4} + 3\frac{2}{3}}$.

TO FIND A NUMBER WHEN A FRACTIONAL PART OF IT IS KNOWN.

40. I. $5\frac{5}{6}$ is $\frac{7}{9}$ of what number?

$$5\frac{5}{6} \div \frac{7}{9} = \frac{\overset{5}{\cancel{35}}}{\underset{2}{\cancel{6}}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{7}} = \frac{15}{2} = 7\frac{1}{2}.$$

$\frac{7}{9}$ of some number means the same as $\frac{7}{9}$ times some number. If $5\frac{5}{6}$ is $\frac{7}{9}$ times a certain number, the whole number is as much as $\frac{7}{9}$ is contained in $5\frac{5}{6}$, which equals $7\frac{1}{2}$.

II. A boy after spending $\frac{2}{5}$ of his money has $\$7\frac{1}{2}$ left; how much had he at first?

$1 - \frac{2}{5} = \frac{3}{5}$.

$$7\frac{1}{2} \div \frac{3}{5} = \frac{\overset{5}{\cancel{15}}}{2} \times \frac{5}{\cancel{3}} = \frac{25}{2} = 12\frac{1}{2}.$$

Ans. $\$12\frac{1}{2}$.

If he spent $\frac{2}{5}$, he had $\frac{3}{5}$ remaining. Hence the total sum is as much as $\frac{3}{5}$ is contained in $\$7\frac{1}{2}$, which is $\$12\frac{1}{2}$.

EXAMPLES.

1. $19\frac{1}{3}$ is $\frac{2}{5}$ of what number?

2. $32\frac{2}{5}$ is $\frac{9}{14}$ of what number?

3. $\frac{7}{16}$ of $\frac{14}{25}$ is $\frac{7}{15}$ of what number?

4. $\frac{9}{14}$ of $\frac{8}{15}$ of $1\frac{5}{16}$ is $\frac{3}{5}$ of what number?

5. $\frac{17}{28}$ of $5\frac{1}{11}$ is $5\frac{2}{3}$ times what number?

6. Of what number is $\frac{3}{5}$ the $\frac{7}{9}$ part?

7. From Boston to Worcester is 44 miles, which is $\frac{4}{9}$ of the distance from Boston to Springfield; find the distance from Boston to Springfield.

8. Find the cost of a barrel of flour when $\frac{7}{13}$ of a barrel costs \$3.50.

9. A man can dig $\frac{12}{17}$ of a ditch in $2\frac{2}{3}$ days; how long will it take him to dig the whole ditch?

10. A man after selling $\frac{4}{7}$ of his farm has $28\frac{1}{2}$ acres remaining; how many acres were there in the entire farm?

11. $\frac{2}{13}$ of a basket of eggs were broken, and there were 66 left; find the original number in the basket.

12. A grocer sold $\frac{3}{8}$ of a barrel of sugar to one customer and $\frac{5}{11}$ of it to another customer, and had 45 pounds left; how many pounds were there in the barrel when full?

13. In an orchard $\frac{1}{3}$ of the trees bear apples, $\frac{1}{4}$ bear pears, $\frac{1}{5}$ bear peaches, and the remainder, 39 in number, bear plums; find the number of trees in the orchard.

14. A boy after losing $\frac{1}{5}$ of his money has 10 cents given him, and then finds that he has $\frac{5}{6}$ of the original amount; what was the original amount?

To Find what Fractional Part one Number is of Another.

41. I. 8 is what part of 13?

$\frac{8}{13}$. 1 is $\frac{1}{13}$ of 13, and 8 is 8 times $\frac{1}{13}$ of 13, which is $\frac{8}{13}$. The number of which a part is taken is the denominator, and the part taken is the numerator.

II. $1\frac{5}{7}$ is what part of $3\frac{2}{3}$?

$$\frac{1\frac{5}{7}}{3\frac{2}{3}}=\frac{12}{7}\times\frac{3}{11}=\frac{36}{77}.$$

We first form a complex fraction expressing the fractional part, and then reduce this complex fraction to a simple one.

EXAMPLES.

What part of

1. 12 is 7?
2. $10\frac{1}{2}$ is 3?
3. 17 is $4\frac{1}{4}$?
4. 12 is $\frac{5}{6}$?
5. $15\frac{3}{4}$ is $\frac{7}{9}$?
6. $\frac{10}{15}$ is $\frac{1}{3}$?
7. $\frac{4}{5}$ is $\frac{2}{3}$?
8. $2\frac{2}{3}$ is $1\frac{1}{2}$?
9. $7\frac{7}{9}$ is $3\frac{1}{3}$?
10. $25\frac{1}{3}$ is $2\frac{3}{8}$?
11. 4 is $\frac{4}{9}$ of 6?
12. $\dfrac{1\frac{3}{5}}{1\frac{1}{2}}$ is $\dfrac{\frac{2}{9}}{\frac{3}{7}}$?
13. $\frac{9}{11}$ of $\frac{10}{27}$ of 12 is $\frac{8}{13}$ of $2\frac{4}{11}$?
14. $\frac{2}{5}+\frac{7}{12}+\frac{1}{3}$ is $\frac{3}{7}-\frac{2}{5}$?
15. $12-7\frac{2}{3}$ is $1\frac{1}{2}+2\frac{2}{3}$?
16. $37\frac{6}{7}$ is $2\frac{1}{4}\times 3\frac{3}{7}+\frac{1}{3}$ of $5\frac{1}{4}$?
17. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$ is $\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}$?
18. $(\frac{1}{2}-\frac{1}{3})\times(4-3\frac{3}{7})$ is $(2+\frac{1}{5})\div(3+\frac{1}{7})$?
19. If a tank can be filled by a pipe in 11 hours, what part can be filled in $3\frac{1}{2}$ hours?
20. If a man can build a wall in $3\frac{1}{3}$ days, what part can he build in $2\frac{1}{2}$ days?

21. A man owning $\frac{5}{6}$ of a ship's cargo sells $\frac{3}{8}$ of the cargo; what part of his share does he sell?

22. A boy had $\$15\frac{1}{4}$ given him, and he spent $\$8$; what part of the money did he spend?

Reduction of Common Fractions to Decimal Fractions.

42. I. Reduce $\frac{7}{8}$ to a decimal fraction.

$$\begin{array}{r} 8\underline{)7.000} \\ 0.875 \end{array}$$

Since $\frac{7}{8}$ equals $7 \div 8$, we can perform the division decimally and obtain a decimal fraction for the value.

If, when the fraction is in its lowest terms, the denominator contains any factor besides 2 and 5, the quotient cannot be obtained exactly. In such cases, as in division of decimals, five decimal places are ordinarily enough for the answer.

EXAMPLES.

Reduce the following common fractions to decimal fractions:

1. $\frac{3}{4}$.	4. $\frac{7}{16}$.	7. $\frac{3}{32}$.	10. $\frac{329}{400}$.
2. $\frac{4}{5}$.	5. $\frac{9}{25}$.	8. $\frac{21}{80}$.	11. $15\frac{32}{625}$.
3. $\frac{3}{16}$.	6. $3\frac{11}{20}$.	9. $10\frac{87}{125}$.	12. $62\frac{1329}{2000}$.

13. Reduce $\dfrac{7\frac{1}{2}}{\frac{1}{7}}$ to a decimal fraction.

14. Express $\dfrac{32\frac{1}{6}}{75\frac{3}{5}}$ decimally to three places.

15. Reduce to decimals and add $\frac{3}{4}$, $\frac{17}{25}$, and $9\frac{37}{40}$.

16. Write $1\frac{1}{32}$ and $2\frac{1}{16}$ in decimal form. Give the division in decimals of the first by the second.

Reduction of Decimal Fractions to Common Fractions.

43. I. Reduce 0.0375 to a common fraction.

$0.0375 = \frac{375}{10000} = \frac{3}{80}$. 0.0375 can be expressed as a common fraction in the form $\frac{375}{10000}$. This common fraction reduced to its lowest terms equals $\frac{3}{80}$.

The denominator of the common fraction is always 1 with as many zeros annexed as there are decimal places in the decimal fraction.

EXAMPLES.

Reduce the following decimal fractions to common fractions:

1. 0.7.	7. 0.0625.	13. 0.00096.
2. 0.24.	8. 0.0806.	14. 21.1875.
3. 0.625.	9. 0.98.	15. 0.05128.
4. 0.440.	10. 12.043.	16. 14.06225.
5. 0.0016.	11. 0.03125.	17. 42.030125.
6. 5.082.	12. 8.65.	18. 0.0007648267.

Reduction of Common Fractions to Circulating Decimals.

44. When the result cannot be obtained exactly in reducing a common fraction to a decimal, if the division be carried far enough, the quotient will be found to contain the repetition of a figure or series of figures. For example, $\frac{2}{3} = 0.6666 +$; $\frac{7}{22} = 0.3181818 +$. Such decimals are known

as **circulating decimals, repeating decimals,** or **infinite decimals.** The figure or series of figures which is repeated is called the **repetend.** In the case of a single figure the repetend is denoted by a dot over the figure, and in the case of a series of figures by dots over the first and last figures. For example, $\frac{2}{3} = 0.\dot{6}$; $\frac{7}{22} = 0.3\dot{1}\dot{8}$.

I. Reduce $\frac{12}{35}$ to a circulating decimal.

$$
\begin{array}{r|l}
 & 0.3\dot{4}2857\dot{1} \\
35 & 12.0 \\
 & \underline{10\,5} \\
 & 1\,50 \\
 & \underline{1\,40} \\
 & 100 \\
 & \underline{70} \\
 & 300 \\
 & \underline{280} \\
 & 200 \\
 & \underline{175} \\
 & 250 \\
 & \underline{245} \\
 & 50 \\
 & \underline{35} \\
 & 15
\end{array}
$$

We must continue the division until the remainder is the same as some preceding remainder; from this point the figures will continue in series as before. In the present example 15 is the same as the second remainder. The repetend begins with 4 and ends with 1; hence dots are placed over these figures.

EXAMPLES.

Reduce the following common fractions to circulating decimals:

1. $\frac{7}{9}$.	5. $\frac{3}{7}$.	9. $8\frac{5}{13}$.	13. $2\frac{21}{74}$.
2. $\frac{8}{11}$.	6. $\frac{1}{13}$.	10. $\frac{3}{220}$.	14. $\frac{3}{140}$.
3. $\frac{5}{6}$.	7. $3\frac{16}{21}$.	11. $\frac{9}{28}$.	15. $12\frac{11}{104}$.
4. $5\frac{7}{12}$.	8. $\frac{19}{70}$.	12. $18\frac{19}{144}$.	16. $5\frac{5}{231}$.

17. What circulating decimal is equivalent to the sum of $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{1}{11}$?

18. Find the sum of $6\frac{1}{2}$, $7\frac{4}{15}$, and $8\frac{72}{125}$, and express the result as a circulating decimal.

19. Reduce $\dfrac{12\frac{1}{7}\left(\frac{17}{18}+\frac{19}{10}\right)}{\frac{2}{3}\div\frac{19}{17}}$ to a repeating decimal.

Reduction of Circulating Decimals to Common Fractions.

45. When the repetend comprises all the decimal places, a circulating decimal is equal to a common fraction which has the repetend for the numerator and as many nines as there are decimal places for the denominator. Take $0.\dot{3}2\dot{4}$ as an example to show this.

$$1000 \text{ times } 0.\dot{3}2\dot{4} = 324.324324+.$$
$$\text{Once } 0.\dot{3}2\dot{4} = \quad 0.324324+.$$

By subtraction we obtain

$$999 \text{ times } 0.\dot{3}2\dot{4} = 324.$$

Hence $0.\dot{3}2\dot{4} = \frac{324}{999}$.

I. Reduce $0.\dot{7}\dot{2}$ to a common fraction.

$0.\dot{7}\dot{2} = \frac{72}{99} = \frac{8}{11}$. The denominator is 99. Then $\frac{72}{99}$ must be reduced to its lowest terms, which is $\frac{8}{11}$.

II. Reduce $0.47\dot{7}\dot{2}$ to a common fraction.

$$0.47\dot{7}\dot{2} = 0.47\tfrac{72}{99} = 0.47\tfrac{8}{11} = \frac{47\frac{8}{11}}{100} = \frac{525}{1100} = \frac{21}{44}.$$

The repetend reduces to the common fraction $\frac{8}{11}$. Then the decimal $0.47\frac{8}{11}$ can be expressed as a complex fraction, which reduces to $\frac{21}{44}$.

When circulating decimals are to be added, subtracted, multiplied, or divided, they should first be reduced to common fractions; then perform the operations indicated, and reduce the resulting fraction to a decimal.

EXAMPLES.

Reduce the following circulating decimals to common fractions:

1. $0.\dot{3}$.	8. $2.00\dot{0}5\dot{4}$.	15. $3.\dot{2}34\dot{3}$.
2. $0.\dot{2}\dot{7}$.	9. $4.\dot{0}08\dot{1}$.	16. $0.00\dot{2}34\dot{3}$.
3. $0.00\dot{2}\dot{7}$.	10. $0.4\dot{0}8\dot{1}$.	17. $0.01\dot{2}34\dot{3}$.
4. $0.01\dot{2}\dot{7}$.	11. $15.\dot{1}0\dot{8}$.	18. $10.\dot{0}0234\dot{3}$.
5. $0.21\dot{6}$.	12. $0.\dot{2}2\dot{5}$.	19. $12.03\dot{4}0\dot{5}$.
6. $7.01\dot{3}\dot{6}$.	13. $0.00\dot{2}2\dot{5}$.	20. $0.81\dot{2}4\dot{7}$.
7. $0.20\dot{0}5\dot{4}$.	14. $0.\dot{8}5714\dot{2}$.	21. $1.1\dot{5}4791\dot{1}$.

22. What common fraction equals the sum of $0.\dot{1}\dot{8}$ and $0.\dot{3}0769\dot{2}$?

23. Add $0.\dot{0}\dot{3}$ to $0.\dot{4}6\dot{2}$, expressing the result as a circulating decimal.

24. Add $0.\dot{0}\dot{7}$ to $0.\dot{3}8\dot{2}$, expressing the result as a circulating decimal.

25. Multiply $0.1\dot{4}\dot{5}$ by $0.\dot{2}9\dot{7}$, and give the answer as a circulating decimal.

26. Multiply $0.3\dot{4}6153\dot{8}$ by $0.\dot{2}8571\dot{4}$, and express the result as a circulating decimal.

27. Multiply $2.60\dot{4}$ by $1.2\dot{3}\dot{4}$, and divide the result by $0.00\dot{4}$.

Greatest Common Divisor of Fractions.

46. I. Find the G.C.D. of $\frac{3}{20}$, $\frac{9}{10}$, and $\frac{12}{25}$.

G.C.D. of 3, 9, and 12 = 3.
L.C.M. of 20, 10, and 25 = 100.
G.C.D. = $\frac{3}{100}$.

The G.C.D. of the numerators is 3. To be a divisor of $\frac{3}{20}$ this number must be divided by 20; to be a divisor of $\frac{9}{10}$ it

must be divided by 10; and to be a divisor of $\frac{12}{25}$ it must be divided by 25. If, however, 3 be divided by the L.C.M. of the denominators, it will be divided by the least number containing all the factors of the denominators; hence this result is the G.C.D. of the fractions.

To find the G.C.D. of several fractions, *write the G.C.D. of the numerators over the L.C.M. of the denominators.* The fractions should be in their lowest terms, and mixed numbers should first be reduced to improper fractions.

EXAMPLES.

Find the G.C.D. of

1. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.
2. $\frac{8}{9}$, $\frac{12}{13}$, and $\frac{20}{27}$.
3. $\frac{7}{30}$, $\frac{21}{40}$, and $\frac{49}{50}$.
4. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$.
5. $3\frac{1}{3}$, $2\frac{1}{2}$, and $\frac{5}{6}$.
6. $6\frac{2}{5}$, $16\frac{2}{3}$, and $5\frac{5}{9}$.
7. $6\frac{1}{4}$, $8\frac{1}{3}$, and $12\frac{1}{2}$.
8. $1\frac{8}{13}$, $1\frac{7}{15}$, and $1\frac{3}{20}$.

9. Find the width of the widest stone that can be used in laying three walks which are respectively $3\frac{1}{4}$ feet, $3\frac{9}{10}$ feet, and $5\frac{1}{5}$ feet wide.

10. What is the largest measure that can be used in measuring the contents of four bins which contain respectively 9, $13\frac{1}{2}$, $10\frac{4}{5}$, and $10\frac{1}{8}$ bushels?

Least Common Multiple of Fractions.

47. I. Find the L.C.M. of $\frac{3}{20}$, $\frac{9}{10}$, and $\frac{12}{25}$.

L.C.M. of 3, 9, and 12 = 36.
G.C.D. of 20, 10, and 25 = 5.
L.C.M. $= \frac{36}{5} = 7\frac{1}{5}$.

The L.C.M. of the numerators is 36, which is also a multiple of the fractions. If this number be divided by 20 or any factor of 20, it is still a multiple of $\frac{3}{20}$; if divided by 10 or any factor of 10, it is still a multiple of $\frac{9}{10}$; if divided by 25 or any factor of 25, it is still a multiple of $\frac{12}{25}$. If, then, 36 be divided by the G.C.D. of 20,

10, and 25, it will be divided by the greatest common factor of the denominators; hence this result is the L.C.M. of the fractions.

To find the L.C.M. of several fractions, *write the L.C.M. of the numerators over the G.C.D. of the denominators.*

EXAMPLES.

Find the L.C.M. of

1. $\frac{2}{7}$, $\frac{3}{14}$, and $\frac{6}{35}$.
2. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.
3. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$.
4. $\frac{22}{18}$, $\frac{7}{82}$, and $\frac{9}{24}$.
5. $2\frac{2}{3}$, $3\frac{5}{9}$, and $4\frac{4}{15}$.
6. $11\frac{1}{9}$, $14\frac{2}{7}$, and $33\frac{1}{3}$.
7. $4\frac{7}{8}$, $5\frac{5}{12}$, and $4\frac{1}{16}$.
8. $\frac{1}{15}$, $\frac{1}{11}$, $2\frac{1}{2}$, 5, and $6\frac{1}{3}$.

9. Find the capacity of the smallest tank whose contents can be exactly measured by either of three measures which contain respectively $1\frac{1}{4}$, $1\frac{5}{8}$, and $2\frac{1}{3}$ quarts.

10. A can travel around a certain island in $2\frac{2}{15}$ days, B in $3\frac{1}{5}$ days, and C in $3\frac{1}{3}$ days. If they set out at the same time from the same point, and travel in the same direction, in how many days will they all come together at the starting point, and how many times will each man have gone around the island?

11. The pendulum of one clock makes 24 beats in 26 seconds; that of another, 36 beats in 40 seconds. If they start at the same time, when first will the beats occur together?

MISCELLANEOUS EXAMPLES.

1. Add $\dfrac{3\frac{1}{4}}{5\frac{2}{3}}$, $\frac{7}{8}$, and $\frac{9}{10}$ of $\frac{6}{7}$.
2. Add $\frac{1}{2}$, $\frac{2}{3}$ of $\frac{4}{9}$, and $\dfrac{7\frac{2}{3}}{8\frac{3}{4}}$.
3. Add $\dfrac{\frac{4}{3}}{1\frac{5}{7}}$ to $\dfrac{8\frac{1}{2}-2\frac{2}{3}}{9}$.
4. Add $\frac{17}{5}$ of $\dfrac{4}{1\frac{6}{11}}$ to $\dfrac{\frac{22}{7}}{1\frac{2}{9}}$.
5. Add $\frac{3}{4}$ of $3\frac{1}{2}$ to $\frac{6}{7}$ of $\dfrac{18\frac{1}{3}}{15\frac{1}{4}}$ of $\frac{9}{10}$.

6. Add $\frac{4}{7}$ of $18\frac{3}{11}$ and $\frac{11}{12}$ of $\frac{4}{3}$ of $6\frac{3}{11}$.

7. Add $\frac{1}{3}$ of $\frac{2}{7}$ of $28\frac{3\frac{6}{7}}{4\frac{2}{7}}$ to $3\frac{39\frac{1}{2}}{105}$.

8. Add $\dfrac{3\frac{1}{4}}{8\frac{2}{3}}$, $\frac{5}{6}$, and $\frac{8}{9}$ of $\frac{3}{16}$.

9. Add $\dfrac{3\frac{1}{4}}{6\frac{7}{8}}$ to $\frac{5}{6}$ of $\frac{12}{35}$ of $\frac{2}{3}$ of $(\frac{3}{4} - \frac{1}{7})$.

10. Add $\dfrac{\frac{2}{3}}{5}$ to $\left(7\frac{1}{5} \div \dfrac{\frac{2}{3} \times 7\frac{1}{2}}{\frac{1}{9}}\right)$.

11. Add $\frac{2}{7}$ of $\dfrac{5\frac{1}{3}}{\frac{1}{8}}$ to $\frac{5}{9}$ of $(4\frac{1}{3} - 2\frac{7}{9})$.

12. Add $\dfrac{0.0007}{0.035}$ and $\frac{3}{25}$.

13. Add $\dfrac{\frac{3}{4} \text{ of } 2\frac{1}{3}}{\frac{7}{8} + 4\frac{1}{5}}$ and $\dfrac{2\frac{1}{2} + 3\frac{1}{3}}{7\frac{1}{3} - 4\frac{2}{5}}$.

14. What is the sum of $\dfrac{\frac{3}{5} \text{ of } \frac{5}{6}}{\frac{1}{2}}$ and $\dfrac{\frac{4}{7} \text{ of } \frac{7}{12}}{\frac{4}{9} \text{ of } 2\frac{1}{4}}$?

15. Add $\dfrac{\frac{1}{5} \text{ of } 2\frac{1}{7}}{0.5 + \frac{3}{8}}$ to $\dfrac{0.06 + 0.3\frac{1}{3}}{3\frac{1}{5} - 2\frac{1}{2}}$.

16. Subtract $\frac{1}{3}$ of $\frac{9}{10}$ from $\dfrac{8\frac{2}{3} + 2\frac{1}{4}}{4\frac{1}{5}}$.

17. From $\frac{3}{4}$ of $\frac{4}{5}$ take $\frac{1}{2}$ of $\frac{2}{3}$.

18. From $\frac{8}{7}$ of $\frac{7}{15}$ take $\frac{3}{40}$ of $1\frac{1}{9}$.

19. From $\dfrac{4\frac{4}{15} \times 2\frac{5}{8}}{5\frac{1}{5} - 4\frac{1}{2}}$ subtract $\dfrac{7\frac{1}{3}}{2\frac{1}{4}}$.

20. Find the difference between $\dfrac{\frac{4}{3}}{1\frac{5}{7}}$ and $\dfrac{8\frac{1}{2} - 2\frac{2}{3}}{7}$.

21. Find the difference between $3\frac{1}{4} \times 6\frac{12}{13}$ and $\dfrac{\frac{5}{6} + \frac{8}{9}}{7\frac{3}{4}}$.

22. Subtract $7\frac{1}{2}+\frac{3}{16}$ of $\frac{3}{11}$ from $15\frac{1}{4}+\dfrac{\frac{7}{8}}{\frac{1}{3}}+0.6\dot{3}$.

23. From $3\frac{1}{2}$ subtract $\left(\dfrac{7}{15}\text{ of }\dfrac{4\frac{1}{6}}{\frac{2}{5}}\text{ of }1\frac{4}{5}\right)\div\dfrac{4\frac{7}{12}}{1\frac{7}{15}}$.

24. From $5\frac{1}{3}$ subtract $\dfrac{3\frac{7}{16}}{3\frac{1}{9}}\div\left(\dfrac{3}{10}\text{ of }\dfrac{4\frac{5}{7}}{2\frac{2}{3}}\text{ of }4\frac{1}{6}\right)$.

25. Find the sum and product of $\frac{7}{5}$, $\frac{1}{3}$, and $\frac{5}{8}$.

26. Divide $\frac{2}{3}$ of $7\frac{3}{4}$ by $\frac{4}{5}$ of $12\frac{11}{12}$.

27. Divide $\left(\dfrac{4}{5\frac{1}{2}}-\dfrac{1}{8}\right)$ by $\dfrac{3}{4}$.

28. Divide $\dfrac{4}{6\frac{4}{7}}-\dfrac{1}{7}$ by $\dfrac{8}{11}$.

29. Divide $\dfrac{4\frac{5}{7}}{2\frac{1}{12}}$ by $\dfrac{2}{5}$ of $\left(\dfrac{2}{1\frac{3}{4}}-\dfrac{1}{5}\right)$.

30. Divide $\frac{14}{5}$ of $\frac{9}{56}$ of $1\frac{2}{5}$ by $\dfrac{\frac{2}{3}}{\frac{20}{77}+\frac{4}{7}\times\frac{5}{99}}$.

31. Divide $3\frac{2}{3}-1\frac{7}{9}$ by $\dfrac{5}{8}$ of $\left(\dfrac{7\frac{1}{4}}{\frac{2}{3}}+\dfrac{1}{2}\right)$.

32. Divide 0.75 by $\dfrac{2\frac{7}{9}}{15}\times 0.081$.

33. Reduce to a common denominator and add $\frac{3}{8}\times\frac{4}{5}\times\frac{5}{9}$, $\frac{7}{15}$, $\frac{3}{4}$, and $\frac{9}{10}$.

34. Find the simplest expression for $\dfrac{1}{3\frac{1}{5}}-\dfrac{2\frac{1}{4}}{9}+\dfrac{3\frac{5}{8}}{2}-\dfrac{\frac{4}{7}}{4\frac{4}{7}}$.

35. Add $\frac{1}{7}$, $\frac{3}{14}$, $\frac{8}{21}$, and $\frac{5}{28}$, and reduce the sum to a decimal fraction carried to three decimal places.

36. Add $\frac{4}{5}$, $2\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$, and divide the sum by fifty-six thousandths.

37. From $\frac{8}{25}$ of $1\frac{2}{3}$ subtract $\frac{10}{27}$ of $\frac{9}{40}$, and reduce the answer to a decimal.

38. From $\frac{2}{4}$ of $\frac{28}{45}$ subtract $\frac{3}{20}$ of $2\frac{7}{9}$, and reduce the answer to a decimal.

39. Divide $(2\frac{1}{7} \times \frac{3}{16})$ by $(2\frac{1}{4} - 1\frac{5}{7})$, and reduce the result to a decimal.

40. Divide $\frac{1}{3}$ of $\frac{6}{7}$ of $\frac{8}{9}$ by $\dfrac{4\frac{5}{6}}{7\frac{3}{4}}$, and add the quotient to $\frac{5}{8}$.

41. Divide $\frac{3}{4}$ of $\frac{5}{16}$ of $\frac{3}{2}$ by $\dfrac{3\frac{3}{4}}{5\frac{7}{10}}$, and add the quotient to $\frac{3}{4} - \frac{7}{15}$.

42. Divide $\left(\dfrac{3}{5\frac{2}{3}} - \dfrac{1}{5} + \dfrac{1}{3}\right)$ by $\frac{3}{4}$, and reduce the result to an equivalent decimal fraction.

43. From $\frac{1}{4}$ of $1\frac{2}{7}$ take $\dfrac{\frac{3}{8}}{2\frac{1}{2}}$, add to the remainder $\frac{2}{3}$, and divide the result by $6\frac{2}{7}$.

44. From the sum of $\dfrac{7\frac{1}{2}}{13\frac{1}{3}}$ and $\dfrac{3\frac{7\frac{1}{2}}{8}}{6\frac{2}{6\frac{2}{3}}}$ subtract $\frac{19}{24}$, and divide the result by the product of $3\frac{1}{6}$ and $2\frac{1}{3}$.

45. To $\frac{3}{4}$ of $\frac{5}{6}$ add $\frac{7}{16} \div \frac{3}{28}$, multiply the sum by $\dfrac{4}{56\frac{1}{2}}$, and divide the product by $\dfrac{\frac{2}{3} \text{ of } \frac{6}{7}}{3\frac{2}{5}}$.

46. Add $\dfrac{4\frac{1}{5}}{6\frac{3}{10}}$ and $\dfrac{2\frac{1}{4}}{7\frac{7}{8}}$; divide the result by $7\frac{13}{21}$, and change the quotient to a decimal.

47. From $\frac{4}{21}$ of $2\frac{4}{5}$ subtract the product of 0.075 and $1\frac{1}{9}$, and divide the remainder by 12; reduce the result to a decimal form.

48. From $\frac{3}{4}$ of $\frac{8}{9}$ subtract $\frac{2}{3}$ of $\dfrac{3\frac{1}{2}}{4\frac{1}{5}}$, add to the remainder $\frac{5}{16}$, divide the result by $6\frac{7}{9}$, and change the quotient to a decimal.

49. Reduce $\frac{3-\frac{2}{3}}{5+\frac{3}{2}}$ of $4\frac{1}{3}$ of $\frac{6}{7}$ to a simple fraction.

50. Reduce $\frac{\frac{2}{3}\text{ of }7\frac{6}{15}}{3\frac{1}{2}+1.125}$ to a decimal fraction.

51. Express as a decimal $\frac{2}{3}\times\frac{\frac{3}{4}-\frac{1}{5}\times\frac{10}{6}}{3\frac{1}{3}-2\frac{5}{6}}$.

52. What decimal is equivalent to $\frac{1}{3}$ of $\frac{2\frac{2}{5}}{7}\times 0.021$?

53. Simplify $\left(1+\frac{1+\frac{1}{5}}{5}\right)\div\left(1+\frac{5}{1+\frac{1}{5}}\right)$.

54. $\frac{\frac{36}{51}}{\frac{3}{17}}+\frac{5.62}{33}-\frac{7\frac{1}{5}}{27}=$ what?

55. $(1\frac{2}{3}+\frac{17}{12}-0.024)\div(15\frac{1}{2}-1.209)=$ what?

56. Reduce $\frac{\frac{0.2}{29}}{1300}\div\frac{12}{41.64}$ to its simplest decimal form.

57. Simplify $\frac{303\frac{2}{5}}{310\frac{3}{7}}\times\left(\frac{1}{4\frac{1}{1\frac{1}{2}}}+\frac{1\frac{2}{3}}{3\frac{1}{3}}\right)$.

58. Reduce $\left(\frac{1\frac{3}{4}}{4\frac{1}{2}}\div\frac{2\frac{1}{3}}{2\frac{1}{4}}\right)\times\frac{4}{5}\times\frac{1}{2}$ to a decimal.

59. Simplify $\frac{1}{2+\frac{3}{4+\frac{5}{6+\frac{7}{8}}}}$.

60. What is the exact value of $\left(2\frac{3}{4}+\frac{5}{2}\text{ of }\frac{7}{2\frac{4}{5}}+\frac{\frac{4}{9}}{\frac{1}{3}}\right)\div 4\frac{85}{228}$?

61. Simplify $\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}$ of $\frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}+\frac{1}{5}}$ of $\frac{\frac{1}{6}-\frac{1}{7}}{\frac{1}{6}+\frac{1}{7}}$ of 585.

62. The sum of $\frac{2}{3}$ and $\frac{4}{15}$ is diminished by $\frac{1}{10}$. How many times does the difference contain $\frac{3}{11}$ of the sum of $\frac{1}{8}$, $\frac{1}{9}$, and $\frac{1}{10}$?

63. The sum of two numbers equals $3\frac{2\frac{1}{2}}{5\frac{1}{3}}$, and one of them is the difference between $\dfrac{47\frac{3}{10}}{11}$ and $\dfrac{22\frac{1}{2}}{9}$; what is the other number?

64. $\frac{1}{6}$ of a number exceeds $\frac{1}{10}$ of it by 15; what is the number?

65. What part of $\dfrac{12\frac{1}{2}}{\frac{4}{7}}$ is $\dfrac{\frac{2}{3}\times\frac{3}{4}}{\frac{1}{2}}$?

66. What part of $2\frac{1}{3}$ is $\dfrac{7\frac{2}{3}}{31\frac{8}{9}} \times \dfrac{7\frac{2}{3}-4\frac{1}{4}}{\frac{1}{2}\times 3\frac{5}{6}}$?

67. Simplify $\dfrac{5\frac{1}{3}-0.042-2.4+7\frac{5}{6}}{16\frac{2}{15}\div 60\frac{1}{2}}$.

68. Simplify $\dfrac{(3.71-1.908)\times 7.03}{2.\dot{2}-\frac{74}{333}}$.

69. Simplify $\dfrac{2.8 \text{ of } 2.\dot{2}\dot{7}}{1.1\dot{3}\dot{6}}$.

70. Simplify $\dfrac{2\frac{2}{3} \text{ of } 5\frac{1}{6}}{13\frac{2}{7}} + \dfrac{4\frac{1}{4}}{3+\dfrac{1}{2\frac{1}{2}}} - \dfrac{5}{12} \div \dfrac{9}{17}$.

71. Simplify $(2\frac{2}{7} \text{ of } 3\frac{1}{16}) + \frac{4}{9} - (1\frac{1}{3} \text{ of } 1\frac{5}{16}) - (1\frac{3}{4} \text{ of } 4\frac{4}{7} \text{ of } \frac{3}{14})$.

72. Find the G.C.D. and the L.C.M. of $\frac{7}{9}$, $1\frac{10}{11}$, and $3.\dot{6}\dot{0}$.

73. The sum of $\dfrac{\frac{3}{5}\times 0.83\frac{1}{3}}{0.5}$ and $\dfrac{2\frac{2}{3}\times\frac{2}{5}}{3\frac{1}{5}}$ is how many times their difference?

74. The sum of $\dfrac{3\frac{3}{5}\times\frac{5}{6}}{0.5}$ and $\dfrac{\frac{4}{7} \text{ of } \frac{7}{12}}{\frac{4}{9}\times 2.25}$ is how many times their difference?

75. What is the value of $\left(\frac{8-0.4}{2}+\frac{16-0.8}{4}-\frac{5}{2}\right)\div 7\frac{7}{10}$?

76. Simplify $\frac{5\frac{5}{8}\div\frac{2}{3}}{1\frac{1}{2}\text{ of }\frac{5}{9}-\frac{1}{3}}\times\frac{2}{5}\text{ of }\frac{1\frac{1}{2}\text{ of }4\frac{1}{9}}{13\frac{7}{8}\text{ of }5\frac{1}{3}}-\frac{1}{2}+\frac{2}{27}$.

77. Simplify $\frac{3\frac{7}{9}\times 1\frac{1}{17}+8\frac{1}{12}-3\frac{9}{16}}{5\frac{1}{9}-7\frac{7}{8}\div 28\frac{7}{20}+\frac{1}{3}}$ and $\frac{3\frac{2}{3}}{4\frac{1}{7}}\times(3\frac{5}{8}\times 5\frac{6}{7})\times 17\frac{3}{6}$, and find their sum.

78. Find the value of $\left(4\frac{3}{8}\div\frac{9\frac{1}{10}\div\frac{2}{11}}{2\frac{6}{7}\div\frac{1}{42}}\right)\times 0.3\dot{6}\times 0.2\dot{3}\dot{6}$.

79. Simplify

$$0.6\text{ of }3.3+\frac{1.75}{2.625}\text{ of }17+0.4\text{ of }5.75-\frac{1.\dot{7}1428\dot{5}}{2.\dot{0}9523\dot{8}}.$$

---

80. By what must $\frac{2\frac{2}{3}}{3\frac{1}{5}}$ be multiplied to give the product 1?

81. What number is that, $\frac{2}{5}$ of which exceeds $\frac{1}{4}$ of it by $11\frac{3}{4}$?

82. Find the cost of $81\frac{1}{3}$ acres of land at $\$28\frac{7}{8}$ per acre.

83. Find the weight of $8\frac{1}{2}$ reams of paper at $14\frac{5}{7}$ pounds per ream.

84. Find the cost of $\frac{4}{5}$ of a ton of coal if 3 tons cost $20.

85. If a man saw $3\frac{2}{3}$ cords of wood in one day, how much will he saw in $\frac{5}{8}$ of a day?

86. Find the price of flour per barrel when $9\frac{7}{8}$ barrels cost $\$65\frac{5}{6}$.

87. At $\$2\frac{1}{5}$ per barrel, how many barrels of apples can be bought for $55?

88. If a man travel $28\frac{4}{7}$ miles in one day, how many days will it take him to travel $177\frac{2}{9}$ miles?

89. If a man walk $3\frac{1}{4}$ miles in $\frac{5}{6}$ of an hour, at what rate does he walk per hour?

90. What number divided by $\frac{9}{13}$ equals $6\frac{6}{17}$?

91. Find the cost of 8 rolls of carpet, $42\frac{1}{2}$ yards in a roll, at $91\frac{2}{3}$ cents a yard.

92. If $\frac{2}{9}$ of a yard of cloth cost $\$3\frac{1}{5}$, what is the cost of $4\frac{5}{6}$ yards?

93. A farmer sold $4\frac{4}{5}$ tons of hay at the rate of $2\frac{2}{3}$ tons for \$44; what did he receive for it?

94. Find the number of square yards in the surface of three floors measuring respectively $16\frac{7}{9}$, $21\frac{13}{16}$, and $28\frac{17}{24}$ square yards.

95. A farm is divided into four fields which contain respectively $18\frac{2}{3}$, $22\frac{7}{8}$, $19\frac{5}{12}$, and $29\frac{8}{15}$ acres; find the number of acres in the entire farm.

96. What number is that, to which if you add $\frac{7}{8}$ of $19\frac{2}{3}$, the sum will be 150?

97. A man bought 95 bushels of corn at $33\frac{1}{3}$ cents a bushel and sold it at $37\frac{1}{2}$ cents a bushel; find the amount gained.

98. What number is that, $\frac{3}{7}$ of which exceeds $2\frac{1}{2}$ by $13\frac{2}{3}$?

99. A merchant sold 38 yards of cloth at the rate of $2\frac{1}{2}$ yards for \$3; what did he receive for it?

100. What is the price of land per acre when $\frac{3}{11}$ of an acre costs \$44.25?

101. The product of three numbers is $453\frac{1}{3}$; two of them are $5\frac{5}{6}$ and $11\frac{1}{3}$; find the third number.

102. If $\frac{7}{8}$ of a ton of hay will pay for 8 barrels of apples worth $\$2\frac{1}{4}$ per barrel, what is the value of the hay per ton?

103. If $\frac{8}{11}$ of a yard of velvet cost $\$7\frac{4}{5}$, how many yards can be bought for $\$68\frac{16}{25}$?

104. A clerk spends \$425 a year for board, which is $\frac{17}{54}$ of his salary; what is his salary?

105. If $5\frac{1}{17}$ tubs of butter cost $\$103\frac{1}{5}$, how many barrels of flour worth $\$8\frac{1}{2}$ per barrel will pay for one tub of butter?

106. If $\frac{5}{8}$ of $\frac{7}{9}$ of a ship cost \$70000, what is $\frac{8}{11}$ of it worth?

107. How many pieces of cloth, each containing $2\frac{3}{32}$ yards, can be cut from a piece $50\frac{1}{4}$ yards in length?

108. Find the cost of $8\frac{1}{4}$ tons of hay when $2\frac{1}{5}$ tons cost $\$31\frac{1}{3}$.

109. A farmer exchanged 10 pounds of butter worth $31\frac{1}{4}$ cents a pound for sugar worth $7\frac{1}{2}$ cents a pound; how much sugar did he obtain?

110. If a certain number is increased by $\frac{5}{9}$ of $\frac{14}{15}$ of itself, the result is 246; find the number.

111. A boy spent $\frac{5}{12}$ of his money one day, and $\frac{2}{9}$ of it the next day, and then had 65 cents left; how much had he at first?

112. A, owning $\frac{7}{8}$ of a farm, sold $\frac{1}{3}$ of his share to B, and $\frac{1}{2}$ of what he then owned to C for \$420; what was the value of the entire farm at the same rate?

113. A tailor has $97\frac{1}{5}$ yards of cloth, from which he wishes to cut an equal number of coats and vests; how many of each can he cut if they contain $4\frac{1}{5}$ and $1\frac{7}{8}$ yards respectively?

114. If $\frac{5}{6}$ of a ton of coal cost $\$6\frac{1}{4}$, what will $\frac{9}{10}$ of a ton cost?

115. A horse and cow were bought for \$180, and the cow cost $\frac{2}{7}$ as much as the horse; find the price of each.

116. If $\frac{2}{3}$ of a bushel of corn be worth $\frac{3}{7}$ of a bushel of wheat, and wheat be worth \$1.40 a bushel, how many bushels of corn can be bought for \$27?

117. $\frac{9}{13}$ of $\frac{17}{25}$ of 28 times what number equals $50\frac{2}{5}$?

118. A man, owning $\frac{3}{7}$ of a mill, sold $\frac{5}{12}$ of his share for \$2750; find the value of the whole mill at the same rate.

119. A man has $\frac{1}{3}$ of his property invested in real estate, $\frac{1}{4}$ in state bonds, $\frac{2}{7}$ in bank stock, and the remainder, \$5500, in business; find the value of his entire property.

120. A owns $\frac{2}{7}$ of a mill, and B the remainder; $\frac{5}{8}$ of the difference between their shares is \$10500; find the value of the whole mill.

121. A farmer sold $21\frac{1}{2}$ dozen eggs at $18\frac{3}{4}$ cents a dozen, and bought $14\frac{1}{4}$ yards of cloth at $12\frac{1}{2}$ cents a yard; how much money did he have left?

122. If 19 pounds of butter cost $\$6.33\frac{1}{3}$, what part of a pound can be bought for 25 cents?

123. What is the smallest sum of money that can be exactly paid either in pieces of money worth $6\frac{1}{4}$ cents or in pieces worth $8\frac{1}{3}$ cents?

124. Find the width of the widest blocks that will exactly fit either of three walks which are respectively $6\frac{1}{4}$, $7\frac{1}{2}$, and 10 feet wide.

125. If 6 be added to both terms of the fraction $\frac{9}{16}$, is the value of the fraction increased or diminished, and how much?

126. If 6 be subtracted from both terms of the fraction $\frac{9}{16}$, is the value of the fraction increased or diminished, and how much?

127. If 6 men can do a piece of work in $\frac{2}{3}$ of $\frac{8}{9}$ of $\frac{18}{5}$ of $6\frac{1}{4}$ days, how many men could do it in one day?

128. $\$48\frac{3}{4}$ are to be divided among 5 men and 3 boys so that each boy will have half as much as a man; how much will each have?

129. A grocer sold $\frac{5}{8}$ of a barrel of flour to one customer, $\frac{2}{3}$ of the remainder to another customer, and had $24\frac{1}{2}$ pounds left; how many pounds were there in the barrel when full?

130. A merchant owned $\frac{11}{16}$ of a stock of goods; $\frac{4}{5}$ of the whole stock was destroyed by fire, and $\frac{7}{12}$ of the remainder damaged by water. How much did the merchant lose, provided the uninjured goods were sold at cost for \$4200, and the damaged at half the cost?

CHAPTER V.

COMPOUND NUMBERS.

48. When the value of anything is expressed in different units of the same nature, it is called a **compound number**; as 3 bushels 2 pecks 5 quarts.

49. **Long** or **Linear Measure** is used in measuring lengths and distances.

TABLE.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	= 1 rod (rd.).
320 rods or 5280 feet	= 1 mile (mi.).

NOTE. A line = $\frac{1}{12}$ in.; a furlong = 40 rd.; a fathom = 6 ft.

50. **Surveyors' Measure** is used in measuring dimensions of land.

TABLE.

7.92 inches	= 1 link (li.).
100 links	= 1 chain (ch.).
80 chains	= 1 mile (mi.).

NOTE. A surveyors' chain is 4 rods long and contains 100 links. Engineers use a chain, or measuring tape, 100 feet long.

51. **Square Measure** is used in measuring the area of surfaces.

TABLE.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards or $272\frac{1}{4}$ square feet	= 1 square rod (sq. rd.).
160 square rods	= 1 acre (A.).
640 acres	= 1 square mile (sq. mi.).

NOTE. A perch (P.) is a square rod, and a rood (R.) = 40 sq. rd.
10 square chains = 1 acre.
A section of land is a square mile; 36 sections = 1 township.

52. Cubic Measure is used in measuring things which have length, breadth, and thickness.

TABLE.

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.).
27 cubic feet	= 1 cubic yard (cu. yd.).

53. Wood Measure is used in measuring wood and other merchandise.

TABLE.

16 cubic feet	= 1 cord foot (cd. ft.).
8 cord feet or 128 cubic feet	= 1 cord (cd.).

NOTE. A cord of wood, as generally piled, is 8 ft. long, 4 ft. wide, and 4 ft. high.

54. Liquid Measure is used in measuring liquids.

TABLE.

4 gills (gi.)	= 1 pint (pt.).
2 pints	= 1 quart (qt.).
4 quarts	= 1 gallon (gal.).

NOTE. A gallon contains 231 cu. in. $31\frac{1}{2}$ gallons are considered a barrel (bbl.), and 63 gallons a hogshead (hhd.); but barrels and hogsheads are made of various sizes.

55. Apothecaries' Fluid Measure is used in compounding medicines.

TABLE.

60 minims (♏)	= 1 fluid dram (f ʒ).
8 fluid drams	= 1 fluid ounce (f ℥).
16 fluid ounces	= 1 pint (O.).

56. Dry Measure is used in measuring dry articles.

TABLE.

2 pints (pt.)	= 1 quart (qt.).
8 quarts	= 1 peck (pk.).
4 pecks	= 1 bushel (bu.).

NOTE. A bushel contains 2150.42 cu. in.

57. Troy Weight is used in weighing gold, silver, and precious stones.

TABLE.

24 grains (gr.)	= 1 pennyweight (pwt.).
20 pennyweights	= 1 ounce (1 oz.).
12 ounces	= 1 pound (lb.).

NOTE. 1 lb. Troy = 5760 grains. In weighing diamonds 1 carat = $3\frac{1}{6}$ Troy grains, and is divided into quarters, which are called carat grains.

The word *carat* applied to gold indicates the number of parts in 24 that are pure gold. For example, 18 carats fine means that $\frac{18}{24}$ is pure gold, while the rest is alloy.

58. Apothecaries' Weight is used in compounding medicines.

TABLE.

20 grains (gr.)	= 1 scruple (℈).
3 scruples	= 1 dram (ʒ).
8 drams	= 1 ounce (℥).
12 ounces	= 1 pound (℔.).

NOTE. The pound, ounce, and grain have the same weight as those of Troy Weight.

59. Avoirdupois Weight is used in weighing all articles except gold, silver, and precious stones.

TABLE.

16 drams (dr.)	= 1 ounce (oz.).
16 ounces	= 1 pound (lb.).
100 pounds	= 1 hundred-weight (cwt.).
20 hundred-weight or 2000 pounds	= 1 ton (T.).

NOTE. 1 lb. Avoirdupois = 7000 gr.

The long ton of 2240 lb., and the long hundred-weight of 112 lb., are used at United States Custom Houses and in wholesale transactions in coal and iron. The ton of 2000 lb. is often called the short ton.

1 quarter (qr.) = 25 lb.; when the long ton is the standard, 1 qr. = 28 lb.

60. Circular or **Angular Measure.** A **circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the **centre.** The bounding line is called the **circumference,** and any part of the circumference is called an **arc.** A straight line passing through the centre and having its extremities in the circumference is called a **diameter**; a straight line drawn from the centre to the circumference is called a **radius,** and it is equal to one half a diameter. The circumference is divided into 360 equal parts, called *degrees,* each degree into 60 *minutes,* and each minute into 60 *seconds.*

The opening between two straight lines which meet at a point is called an **angle,** and the point where the lines meet is called the **vertex** of the angle. An angle with its vertex at the centre of a circle is measured by the arc included between its sides. The length of an arc of one degree varies with the size of the circle, but an angle of one degree always has the same size opening between the two lines.

The annexed diagram represents a circle; C is the centre, DB an arc, AB a diameter, and CD a radius. The angle DCB contains the same number of degrees as the arc DB.

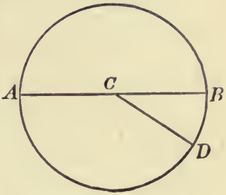


TABLE.

60 seconds (″)	=	1 minute (′).
60 minutes	=	1 degree (°).
360 degrees	=	1 circumference.

NOTE. An arc of 90° is called a quadrant, and an angle of 90° is called a right angle.

A degree of longitude at the equator, or a degree of latitude, equals 69.16 miles.

61. The measures of **time** are determined by the revolution of the earth on its axis and around the sun.

TABLE.

60 seconds (sec.)	=	1 minute (min.).
60 minutes	=	1 hour (hr.).
24 hours	=	1 day (da.).
7 days	=	1 week (wk.).
365 days	=	1 year (yr.).
366 days	=	1 leap year.
100 years	=	1 century.

The length of a **solar day** is the interval between two successive transits of the sun over the same meridian. The length of a **civil day** is the interval between two successive midnights, and is the average length of all the solar days in the year.

The exact time in which the earth revolves about the sun is 365 da. 5 hr. 48 min. 49.7 sec. For convenience in reckoning it is necessary to have an integral number of days in

a year, so it has been arranged to let the common year consist of 365 days, while certain years, called leap years, consist of 366 days. *When the number denoting the year is divisible by 4 and not by 100, or is divisible by 400, the year is a leap year.* For example, 1884 and 2000 are leap years, but 1885 and 1900 are common years. By this method of reckoning the error is less than 1 day in 3600 years.

The year is divided into 12 months (mo.). Their names and the number of days in each are given in the following table:

January (Jan.)	31.
February (Feb.)	28; in leap year 29.
March (Mar.)	31.
April (Apr.)	30.
May	31.
June	30.
July	31.
August (Aug.)	31.
September (Sep. or Sept.)	30.
October (Oct.)	31.
November (Nov.)	30.
December (Dec.)	31.

In business it is customary to reckon 30 days to a month, which makes an error of 5 days a year.

NOTE. The number of days in each month can easily be remembered by the following stanza:

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except February alone,
To which we twenty-eight assign,
Till leap year gives it twenty-nine.

62. **English** or **Sterling Money** is the currency of Great Britain and many of its colonies.

TABLE.

4 farthings (far.)	=	1 penny (d.).
12 pence	=	1 shilling (s.).
20 shillings	=	1 pound (£).

NOTE. A florin = 2 s.; a crown = 5 s.; a sovereign = 20 s.; a guinea = 21 s.

63. MISCELLANEOUS TABLES.

NUMBERS.

12 units	=	1 dozen (doz.).
12 dozen	=	1 gross (gro.).
12 gross	=	1 great gross.
20 units	=	1 score.

PAPER.

24 sheets	=	1 quire.
20 quires	=	1 ream.
2 reams	=	1 bundle.
5 bundles	=	1 bale.

BOOKS.

A book formed of sheets folded
- in 2 leaves is a folio.
- in 4 leaves is a quarto or 4to.
- in 8 leaves is an octavo or 8vo.
- in 12 leaves is a 12mo.
- in 16 leaves is a 16mo.
- in 18 leaves is an 18mo.
- in 24 leaves is a 24mo.

NOTE. These names are based on sheets measuring about 18 in. × 24 in.

REDUCTION DESCENDING.

64. The process of changing a compound number from one denomination to another without altering its value is called **reduction**. When the reduction is from a higher to a lower denomination, it is called **reduction descending**.

I. Reduce 8 lb. 6 oz. 8 pwt. 21 gr. to grains.

```
 8 lb. 6 oz. 8 pwt. 21 gr.
 12
 ---
 96
  6
----
102 oz.
 20
----
2040
   8
----
2048 pwt.
  24
----
 8192
4096
-----
49152
   21
-----
49173 gr.
```

Since there are 12 oz. in 1 lb., in 8 lb. there are 8 times 12 oz., which equals 96 oz.; 6 oz. added to this gives 102 oz. In 1 oz. there are 20 pwt.; in 102 oz. there are 102 times 20 pwt., or 2040 pwt.; 8 pwt. added gives 2048 pwt. In 1 pwt. there are 24 gr.; in 2048 pwt. there are 2048 times 24 gr., or 49152 gr.; 21 gr. added gives 49173 gr.

EXAMPLES.

1. Reduce 5 yd. 2 ft. 7 in. to inches.
2. Reduce 27 gal. 2 qt. 1 pt. 3 gi. to gills.
3. Reduce 8 bu. 3 pk. 4 qt. 1 pt. to pints.
4. Reduce 29 cu. yd. 8 cu. ft. 999 cu. in. to cubic inches.
5. Reduce 145° 6′ 33″ to seconds.
6. Reduce £24 18 s. 9 d. 2 far. to farthings.
7. Reduce 19 lb. 6 oz. 3 pwt. 20 gr. to grains.
8. Reduce 11 ℔. 4 ℥ 4 ʒ 1 ℈ 15 gr. to grains.
9. Reduce 4 T. 2 cwt. 1 qr. 11 lb. to ounces.
10. Reduce 3 O. 7 f ℥ 4 f ʒ 40 ♏ to minims.
11. Reduce 8 cd. 2 cd. ft. 13 cu. ft. to cubic feet.
12. Reduce 2 mi. 51 rd. 4 yd. 2 ft. 7 in. to inches.
13. Reduce 8 mi. 3 fur. 15 rd. 1 ft. 9 in. to inches.

14. Reduce 5 A. 101 sq. rd. 25 sq. yd. 112 sq. in. to square inches.

15. Reduce 8 A. 2 R. 21 P. 17 sq. yd. 6 sq. ft. 89 sq. in. to square inches.

16. Reduce 4 mi. 65 ch. 72 li. 5 in. to inches.

17. Reduce 3 yr. 7 wk. 6 da. 21 hr. to seconds.

18. Reduce 11 yr. 3 wk. 4 da. 18 hr. to minutes, allowing for three leap years.

19. How many units are there in 8 gro. 8 doz.?

20. How many sheets are there in 2 bundles 1 ream 15 quires 10 sheets?

21. Find the number of ounces in a long ton.

22. Find the number of gills of molasses in a barrel which contains 86 gal.

23. What is the value of a silver cup weighing 10 oz. 16 pwt. at $12\frac{1}{2}$ cents a pennyweight?

24. What is the value of 50 lb. 8 oz. of gold at \$20.59$\frac{1}{4}$ per ounce?

Reduction Ascending.

65. When a compound number is reduced from a lower to a higher denomination, the process is called **reduction ascending.**

I. Reduce 766 gi. to higher denominations.

$$\begin{array}{l} 4\underline{)766} \text{ gi.} \\ 2\underline{)191} \text{ pt. } 2 \text{ gi.} \\ \quad 4\underline{)95} \text{ qt. } 1 \text{ pt.} \\ \quad\quad 23 \text{ gal. } 3 \text{ qt.} \end{array}$$

Ans. 23 gal. 3 qt. 1 pt. 2 gi.

Since there are 4 gi. in 1 pt., in 766 gi. there are as many pints as 4 is contained times in 766, which equals 191 pt and 2 gi. remaining. There are 2 pt. in 1 qt.; in 191 pt. there are as many quarts as 2 is contained times in 191, which equals 95 qt. and 1 pt. remaining. There are 4 qt. in 1 gal.; in 95 qt.

there are as many gallons as 4 is contained times in 95, which equals 23 gal. and 3 qt. remaining. The entire result is 23 gal. 3 qt. 1 pt. 2 gi.

II. Reduce 104037 in. to higher denominations.

$$\begin{array}{l}
12\underline{)104037} \text{ in.} \\
\quad 3\underline{)8669} \text{ ft. } 9 \text{ in.} \\
5\tfrac{1}{2})2889 \text{ yd. } 2 \text{ ft.} \\
\underline{2 \qquad\quad 2} \\
11\underline{)5778} \\
320\underline{)525} \text{ rd. } \tfrac{3}{2} \text{ yd.} = 1\tfrac{1}{2} \text{ yd.} \\
\qquad\quad 1 \text{ mi. } 205 \text{ rd.}
\end{array}$$

$$\begin{array}{r}
1 \text{ mi. } 205 \text{ rd. } 1 \text{ yd. } 2 \text{ ft. } 9 \text{ in.} \\
\underline{1 \text{ ft. } 6 \text{ in.}} \\
1 \text{ mi. } 205 \text{ rd. } 2 \text{ yd. } 1 \text{ ft. } 3 \text{ in.}
\end{array}$$

The method is the same as that used in the preceding example. When the divisor is $5\frac{1}{2}$, both dividend and divisor are multiplied by 2 to avoid fractions. The dividend and divisor thus obtained are half-yards; hence the remainder is 3 half-yards, which equals $1\frac{1}{2}$ yd.

$\frac{1}{2}$ yd = 1 ft. 6 in., which must be added to the rest of the answer. The sum of 9 in. and 6 in. is 15 in., which equals 1 ft. 3 in. Write the 3 in., and carry 1 ft. to the column of feet. The sum of 2 ft., 1 ft., and 1 ft. is 4 ft., which equals 1 yd. 1 ft. Write the 1 ft. and carry 1 yd. to the column of yards. The sum of 1 yd. and 1 yd. is 2 yd.

NOTE. In square measure, when the divisor is $30\frac{1}{4}$, multiply both dividend and divisor by 4.

III. Reduce 4840371 min. to higher denominations.

$$\begin{array}{l}
60\underline{)4840371} \text{ min.} \\
\;24\underline{)80672} \text{ hr. } 51 \text{ min.} \\
365\underline{)3361} \text{ da. } 8 \text{ hr.} \\
\qquad\quad 9 \text{ yr. } 76 \text{ da.} \\
\qquad\qquad\quad \underline{2} \\
\qquad\qquad\; 74 \text{ da.}
\end{array}$$

Ans. 9 yr. 74 da. 8 hr. 51 min.

When the subject of time is being considered, proper allowance must be made for leap years. Since every fourth year is a leap year, in 9 years there are at least 2 leap years, and hence 2 da. must be taken from the 76 da. remaining, which leaves 74 da.

In order to ensure absolute accuracy with regard to the number of days, the exact number of leap years in the given time must be known.

NOTE. When the divisor is a large number, it is more convenient to perform the long division at one side of the work and then tabulate the results as if it had all been done by short division.

EXAMPLES.

1. Reduce 34718 far. to higher denominations.

2. Reduce 2763 gi. to higher denominations.

3. Reduce 935923 cu. in. to higher denominations.

4. Reduce 67421″ to higher denominations.

5. Reduce 49328 ♏ to higher denominations.

6. Reduce 677653 in. to higher denominations.

7. Reduce 10075 li. to higher denominations.

8. Reduce 147655 sq. yd. to higher denominations.

9. Reduce 1286 pt. to bu., pk., etc.

10. Reduce 54321 gr. to lb., oz., etc. (Troy Weight).

11. Reduce 87634 gr. to ℔., ℥, etc.

12. How many cords and cord feet are there in 2224 cubic feet?

13. How many bales, bundles, etc., are there in 10379 sheets?

14. Reduce 8256120 sec. to higher denominations.

15. In 372483 oz. how many T., cwt., qr., etc.?

16. Change 106760 ft. to mi., rd., etc.

17. Reduce 8868097 sq. ft. to A., R., P., etc.

18. In 8476321 in. how many mi., fur., rd., etc.?

19. In 1320765 sq. in. how many sq. rd., sq. yd., etc.?

20. In 80937864 sq. in. how many A., sq. rd., etc.?

21. Reduce 7963721 min. to yr., da., etc.

22. A box contains 12579 buttons; find the number estimated in great gross, gross, etc.

Addition of Compound Numbers.

66. I. Find the sum of 4 rd. 2 yd. 1 ft. 10 in., 6 rd. 2 ft. 7 in., 14 rd. 2 yd. 1 ft. 9 in., and 21 rd. 3 yd. 2 ft. 6 in.

4 rd.	$\cancel{2}$ yd.	1 ft.	10 in.
6	0	2	7
14	2	1	9
21	3	2	6
46	$3\cancel{\frac{1}{2}}$	2	8
		1	6
46 rd.	4 yd.	1 ft.	2 in.

Since only units of the same kind can be added, write the numbers so that units of the same kind shall be in the same column. Then begin at the right to add. The sum of the inches is 32 in., which equals 2 ft. 8 in. Write the 8 in., and carry 2 ft. to the column of feet. The sum of the feet, including 2 ft. previously obtained, is 8 ft., which equals 2 yd. 2 ft. Write the 2 ft., and carry 2 yd. to the column of yards. The sum of the yards, including 2 yd. previously obtained, is 9 yd., which equals 1 rd. $3\frac{1}{2}$ yd. Write the $3\frac{1}{2}$ yd., and carry 1 rd. to the column of rods. The sum of the rods, including 1 rd. previously obtained, is 46 rd. Cross out the $\frac{1}{2}$ yd., and write its equivalent 1 ft. 6 in. Then add again, and the entire result is 46 rd. 4 yd. 1 ft. 2 in.

EXAMPLES.

1. Find the sum of 18 gal. 3 qt. 1 pt. 1 gi., 5 gal. 1 pt. 3 gi., 16 gal. 2 qt. 2 gi., and 4 gal. 1 qt. 1 pt.

2. Find the sum of 101 bu. 3 pk. 5 qt., 27 bu. 2 pk. 6 qt. 1 pt., 14 bu. 1 qt. 1 pt., and 33 bu. 3 pk. 7 qt.

3. Find the sum of 27° 30′ 54″, 32° 24′ 58″, 62° 47′ 25″, and 75° 29′ 47″.

4. Find the sum of 16 cd. 6 cd. ft. 14 cu. ft., 22 cd. 2 cd. ft. 4 cu. ft., and 19 cd. 1 cd. ft. 2 cu. ft.

5. Find the sum of 74 cu. yd. 20 cu. ft. 918 cu. in., 29 cu. yd. 15 cu. ft. 1000 cu. in., and 14 cu. yd. 2 cu. ft. 323 cu. in.

6. Find the sum of 4 wk. 2 da. 17 hr. 48 min. 37 sec., 6 da. 2 hr. 29 min. 13 sec., and 12 wk. 1 da. 11 hr. 16 min. 4 sec.

7. Find the sum of £5 15 s. 6 d. 2 far., £7 8 s. 1 far., 19 s. 7 d. 3 far., and £12 10 s.

8. Find the sum of 3 lb. 9 oz. 15 pwt. 12 gr., 6 lb. 16 pwt. 8 gr., and 3 lb. 11 oz. 7 pwt. 4 gr.

9. Find the sum of 5 ℔. 9 ℥ 6 ʒ 2 ℈ 15 gr., 7 ℔. 11 ℥ 7 ʒ 2 ℈ 8 gr., and 11 ℔. 7 ℥ 6 ʒ 1 ℈ 3 gr.

10. Find the sum of 18 T. 7 cwt. 1 qr. 18 lb. 12 oz., 18 cwt. 3 qr. 21 lb. 6 oz., and 9 T. 14 cwt. 15 lb. 15 oz.

11. Find the sum of 5 T. 18 cwt. 52 lb. 8 oz. 6 dr., 15 T. 7 cwt. 44 lb. 10 oz. 12 dr., and 15 cwt. 78 lb. 12 oz. 14 dr.

12. Find the sum of 3 mi. 7 fur. 19 rd. 4 yd. 1 ft., 7 mi. 1 fur. 32 rd. 1 yd. 2 ft., and 9 mi. 6 fur. 25 rd. 3 yd.

13. Find the sum of 15 mi. 110 rd. 4 yd. 1 ft. 6 in., 22 mi. 15 rd. 5 yd. 2 ft. 10 in., 17 mi. 214 rd. 2 yd. 7 in., and 63 rd. 1 yd. 1 ft. 11 in.

14. Find the sum of 28 A. 120 sq. rd. 15 sq. yd. 7 sq. ft. 120 sq. in., 6 A. 91 sq. rd. 21 sq. yd. 4 sq. ft. 32 sq. in., and 65 A. 11 sq. rd. 12 sq. yd. 6 sq. ft. 14 sq. in.

15. Find the sum of 3 A. 2 R. 17 P. 22 sq. yd. 8 sq. ft. 28 sq. in., 4 A. 1 R. 35 P. 17 sq. yd. 4 sq. ft. 92 sq. in., and 11 A. 26 P. 26 sq. yd. 7 sq. ft. 116 sq. in.

Subtraction of Compound Numbers.

67. I. Subtract 7 T. 13 cwt. 78 lb. 5 oz. from 18 T. 6 cwt. 31 lb. 12 oz.

18 T.	6 cwt.	31 lb.	12 oz.
7	13	78	5
10 T.	12 cwt.	53 lb.	7 oz.

Write the subtrahend under the minuend so that units of the same kind shall be in the same column. Then begin at the right to subtract. Subtracting 5 oz. from 12 oz., we have 7 oz. remaining. We cannot subtract 78 lb. from 31 lb., so we take 1 cwt. from 6 cwt. and add it,

reduced to pounds, to 31 lb., making 131 lb.; subtracting 78 lb. from 131 lb., we have 53 lb. We cannot subtract 13 cwt. from 5 cwt.; so we take 1 T. from 18 T. and add it, reduced to hundred-weight, to 5 cwt., making 25 cwt.; subtracting 13 cwt. from 25 cwt., we have 12 cwt. Subtracting 7 T. from 17 T., we have 10 T. The entire result is 10 T. 12 cwt. 53 lb. 7 oz.

EXAMPLES.

1. Subtract 8 cwt. 72 lb. 9 oz. from 13 cwt. 90 lb. 14 oz.

2. Subtract 5 cd. 6 cd. ft. 8 cu. ft. from 76 cd. 3 cd. ft. 12 cu. ft.

3. Subtract 35 gal. 3 qt. 2 gi. from 57 gal. 2 qt. 1 pt. 1 gi.

4. Subtract 2 bu. 3 pk. 2 qt. 1 pt. from 7 bu. 3 pk.

5. Subtract £13 12s. 5d. 3far. from £21 9s. 7d. 2far.

6. Subtract 54° 42′ 30″ from 90°.

7. Subtract 12 lb. 5 oz. 16 pwt. 15 gr. from 18 lb. 6 oz. 14 pwt. 3 gr.

8. Subtract 9 ℔. 10 ℥ 5 ʒ 1 ℈ 16 gr. from 12 ℔. 8 ℥ 4 ʒ 2 ℈ 7 gr.

9. Subtract 10 lb. 8 oz. from 5 T.

10. Subtract 2 yr. 202 da. 21 hr. 54 min. 25 sec. from 5 yr. 71 da. 15 hr. 45 min. 45 sec.

11. Subtract 3 da. 16 hr. 18 min. 45 sec. from 1 wk.

12. Subtract 4 mi. 220 rd. 4 yd. 2 ft. 7 in. from 5 mi. 115 rd. 2 yd.

13. Subtract 6 fur. 32 rd. 5 yd. 1 ft. 6 in. from 3 mi. 4 fur. 32 rd. 3 yd. 10 in.

14. Subtract 7 sq. yd. 139 sq. in. from 1 sq. rd. 5 sq. ft.

15. Subtract 3 A. 3 R. 12 P. 18 sq. yd. 8 sq. ft. 87 sq. in. from 4 A. 2 R. 16 P. 11 sq. yd. 7 sq. ft. 54 sq. in.

16. What is the difference between 28 mi. and 27 mi. 7 fur. 39 rd. 5 ft. 11.9 in.?

Difference between Dates.

68. I. Find the difference of time between Aug. 25th, 1872 and Mar. 12th, 1887.

1887	— 3	— 12	
1872	— 8	— 25	
14 yr.	6 mo.	17 da.	

In subtracting dates it is customary to reckon 30 da. to a month. Since March and August are respectively the third and eighth months, we write 3 and 8 instead of the names of the months. The subtraction is then performed like that of ordinary compound numbers.

II. Find the exact number of days from Dec. 23rd, 1887 to Apr. 13th, 1888.

8
31
29
31
13
112 da.

When the exact number of days is wanted, we must take the actual number of days in each month. We must not count in both dates; it is customary to omit the former and count the latter. From Dec. 23rd to the end of the month there are 8 da. January has 31 da. Since 1888 is a leap year, February has 29 da. March has 31 da. There are 13 da. to be counted in April. By addition we find the result to be 112 da.

EXAMPLES.

1. Find the difference of time between Feb. 18th, 1856 and Oct. 25th, 1874.

2. Find the difference of time between Nov. 19th, 1874 and Apr. 3rd, 1888.

3. Find the difference of time between Aug. 31st, 1876 and May 10th, 1886.

4. Washington was born Feb. 22nd, 1732, and died Dec. 14th, 1799; what was his age?

5. A ship started on a whaling voyage Apr. 24th, 1875, and returned Sept. 8th, 1880; how long was it gone?

6. A mortgage was dated Oct. 21st, 1879, and was paid July 1st, 1887; how long did it run?

7. On the 1st of January, 1888, how much time had passed since the Declaration of Independence, July 4th, 1776?

8. Find the exact number of days from Apr. 5th, 1883 to Dec. 9th, 1883.

9. Find the exact number of days from Aug. 19th, 1883 to June 25th, 1884.

10. Find the exact number of days from May 14th, 1884 to Nov. 5th, 1884.

11. Find the exact number of days from Dec. 27th, 1884 to July 5th, 1885.

12. A note was dated July 30th, 1886, and was paid May 9th, 1887; how many days did it run?

13. A man started on a business trip Nov. 14th, 1887, and returned Mar. 9th, 1888; how many days was he gone?

14. If the spring term of a school ends June 22nd, and the fall term begins Sept. 19th, how many days are there in the summer vacation?

Multiplication of Compound Numbers.

69. I. Multiply £4 13s. 10d. 3far. by 7.

£4	13s.	10d.	3far.
			7
£32	17s.	3d.	1far.

Write the multiplier under the lowest denomination of the multiplicand, and begin at the right to multiply. 7 times 3 far. are 21 far., which equals 5 d. 1 far. Write the 1 far., and reserve 5 d. to be added to the product of the pence. 7 times 10 d. are 70 d.; the sum of 70 d. and 5 d. is 75 d., which equals 6 s. 3 d. Write the 3 d., and reserve 6 s. to be added to the product of the shillings. 7 times 13 s. are 91 s.; the sum of 91 s. and 6 s. is 97 s.

which equals £4 17 s. Write the 17 s., and reserve £4 to be added to the product of the pounds. 7 times £4 are £28; the sum of £28 and £4 is £32. The entire result is £32 17 s. 3 d. 1 far.

EXAMPLES.

1. Multiply 12 T. 14 cwt. 1 qr. 18 lb. 10 oz. by 4.
2. Multiply 4 yr. 78 da. 18 hr. 15 min. 30 sec. by 6.
3. Multiply 8 A. 2 R. 22 P. 6 sq. yd. 5 sq. ft. 42 sq. in. by 8.
4. Multiply 18 bu. 3 pk. 5 qt. 1 pt. by 9.
5. Multiply 3 A. 104 sq. rd. 25 sq. yd. 8 sq. ft. by 12.
6. Multiply 37 gal. 3 qt. 1 pt. 2 gi. by 14.
7. Multiply $24° 36' 50''$ by 15.
8. Multiply 15 cu. yd. 12 cu. ft. 227 cu. in by 18.
9. Multiply 21 ℔. 9 ℥ 2 ʒ 1 ℈ 16 gr. by 20.
10. Multiply 15 mi. 128 rd. 1 ft. by 23.
11. Multiply £9 17 s. 6 d. 1 far. by 28.
12. Multiply 5 T. 8 cwt. 64 lb. 8 oz. 6 dr. by 37.
13. Multiply 8 mi. 5 fur. 16 rd. 3 yd. 2 ft. 8 in. by 48.
14. Multiply 4 lb. 8 oz. 16 pwt. 20 gr. by 72.

Division of Compound Numbers.

70. I. Divide 41 bu. 1 pk. 7 qt. 1 pt. by 9.

9)41 bu. 1 pk. 7 qt. 1 pt.
   4 bu. 2 pk. 3 qt. 1 pt.

Write the divisor at the left of the dividend, and begin at the left to divide. 41 bu. divided by 9 equals 4 bu., with a remainder of 5 bu. Write the 4 bu., and reduce 5 bu. to pecks; the result, after adding 1 pk., is 21 pk. 21 pk. divided by 9 equals 2 pk., with a remainder of 3 pk. Write the 2 pk., and reduce 3 pk. to quarts; the result, after adding 7 qt., is 31 qt. 31 qt. divided by 9 equals 3 qt., with a remainder of 4 qt. Write the 3 qt., and reduce 4 qt. to pints; the result, after adding 1 pt., is 9 pt. 9 pt. divided by 9 equals 1 pt. The entire result is 4 bu. 2 pk. 3 qt. 1 pt.

II. Divide 161 cd. 4 cd. ft. 13 cu. ft. by 21.

```
21)161 cd. 4 cd. ft. 13 cu. ft.(7 cd.
   147
   ---
    14
     8
   ---
   112
     4
   ---
21)116 cd. ft. (5 cd. ft.
   105
   ---
    11
    16
   ---
   176
    13
   ---
21)189(9 cu. ft.
   189
   ---
```

The arrangement of work here given can be used when the divisor is a large number.

Ans. 7 cd. 5 cd. ft. 9 cu. ft.

III. Divide 245° 34′ 12″ by 9° 26′ 42″.

```
  9° 26′ 42″        245° 34′ 12″
  60                 60
 ----              -----
 540               14700               34002)884052(26
  26                  34                     68004
 ----              -----                     ------
 566′              14734′                    204012
   60                 60                     204012
 -----             ------                    ------
 33960             884040
    42                 12
 ------            -------
 34002″            884052″
```

When both dividend and divisor are compound numbers, reduce both to the lowest denomination mentioned in either, and divide as in simple numbers. Notice that the answer is 26, not 26″.

EXAMPLES.

1. Divide 3 wk. 6 da. 14 hr. 17 min. 57 sec. by 3.

2. Divide 38 A. 114 sq. rd. 11 sq. yd. 4 sq. ft. 72 sq. in. by 5.

3. Divide £32 16 s. 3 d. by 7.

4. Divide 5 cwt. 12 lb. 4 oz. by 7.

5. Divide 85 gal. 2 qt. 1 pt. 3 gi. by 11.

6. Divide 32 lb. 10 oz. 7 pwt. 18 gr. by 13.

7. Divide $4° 3' 17''.06$ by 15.

8. Divide 347 bu. 1 pk. 7 qt. 1 pt. by 19.

9. Divide 6 mi. 7 fur. 30 rd. 2 ft. by 48.

10. Divide 152 ℔. 5 ℥ 1 ʒ 2 ℈ 14 gr. by 61.

11. Divide 3 rd. 3 yd. 2 ft. 10 in. by 2 ft. 8 in.

12. Divide 35 gal. 3 qt. 1 pt. 1 gi. by 4 qt. 1 pt. 3 gi.

13. Divide 15 cwt. 2 qr. 19 lb. 12 oz. by 1 qr. 12 lb. 6 oz.

14. Divide 92 cd. 1 cd. ft. by 5 cd. 5 cd. ft. 6 cu. ft.

15. Divide £131 1s. 10d. 2 far. by £8 14s. 9d. 2 far.

16. How many house lots, each containing 1 A. 89 sq. rd., are there in a piece of land containing 56 A. 4 sq. rd.?

17. How many coins, each weighing 15 pwt. 18 gr., can be made out of 16 lb. 10 oz. 7 pwt. 18 gr. of metal?

18. How many bags, each containing 2 bu. 1 pk. 3 qt., will be required to hold 111 bu. 2 pk. 4 qt. of grain?

To Multiply or Divide a Compound Number by a Fraction.

71. I. Multiply 4 gal. 1 pt. 3 gi. by $\frac{3}{5}$.

```
  4 gal. 0 qt. 1 pt. 3 gi.
                       3
5)12 gal. 2 qt. 1 pt. 1 gi.
  2 gal. 2 qt.        1 gi.
```

Multiplying by $\frac{3}{5}$ is the same as multiplying by 3 and dividing the result by 5.

To multiply a compound number by a fraction, *multiply by the numerator and divide by the denominator.*

If the multiplier is a mixed number, multiply by the integral and fractional parts separately, and add the results.

II. Divide 2 lb. 8 oz. 2 pwt. 6 gr. by $\frac{7}{9}$.

```
  2 lb. 8 oz. 2 pwt.  6 gr.
                      9
7)24 lb. 1 oz. 0 pwt.  6 gr.
  3 lb. 5 oz. 5 pwt. 18 gr.
```

Dividing by $\frac{7}{9}$ is the same as multiplying by $\frac{9}{7}$.

To divide a compound number by a fraction, *multiply by the denominator and divide by the numerator.*

If the divisor is a mixed number, reduce the mixed number to an improper fraction, and proceed as before.

EXAMPLES.

1. Multiply 5 T. 6 cwt. 48 lb. 5 oz. by $\frac{4}{7}$.
2. Multiply 13 cd. 2 cd. ft. 9 cu. ft. by $\frac{8}{15}$.
3. Multiply 7 bu. 2 pk. 3 qt. 1 pt. by $\frac{5}{12}$.
4. Multiply 14 ℔. 8 ℥ 5 ʒ 1 ℈ 12 gr. by $\frac{29}{36}$.
5. Multiply 4 mi. 112 rd. 1 yd. 1 ft. 8 in. by $5\frac{5}{8}$.
6. Multiply 23° 6′ 41″ by $13\frac{3}{19}$.
7. Multiply 2 wk. 4 da. 19 hr. 7 min. 24 sec. by $8\frac{17}{48}$.
8. Divide £4 14 s. 3 d. 3 far. by $\frac{9}{13}$.
9. Divide 7 sq. rd. 4 sq. yd. 6 sq. ft. 41 sq. in. by $\frac{17}{20}$.
10. Divide 23 gal. 3 qt. 1 pt. 3 gi. by $\frac{13}{35}$.
11. Divide 2 mi. 3 fur. 16 rd. 2 yd. 2 ft. 6 in. by $3\frac{3}{7}$.
12. Divide 13 T. 13 cwt. 3 qr. 9 lb. 6 oz. by $8\frac{5}{8}$.
13. Divide 14° 58′ 8″ by $11\frac{7}{11}$.

To Reduce a Fraction of one Denomination to Lower Denominations.

72. I. Reduce $\frac{1}{48}$ of a yard to the fraction of an inch.

$$\frac{1}{\cancel{48}_{4}}\times 3\times \cancel{12}=\frac{3}{4}\text{ in.}$$

3 times the number of yards equals the number of feet, and 12 times the number of feet equals the number of inches. Hence $\frac{1}{48}$ yd. equals $\frac{1}{48}\times 3\times 12$ in., which equals $\frac{3}{4}$ in.

II. Reduce $\frac{7}{24}$ cwt. to lower denominations.

$$\frac{7}{\cancel{24}_{6}}\times \overset{25}{\cancel{100}}=\frac{175}{6}=29\tfrac{1}{6}\text{ lb.}$$

$$\frac{1}{\cancel{6}_{3}}\times \overset{8}{\cancel{16}}=\frac{8}{3}=2\tfrac{2}{3}\text{ oz.}$$

$$\frac{2}{3}\times 16=\frac{32}{3}=10\tfrac{2}{3}\text{ dr.}$$

Ans. 29 lb. 2 oz. $10\frac{2}{3}$ dr.

Since there are 100 lb. in 1 cwt., in $\frac{7}{24}$ cwt. there are $\frac{7}{24}$ of 100 lb., which equals $29\frac{1}{6}$ lb. In 1 lb. there are 16 oz.; in $\frac{1}{6}$ lb. there is $\frac{1}{6}$ of 16 oz., which equals $2\frac{2}{3}$ oz. In 1 oz. there are 16 dr.; in $\frac{2}{3}$ oz. there are $\frac{2}{3}$ of 16 dr., which equals $10\frac{2}{3}$ dr. The entire result is 29 lb. 2 oz. $10\frac{2}{3}$ dr.

When fractions of different denominations are to be added or subtracted, reduce the fractions to lower denominations, and then perform the operations indicated.

EXAMPLES.

1. Reduce $\frac{1}{160}$ of a cord to the fraction of a cubic foot.
2. Reduce $\frac{5}{304}$ of a shilling to the fraction of a farthing.
3. Reduce $\frac{1}{22}$ of a gallon to the fraction of a gill.
4. Reduce $\frac{1}{6}$ of a bushel to the fraction of a pint
5. Reduce $\frac{18}{70}$ of a week to minutes.
6. Reduce $\frac{8}{11}$ of a furlong to inches.
7. Reduce $\frac{35}{144}$ of a pound Troy to grains.
8. Reduce $\frac{5}{132}$ of a rood to square feet.

9. Reduce $\frac{4}{7}$ bu. to lower denominations.

10. Reduce $\frac{3}{5}$ ℔. to lower denominations.

11. Reduce $\frac{5}{8}$ rd. to lower denominations.

12. Reduce $\frac{6}{11}$ mi. to lower denominations.

13. Reduce $\frac{1}{9}$ yr. to lower denominations.

14. Reduce $\frac{53}{160}$ bu. to lower denominations.

15. Reduce $\frac{7}{11}$ A. to lower denominations.

16. Reduce $\frac{9}{25}$ A. to lower denominations.

17. Reduce $\frac{53}{97}$ gal. to lower denominations.

18. Find the value of $\frac{3\frac{4}{5}}{5\frac{9}{10}}$ of $\frac{3}{4}$ of an acre at $1.36 per square foot.

19. Add $\frac{1}{6}$ of a furlong, $\frac{1}{2}$ of a rod, and $\frac{1}{3}$ of a yard.

20. From $\frac{3}{4}$ of a gallon subtract $1\frac{4}{5}$ of a pint.

21. Add $\frac{5}{9}$ of a pound, $\frac{3}{8}$ of a shilling, and $\frac{5}{7}$ of a penny.

22. Add $\frac{5}{18}$ lb., $4\frac{11}{12}$ oz., and $9\frac{4}{9}$ pwt.

23. Express in rods, yards, etc., $\frac{3}{16}$ mi. + $\frac{2}{3}$ of 40 rd. + $\frac{3}{8}$ yd.

24. From $\frac{1}{3}$ of $4\frac{3}{8}$ bu. subtract $\frac{6}{7}$ of $2\frac{1}{2}$ pk.

25. Add together $\frac{3}{7}$ of £13, $\frac{1}{3}$ of $\frac{1}{2\frac{4}{5}}$ of $\frac{3}{5}$ of £2 12 s., and $\frac{5}{7}$ of 9 d.

To Reduce Lower Denominations to a Fraction of a Higher Denomination.

73. I. Reduce $\frac{2}{3}$ of a grain to the fraction of an ounce Troy.

$$\frac{2}{3} \times \frac{1}{24} \times \frac{1}{\underset{10}{\cancel{20}}} = \frac{1}{720} \text{ oz.}$$

$\frac{1}{24}$ of the number of grains equals the number of pennyweights, and $\frac{1}{20}$ of the number of pennyweights equals the number of ounces. Hence $\frac{2}{3}$ gr. equals $\frac{2}{3} \times \frac{1}{24} \times \frac{1}{20}$ oz., which equals $\frac{1}{720}$ oz.

II. Reduce 2 da. 5 hr. 20 min. to the fraction of a week.

$\frac{20}{60}=\frac{1}{3}$ hr.

$\frac{5\frac{1}{3}}{24}=\frac{\overset{2}{\cancel{16}}}{3}\times\frac{1}{\underset{3}{\cancel{24}}}=\frac{2}{9}$ da.

$\frac{2\frac{2}{9}}{7}=\frac{20}{9}\times\frac{1}{7}=\frac{20}{63}$ wk.

Since there are 60 min. in 1 hr., 20 min. equal $\frac{20}{60}$ of an hour, which equals $\frac{1}{3}$ hr. In 1 da. there are 24 hr.; $5\frac{1}{3}$ hr. equals $\frac{5\frac{1}{3}}{24}$ of a day, which equals $\frac{2}{9}$ da. In 1 wk. there are 7 da.; $2\frac{2}{9}$ da. equal $\frac{2\frac{2}{9}}{7}$ of a week which equals $\frac{20}{63}$ wk.

EXAMPLES.

1. Reduce $\frac{8}{11}$ of a gill to the fraction of a gallon.
2. Reduce $\frac{27}{32}$ of an inch to the fraction of a rod.
3. Reduce $\frac{75}{76}$ of a pound to the fraction of a ton.
4. Reduce $2\frac{2}{3}$ qt. to the fraction of a bushel.
5. Reduce $9\frac{9}{10}$ sq. ft. to the fraction of a rood.
6. What part of a mile is one inch?
7. Reduce 12 s. 6 d. to the fraction of a pound.
8. Reduce 9 hr. 20 min. to the fraction of a week.
9. Reduce 7 qt. $1\frac{1}{3}$ pt. to the fraction of a bushel.
10. Reduce 9 rd. 1 ft. 6 in. to the fraction of a furlong.
11. Reduce 6 cwt. 2 qr. 24 lb. to the fraction of a ton.
12. Reduce 1 ℥ 5 ʒ 1 ℈ 16 gr. to the fraction of a pound.
13. Reduce 6 rd. 5 ft. 9 in. to the fraction of a mile.
14. Reduce 40 sq. rd. 27 sq. yd. 4 sq. ft. 72 sq. in. to the fraction of an acre.
15. Reduce 3 bundles 9 quires 4 sheets to the fraction of a bale.
16. What part of a hogshead is 3 gal. 1 qt. $2\frac{2}{19}$ gi.?

To Find what Fractional Part one Compound Number is of Another.

74. I. What part of 32 bu. 2 pk. 4 qt. is 8 bu. 3 pk. 2 qt.?

$\frac{4}{8}=\frac{1}{2}$ pk. $\frac{2}{8}=\frac{1}{4}$ pk.

$\frac{2\frac{1}{2}}{4}=\frac{5}{8}$ bu. $\frac{3\frac{1}{4}}{4}=\frac{13}{16}$ bu.

$$\frac{8\frac{13}{16}}{32\frac{5}{8}}=\frac{\overset{47}{\cancel{141}}}{\underset{2}{\cancel{16}}}\times\frac{\cancel{8}}{\underset{87}{\cancel{261}}}=\frac{47}{174}.$$

Reduce both compound numbers to the same denomination, and then by the method shown in § 41, find what fractional part one number is of the other.

EXAMPLES.

1. What part of 6 A. 1 R. is 3 R. 5 P.?

2. What part of 3 bu. 2 pk. is 5 pk. 6 qt.?

3. What part of 51° 25′ 20″ is 3° 51′ 24″?

4. What part of 37 sq. yd. 2 sq. ft. 116 sq. in. is 10 sq. yd. 5 sq. ft. 136 sq. in.?

5. What part of 2 A. 71 sq. rd. is 1 A. 7 sq. ch.?

6. What part of 4 da. is 1 da. 9 hr. 13 min. $50\frac{10}{13}$ sec.?

7. What part of 11 mi. 156 rd. 5 yd. is 1 mi. 69 rd. 1 ft. 6 in.?

8. What part of 22 gal. 3 qt. 1 gi. is 4 gal. 2 qt. 1 pt. 2 gi.?

9. What part of 4 lb. 9 ℥ 1 ʒ 12 gr. is 10 ℥ 6 ʒ 6 gr.?

10. What part of £18 15 s. 4 d. 2 far. is £4 3 s. 10 d. 2 far.?

11. What part of 5 fur. 3 rd. 3 yd. 1 ft. 6 in. is 4 fur. 17 rd. 4 yd. 10 in.?

12. What part of 8 long tons is $\frac{5}{9}$ of 720 lb.?

13. What part of $\frac{6\frac{2}{5}}{\frac{35}{11}}$ yards is $\frac{7}{8}$ of an inch?

14. What part 12 yd. 1 ft. 6 in. is $\frac{1}{2112}$ of a mile?

To Reduce a Decimal of one Denomination to Lower Denominations.

75. I. Reduce 0.4375 gal. to lower denominations.

$$\begin{array}{l} 0.4375 \text{ gal.} \\ \underline{\quad\quad 4} \\ 1.75\cancel{0}\cancel{0} \text{ qt.} \\ \underline{\quad 2} \\ 1.5\cancel{0} \text{ pt.} \\ \underline{\quad 4} \\ 2.\cancel{0} \text{ gi.} \end{array}$$

Ans. 1 qt. 1 pt. 2 gi.

Since there are 4 qt. in 1 gal., in 0.4375 gal. there are 0.4375 of 4 qt., which equals 1.75 qt. In 1 qt. there are 2 pt.; in 0.75 qt. there are 0.75 of 2 pt., which equals 1.5 pt. In 1 pt. there are 4 gi.; in 0.5 pt. there are 0.5 of 4 gi., which equals 2 gi. The entire result is 1 qt. 1 pt. 2 gi.

When decimals of different denominations are to be added or subtracted, reduce them to the same denomination, and then perform the operations indicated.

EXAMPLES.

1. Reduce 0.7375 lb. Troy to lower denominations.
2. Reduce 0.5625 da. to lower denominations.
3. Reduce 0.3125 bu. to lower denominations.
4. Reduce 0.4125 ℔. to lower denominations.
5. Reduce 0.0625 A. to lower denominations.
6. Reduce 0.795 wk. to lower denominations.
7. Reduce 0.445 A. to lower denominations.
8. Reduce 0.845 mi. to lower denominations.
9. Reduce 0.428 T. to lower denominations.
10. Reduce 0.333 A. to lower denominations.
11. Reduce 0.984375 bu. to lower denominations.
12. Reduce 0.758762 A. to lower denominations.
13. Subtract 10.869 oz. from 1.203 lb. Troy.

14. Add together 1.001 cwt. and 0.039 qr., and give the answer in ounces and the decimal of an ounce.

15. Subtract 0.335 gal. from 12.51 qt., and give the answer in pints and the decimal of a pint.

16. Subtract 7.3125 fur. from 1.03125 mi., and give the answer in yards and the decimal of a yard.

17. What is the cost of 0.33 bbl. of wine at $1.15 per pint?

18. A man bought a piece of ground containing 0.316 A. at 53 cents a square foot; what did he pay for the piece?

To Reduce Lower Denominations to a Decimal of a Higher Denomination.

76. I. Reduce 7 ℥ 7 ʒ 1 ℈ 16 gr. to the decimal of a pound.

20	16.0 gr.
3	1.8 ℈
8	7.60 ʒ
12	7.9500 ℥
	0.6625 ℔.

Since there are 20 gr. in 1 ℈, the number of scruples equals $\frac{1}{20}$ of the number of grains; $\frac{1}{20}$ of 16 is 0.8, which, added to 1 ℈, equals 1.8 ℈. Since there are 3 ℈ in 1 ʒ, the number of drams equals $\frac{1}{3}$ of the number of scruples; $\frac{1}{3}$ of 1.8 is 0.6, which, added to 7 ʒ, equals 7.6 ʒ. Since there are 8 ʒ in 1 ℥, the number of ounces equals $\frac{1}{8}$ of the number of drams; $\frac{1}{8}$ of 7.6 is 0.95, which, added to 7 ℥, equals 7.95 ℥. Since there are 12 ℥ in 1 ℔., the number of pounds equals $\frac{1}{12}$ of the number of ounces; $\frac{1}{12}$ of 7.95 is 0.6625.

EXAMPLES.

1. Reduce 4 s. 9 d. to the decimal of a pound.
2. Reduce 5 yd. 2 ft. 6 in. to the decimal of a rod.
3. Reduce 3 R. 13 P. 8 sq. ft. to the decimal of an acre.
4. Reduce 12 s. 9 d. 2 far. to the decimal of a pound.
5. Reduce 15 lb. 5 oz. 4 dr. to the decimal of a ton.

6. Reduce 5 fur. 33 rd. 9 ft. 10.8 in. to the decimal of a mile.

7. Reduce 2 cwt. 3 qr. 3 lb. 8 oz. to the decimal of a ton.

8. Reduce 1 hr. 25 min. 30 sec. to the decimal of a day.

9. Reduce 1 oz. 8 pwt. 19.2 gr. to the decimal of a pound.

10. Reduce 6 fur. 30 rd. 6 ft. 7.2 in. to the decimal of a mile.

11. Reduce 38 sq. rd. 21 sq. yd. 5 sq. ft. 108 sq. in. to the decimal of an acre.

12. Reduce 30 rd. 4 yd. 2 ft. 10 in. to the decimal of a mile.

13. Reduce 71 sq. rd. 54 sq. ft. 64.8 sq. in. to the decimal of an acre.

14. What decimal part of a degree is $52' 43''.5$?

15. Reduce 1 fur. 25 rd. 12 ft. 11 in. to the decimal of a mile.

16. Express to the nearest millionth 34 A. 7 sq. rd. 3 sq. yd. 7 sq. ft. 104 sq. in. as a decimal of a square mile.

17. Reduce 2 yr. 5 mo. 12 da. to years and the decimal of a year.

18. Reduce 18216 ft. to miles and the decimal of a mile.

19. Reduce $\frac{1}{3}$ of a farthing to the decimal of a pound.

20. At 6 cents a pound, what decimal part of a ton of nails can be bought for $4.20?

21. Find the value of 21 A. 3 R. 12 P. of land at $45 per acre.

22. Find the value of 7 A. 35 sq. rd. 127 sq. ft. of land at $108.15 per acre.

23. What is the value, at $4500 an acre, of a piece of land containing 30 sq. rd. 19 sq. ft. 89 sq. in.?

To Find what Decimal one Compound Number is of Another.

77. I. What decimal of 8 bu. 3 pk. is 2 bu. 1 pk. 5 qt.?

8 bu. 3 pk.	2 bu. 1 pk. 5 qt.	
4	4	280)77.000(0.275
32	8	560
3	1	2100
35 pk.	9 pk.	1960
8	8	1400
280 qt.	72	1400
	5	
	77 qt.	

8 bu. 3 pk. = 280 qt. 2 bu. 1 pk. 5 qt. = 77 qt. 77 qt. is $\frac{77}{280}$ of 280 qt., and this can be expressed as a decimal by dividing 77 by 280.

Note. It is merely necessary to reduce both compound numbers to the same denomination, and then divide. Choose the denomination in which the two numbers can be most simply expressed.

EXAMPLES.

1. What decimal of 132 bu. is 8 bu. 1 pk.?
2. What decimal of 2 gal. is 1 qt. 1 pt. 2 gi.?
3. What decimal of 19 s. 6 d. is 13 s. $4\frac{1}{2}$ d.?
4. What decimal of 12° 51′ 20″ is 3° 51′ 24″?
5. What decimal of 2 lb. 1 oz. 6 pwt. is 1 lb. 16 pwt.?
6. What decimal of 4 da. 14 hr. 36 min. is 9 hr. 5 min.?
7. What decimal of 4 T. 14 cwt. 56 lb. is 11 cwt. 82 lb.?
8. What decimal of 300 yd. is 1 fur. 2 rd. 6 yd. 2 ft.?

Comparison of Weights.

78. The Troy pound and the Apothecaries' pound are equal in weight, and each contains 5760 grains; the Avoirdupois pound contains 7000 grains. The grain is the only

denomination that is the same in all three weights; hence, when Avoirdupois Weight is to be compared with either Troy Weight or Apothecaries' Weight, the comparison can be made by first reducing the given denominations to grains.

EXAMPLES.

1. What part of a pound Avoirdupois is a pound Troy?

2. What part of an ounce Avoirdupois is an ounce Troy?

3. What part of 1 ℥ is 1 dr.?

4. Express 4 lb. Avoirdupois as the decimal of 8 lb. Troy.

5. Find the number of pennyweights in a pound Avoirdupois.

6. Reduce 5 lb. 6 oz. Troy to Apothecaries' Weight.

7. Reduce 8 lb. Avoirdupois to Apothecaries' Weight.

8. Reduce 12 lb. Troy to Avoirdupois Weight.

9. Reduce 18 lb. 4 oz. Avoirdupois to Troy Weight.

10. A miner obtained $9600 worth of gold. At $16 an ounce Troy, what was the weight of the metal in Avoirdupois Weight?

11. Find the value of a silver cup, weighing 1 lb. 11 oz. Avoirdupois, at $1.95 per ounce Troy.

12. Find the amount gained by a druggist, who buys 15 lb. Avoirdupois of drugs at $2.75 per pound, and sells the same at 20 cts. per dram, Apothecaries' Weight.

Comparison of Money.

79. The relations between United States Money and the moneys of other countries vary from time to time. It is customary for the Secretary of the Treasury to publish annually a table giving the values of the standard coins in United States Money. The values given Jan. 1st, 1889 are as follows:

COUNTRY.	Monetary Unit.	Value in U.S. Money.	Divisions of Units.
Argentine Rep.	Peso	$0.965	100 centavos = 1 peso.
Austria	Florin	.336	100 kreuzers = 1 florin.
Belgium	Franc	.193	100 centimes = 1 franc.
Bolivia	Boliviano	.68	100 centavos = 1 boliviano
Brazil	Milreis	.546	1000 reis = 1 milreis.
British Possessions N.A.	Dollar	1.00	100 cents = 1 dollar.
Chili	Peso	.912	100 centavos = 1 peso.
Cuba	Peseta	.926	100 centimos = 1 peseta.
Denmark	Crown	.268	100 ore = 1 crown.
Ecuador	Sucre	.68	100 centavos = 1 sucre.
Egypt	Pound	4.943	100 piastres = 1 pound.
France	Franc	.193	100 centimes = 1 franc.
German Empire	Mark	.238	100 pfennig = 1 mark.
Great Britain	Pound Sterling	4.8665	20 shillings = 1 pound.
Greece	Drachma	.193	100 lepta = 1 drachma.
Guatemala	Peso	.68	100 centavos = 1 peso.
Hayti	Gourde	.965	100 centavos = 1 gourde.
Honduras	Peso	.68	100 centavos = 1 peso.
India	Rupee	.323	16 annas = 1 rupee.
Italy	Lira	.193	100 centesimi = 1 lira.
Japan	Yen { Gold / Silver	.997 / .734	100 sens = 1 yen.
Liberia	Dollar	1.00	100 cents = 1 dollar.
Mexico	Dollar	.739	100 centavos = 1 dollar.
Netherlands	Florin	.402	100 cents = 1 florin.
Nicaragua	Peso	.68	100 centavos = 1 peso.
Norway	Crown	.268	100 ore = 1 crown.
Peru	Sol	.68	100 centavos = 1 sol.
Portugal	Milreis	1.08	1000 reis = 1 milreis.
Russia	Rouble	.544	100 copecks = 1 rouble.
Spain	Peseta	.193	100 centimos = 1 peseta.
Sweden	Crown	.268	100 ore = 1 crown.
Switzerland	Franc	.193	100 centimes = 1 franc.
Tripoli	Mahbub	.614	20 piastres = 1 mahbub.
Turkey	Piastre	.044	40 paras = 1 piastre.
U.S. of Columbia	Peso	.68	100 centavos = 1 peso.
Venezuela	Bolivar	.136	100 centimos = 1 bolivar.

NOTE. In all answers in English Money reject fractions of a farthing less than one half, and when the fraction of a farthing equals or exceeds one half, reckon it as another farthing. In the moneys of all countries having a decimal system, treat like United States Money, retaining only two decimal places in the answer (for Brazil and Portugal, three decimal places).

EXAMPLES.

1. Reduce £8 12s. 6d. to United States money.
2. Reduce 16 guineas to United States money.
3. Reduce $128.42 to English money.
4. Reduce 172.46 francs to United States money.
5. Reduce $45.36 to French money.
6. Reduce 64.35 marks to United States money.
7. Reduce $75.50 to German money.
8. Reduce 32 roubles to United States money.
9. Reduce 75 crowns to United States money.
10. Reduce $114.25 to Austrian money.
11. Reduce £16 15s. to French money.
12. Reduce 22.25 marks to English money.
13. Reduce £17 9s. 3d. to Federal money, taking 4s. 6d. = $1.00.
14. If the value of a pound sterling is $4.85, and of a franc is 19½ cts., what is the equivalent in francs of 2s. 4d.?

Rectangular Surfaces.

80. Any part of a flat surface taken by itself is called a **plane figure**. The extent of surface of a plane figure is called the **area**, and the distance around it is called the **perimeter**.

A **rectangle** is a plane figure having four straight sides and four right angles. When the four sides of a rectangle are all equal, it is called a **square**.

A rectangle has two dimensions — length and breadth.

The unit of surface is a square, each side of which is a unit of length. For example, a square foot is a square 1 ft. long and 1 ft. wide.

Suppose we have a rectangle 4 in. long and 3 in. wide.

Divide the length into four equal parts, and the width into three equal parts, and draw lines through the points of division as represented in the figure. The rectangle is thus divided into square inches. Upon each inch of length there is constructed a square inch, making a row of 4 sq. in.; since the rectangle is 3 in. wide, there are 3 rows, each containing 4 sq. in., making 3 times 4 sq. in., which equals 12 sq. in. Hence, to find the area of a rectangle, *multiply together the length and breadth expressed in the same linear units, and the result is the area expressed in square units of the same name.*

The quotient arising from dividing the product of two factors by one of the factors is the other factor; hence, *if the area of a rectangle be divided by one dimension, the quotient is the other dimension.*

I. Find the area of a rectangular table whose length is 6 ft. 4 in. and width 4 ft. 9 in.

6 ft. 4 in.	4 ft. 9 in.	76
12	12	57
72	48	532
4	9	380
76 in.	57 in.	144)4332(30 sq. ft.
		432
Ans. 30 sq. ft. 12 sq. in.		12 sq. in.

The two dimensions must be reduced to inches, and their product denotes the number of square inches. Since the dimensions are given in feet and inches, the area should be expressed in square feet and square inches.

II. The area of a rectangular floor is 224 sq. ft., and its length is 16 ft.; find its width.

$$16\overline{)224}$$
$$14 \text{ ft.}$$

The number of square feet in the area divided by the number of feet in length equals the number of feet in breadth.

EXAMPLES.

1. How many square yards are there in a floor 24 ft. long and 14 ft. wide?

2. Find the cost of oil-cloth to cover a floor 15 ft. long and $10\frac{1}{2}$ ft. wide at 45 cts. per square yard.

3. If a floor contains 35 sq. yd., and is 21 ft. long, what is its width?

4. How many blocks 1 ft. square will it take to pave an alley 54 rd. long and 8 ft. wide?

5. How many acres are there in a field 72 rd. long and 60 rd. wide?

6. Find the area of a square field each of whose dimensions is 65 rd.

7. Find the value of a field 180 rd. long and $94\frac{1}{2}$ rd. wide at $18 an acre.

8. The area of a field is 10 A., and its width is 20 rd.; what is its length?

9. Find the cost of paving a street 1028 ft. long and $63\frac{1}{2}$ ft. wide at $3.25 a square yard.

10. What length of road $38\frac{1}{2}$ ft. wide will contain 3 A.?

11. A field is $38\frac{7}{50}$ rd. long and $37\frac{1}{2}$ rd. wide; find its area in acres and square rods.

12. A path is 26 ft. 8 in. long and 5 ft. 3 in. wide; find its area in square feet.

13. A garden, containing $\frac{3}{4}$ of an acre, measures 198 ft. on one side; find the length of the other side.

14. How many acres are there in a field 15.72 ch. long and 8.95 ch. wide?

15. A building lot, containing $\frac{5}{8}$ of an acre, has a frontage of 90 ft.; how far back does it extend?

16. Find the difference in area between two lots of land, one of which is 30 rd. square, and the other contains 30 sq. rd.

17. A field, containing 14 A., is 56 rd. long; what is its width? Find the cost of building a fence around it at 45 cts. a rod.

18. Find the cost of slating a roof 40 ft. long and each of the two sides 20 ft. wide, at $10 per square of 100 sq. ft.

19. At $37.50 per acre, find the cost of a field 55.33 ch. long and 148 rd. 3 yd. 1 ft. 6 in. wide.

20. How many bricks $7\frac{1}{2}$ in. long and $3\frac{1}{2}$ in. wide will it take to lay a walk 462 ft. long and $6\frac{1}{4}$ ft. wide?

21. How many tiles 9 in. square will it take to pave a court 114 ft. long and 48 ft. wide?

22. How many boards, each 14 ft. long and $7\frac{1}{2}$ in. wide, will it take to build a platform 42 yd. long and 30 yd. wide?

23. The area of a field is 49 sq. rd. 22 sq. yd. 6 sq. ft. 108 sq. in., and the length is 7 rd. 4 yd. 1 ft. 6 in.; find the width.

24. Find the cost, at 60 cents a square yard, of making a gravel path 5 ft. wide around a garden 78 ft. long and 42 ft. wide:

(i) when the path is outside the garden.

(ii) when the given dimensions include both garden and path.

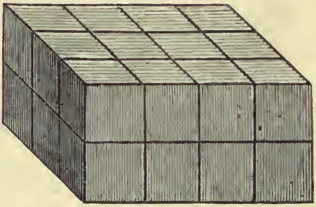
Rectangular Volumes.

81. A **rectangular parallelopiped** is a volume bounded by six rectangular surfaces. The bounding surfaces are called **faces**, and the bounding lines are called **edges**. The faces taken together constitute the **surface**, and the lower face is called the **base**. When the faces are six equal squares, the volume is called a **cube**.

A rectangular parallelopiped has three dimensions — length, breadth, and thickness.

The unit of volume is a cube, each dimension of which is a unit of length. For example, a cubic foot is a cube 1 ft. long, 1 ft. wide, and 1 ft. thick.

Suppose we have a rectangular parallelopiped 4 in. long, 3 in. wide, and 2 in. thick. The upper face contains 3 times 4 sq. in., or 12 sq. in.; and if the parallelopiped were 1 in. thick, there would be as many cubic inches as there are square inches on the upper face. But the parallelopiped is 2 in. thick, and must, therefore, contain twice as many cubic inches as it would if it were only 1 in. thick, or 2 times 3 times 4 cu. in., which equals 24 cu. in. Hence, to find the cubic contents of a rectangular parallelopiped, *multiply together the three dimensions expressed in the same linear units*, *and the result is the cubic contents expressed in cubic units of the same name.*



The quotient arising from dividing the product of three factors by the product of two of the factors is the third factor; hence, *if the cubic contents of a rectangular parallelopiped be divided by the product of two dimensions, the quotient is the third dimension.*

I. Find the number of gallons in a cistern 7 ft. long, 6 ft. wide, and 5 ft. deep.

$7 \times 6 \times 5 = 210$ cu. ft.

$$\begin{array}{r}
1728 \\
\underline{210} \\
17280 \\
\underline{3456} \\
231)362880 \text{ cu. in.}(1570\tfrac{10}{11} \text{ gal.} \\
\underline{231} \\
1318 \\
\underline{1155} \\
1638 \\
\underline{1617} \\
\frac{210}{231} = \frac{10}{11}
\end{array}$$

The product of the three dimensions gives 210 cu. ft. as the cubic contents, and this equals 362880 cu. in. Since there are 231 cu. in. in 1 gal., there are as many gallons in 362880 cu. in. as 231 is contained times in 362880, which equals $1570\frac{10}{11}$ gal.

II. A rectangular solid, whose cubic contents are 924 cu. ft., is 22 ft. long and 7 ft. wide; what is its thickness?

$22 \times 7 = 154$

$$\begin{array}{l}
154)924(6 \text{ ft.} \\
\underline{924}
\end{array}$$

The product of the two known dimensions is 154, and dividing 924, the cubic contents, by 154 gives 6, the number of feet in thickness.

EXAMPLES.

1. Find the number of cubic feet of air in a room 24 ft. long, 18 ft. wide, and $10\frac{1}{2}$ ft. high.

2. Find the cost, at 30 cts. a cubic yard, of digging a cellar 56 ft. long, 28 ft. wide, and 9 ft. deep.

3. A reservoir 30 ft. wide and 12 ft. deep contains 960 cu. yd.; what is its length?

4. A vat 12 ft. square contains 1368 cu. ft.; find its depth.

5. Find the volume of a cube whose edge measures 2 ft. 9 in.

6. How many cubic feet are there in a stick of timber 17 ft. long, 15 in. wide, and 8 in. thick?

7. What must be the length of a stick of timber, $1\frac{1}{4}$ ft. square at the end, to contain 100 cu. ft.?

8. If a box 5 ft. 4 in. high contains 36 cu. ft., what is the area of the base?

9. How many cords of wood are there in a pile 36 ft. long, 6 ft. high, and 4 ft. wide?

10. Find the value of a pile of wood 28 ft. long, $5\frac{1}{2}$ ft. high, and 4 ft. wide, at $3.25 a cord.

11. What must be the length of a pile of wood, $4\frac{1}{2}$ ft. high and $3\frac{1}{2}$ ft. wide, to contain 2 cords?

12. If a cubic foot of ice weighs 58.1 lb., how many tons will be contained in an ice-house 45 ft. long, 32 ft. wide, and 20 ft. high?

13. Find the number of gallons in a tank 3 ft. 6 in. long, 2 ft. 4 in. wide, and 1 ft. 10 in. deep.

14. Find the number of gallons in a cistern $5\frac{1}{2}$ ft. square and 7 ft. deep.

15. Find the value, at 90 cts. a bushel, of the grain that will be contained in a bin 14 ft. long, 10 ft. wide, and 5 ft. deep.

16. Find the depth of a bin necessary to hold 160 bu., if its length is 9 ft. and its width 5 ft.

17. How many bricks will it take to build a wall 56 ft. long, 9 ft. high, and 4 ft. thick, each brick being 8 in. long, $4\frac{1}{2}$ in. wide, and $2\frac{1}{4}$ in. thick?

18. How many stones, 10 in. long, 9 in. broad, and 4 in. thick, would it require to build a wall 80 ft. long, 20 ft. high, and $2\frac{1}{4}$ ft. thick?

19. How many square feet are there on the surface of a box $2\frac{1}{2}$ ft. long, 2 ft. wide, and 3 ft. deep?

20. How many square feet are there on the surface of a cubical box, each of whose dimensions is $2\frac{1}{2}$ ft.?

21. A river, 30 ft. deep and 20 yd. wide, flows 4 mi. an hour. Find the number of cubic feet of water which pass a given point in a minute.

22. How many cords of stone will it take to build a wall, 2 ft. thick and 6 ft. high, about a cellar whose interior dimensions, when the wall is completed, shall be 20 ft. long and 16 ft. wide?

MISCELLANEOUS EXAMPLES.

1. What is the cost of 5 T. 7 cwt. 24 lb. of hay at $16 per ton?

2. Find the cost of 1 qt. of olive oil when 1 doz. pints cost $3.45.

3. What is the value of 1 doz. silver spoons, each weighing 2 oz. 16 pwt. 16 gr., at $1.15 per ounce?

4. How many times will a wheel, 9 ft. 4 in. in circumference, turn in crossing a bridge, the length of which is 54 rd. 4 yd. 2 ft. 4 in.?

5. Reduce 44920.9025 hr. to years (of 365 days), days, hours, minutes, and seconds.

6. When coal is worth $6.25 a long ton, what is the expense of a coal fire for the month of January, allowing 35 lb. a day?

7. If one man performs a piece of labor in 2 da. 13 hr. 41 min., how long would it take 10 men to perform the same work?

8. A rectangular field measures 30 rd. 6 ft. by 21 rd. 11 ft.; find the area in acres, square rods, and square feet.

9. What is the value of a piece of ground, $16\frac{7}{11}$ rd. long and $27\frac{1}{3}$ yd. wide, at 1 s. 4 d. per square foot?

10. If $12\frac{1}{2}$ yd. of silk that is $\frac{3}{4}$ yd. wide will make a dress, how many yards of muslin that is $1\frac{3}{8}$ yd. wide will be required to line it?

11. A cellar is to be dug 30 ft. long and 20 ft. wide; at what depth will 50 cu. yd. of earth have been removed?

12. It takes 8 hr. 40 min. to fill a certain cistern; what part of it has been filled after water has been running in 2 hr. 45 min. 45 sec.?

13. How long will it take a man to walk 48 mi. 210 rd. 12 ft., if he walks 15 mi. in 4 hr. 15 min.?

14. How many silver dollars, each weighing $412\frac{1}{2}$ gr., can be coined from a bar of silver weighing $8\frac{1}{4}$ lb. Avoirdupois?

15. A man earns $325 in $2\frac{1}{2}$ months, and spends in 6 months what he earns in $4\frac{1}{2}$ months; what does he save in a year?

16. A regiment of troops enlisted for 9 months and was discharged May 25th, 1863, which was 1 mo. 12 da. after the term of service had expired. Find the date when they enlisted.

17. A cable that weighs one ton per mile weighs how much per foot?

18. The velocity of a body is 40 mi. per hour; what is it expressed in feet per second?

19. If a train travels 40 ft. in a second, how far will it travel in 1 hr. 31 min. 18 sec.?

20. Divide 2 gal. 1 qt. 1.02 pt. by 17. Express the result in pints; also in the decimal of a barrel.

21. Find $\frac{7}{12}$ of 3 mi. 2 fur. 25 rd. 3 yd. 2 ft. 6 in. as a compound number; reduce it to chains and the decimal of a chain.

22. Express as a fraction of an acre the ground taken up by a path 3 ft. broad round a house, the front of which is 57 ft., and side is 37 ft.

23. If 2 cu. in. of iron weigh as much as 15 cu. in. of water, and 1 cu. ft. of water weighs 1000 oz., find the weight in tons of 1 cu. yd. of iron.

24. If a grocer's scales give only 15 oz. 4 dr. for a pound, out of how much money is a customer cheated who buys sugar to the amount of $55.04?

25. If 2 A. 3 R. 4 P. be multiplied by $2\frac{3}{4}$, what part is the product of 15 A. 1 R. 2 P.?

26. A owns $\frac{5}{13}$ of a field, and B owns the remainder; $\frac{3}{4}$ of the difference between their shares is 5 A. 3 R. $16\frac{1}{2}$ P. What is B's share in acres?

27. Thirty-six persons buy 2766 A. 3 R. 12 P. of land on equal shares. What does one man receive, who sells $\frac{2}{3}$ of his share at 1 s. 9 d. 2 far. per square rod? [Give the answer in pounds and the decimal of a pound.]

28. Find the weight of 500000 bricks at 4 lb. 2 oz. each, and the cost in dollars and cents, at 27 s. 6 d. a thousand, allowing 4 s. 2 d. to make a dollar.

29. Reduce 12 T. 8 cwt. 55 lb. 3 oz. $3\frac{1}{5}$ dr. Avoirdupois Weight to pounds and the decimal of a pound; then reduce to Troy Weight.

30. If a body revolves uniformly in the circumference of a circle at the rate of $12° 15' 25''$ per minute, how long is it in performing a complete revolution?

CHAPTER VI.

THE METRIC SYSTEM.

82. The **metric system** is a system of weights and measures based upon the decimal system of notation. It has been adopted by nearly all civilized nations, and its use has been legalized in the United States and Great Britain.

The unit of length is the **meter,** and from it are derived the units of surface, volume, capacity, and weight. The length of the meter is defined by a bar kept at Paris. This length was adopted in 1799, and is one ten-millionth of the distance from the equator to either pole, as calculated at that time. However, later calculations have proved that the meter is a very small fraction shorter than one ten-millionth of this distance on the earth's surface.

From the different units are derived other denominations by adding prefixes. The prefixes for the fractional parts of the unit are derived from Latin numerals, and those for the multiples of the unit are derived from Greek numerals.

Deci	means	tenth.	Deka	means	ten.
Centi	"	hundredth.	Hekto	"	hundred.
Milli	"	thousandth.	Kilo	"	thousand.
			Myria	"	ten thousand.

In the following tables the denominations in common use are printed in full-faced type. Abbreviations beginning with a small letter denote a fractional part of the principal unit; abbreviations beginning with a capital letter denote a multiple of the unit.

Measures of Length.

83. TABLE.

10 **millimeters** (mm)	= 1 **centimeter** (cm).
10 centimeters	= 1 decimeter (dm).
10 decimeters	= 1 **meter** (m).
10 meters	= 1 dekameter (Dm).
10 dekameters	= 1 hektometer (Hm).
10 hektometers	= 1 **kilometer** (Km).
10 kilometers	= 1 myriameter (Mm).

Measures of Surface.

84. The units of surface are squares whose dimensions are the corresponding linear units; hence it takes 10 times 10, or 100, of one denomination to make one of the next higher. For measuring small surfaces the principal unit is the **square meter.**

TABLE.

100 square millimeters (sq mm)	= 1 **square centimeter** (sq cm).
100 square centimeters	= 1 square decimeter (sq dm).
100 square decimeters	= 1 **square meter** (sq m).
100 square meters	= 1 square dekameter (sq Dm).
100 square dekameters	= 1 square hektometer (sq Hm).
100 square hektometers	= 1 **square kilometer** (sq Km).

In measuring land the square meter is called a **centar** (ca), the square dekameter is called an **ar** (a), and the square hektometer is called a **hektar** (Ha).

Measures of Volume.

85. The units of volume are cubes whose dimensions are the corresponding linear units; hence it takes 10 times 10

times 10, or 1000, of one denomination to make one of the next higher. The principal unit is the **cubic meter.**

TABLE.

1000 cubic millimeters ($^{cu\ mm}$)	= 1 **cubic centimeter** ($^{cu\ cm}$).
1000 cubic centimeters	= 1 cubic decimeter ($^{cu\ dm}$).
1000 cubic decimeters	= 1 **cubic meter** ($^{cu\ m}$).

In measuring wood the cubic meter is called a **ster** (st); one tenth of a cubic meter is a **decister** (^{dst}), and ten cubic meters are a **dekaster** (^{Dst}).

MEASURES OF CAPACITY.

86. The unit of capacity is a **liter,** which equals a cubic decimeter.

TABLE.

10 milliliters (^{ml})	= 1 **centiliter** (^{cl}).
10 centiliters	= 1 deciliter (^{dl}).
10 deciliters	= 1 **liter** (^{l}).
10 liters	= 1 dekaliter (^{Dl}).
10 dekaliters	= 1 **hektoliter** (^{Hl}).
10 hektoliters	= 1 kiloliter (^{Kl}).

WEIGHT.

87. The unit of weight is a **gram,** which equals the weight of a cubic centimeter of water at its greatest density.

TABLE.

10 **milligrams** (^{mg})	= 1 **centigram** (^{cg}).
10 centigrams	= 1 decigram (^{dg}).
10 decigrams	= 1 **gram** (^{g}).

10 grams	= 1 dekagram (^{Dg}).
10 dekagrams	= 1 hektogram (^{Hg}).
10 hektograms	= 1 **kilogram** (^{Kg}) or **kilo** (^{K}).
10 kilograms	= 1 myriagram (^{Mg}).
10 myriagrams	= 1 quintal (^{Q}).
10 quintals	= 1 **tonneau** or **ton** (^{T}).

A cubic centimeter or milliliter of water weighs a gram.
A cubic decimeter or liter of water weighs a kilogram.
A cubic meter or kiloliter of water weighs a ton.

88. A metric number can be reduced to another denomination by simply moving the decimal point. For example, $1945.2^{g} = 1.9452^{Kg}$, because dividing by 10 three times is the same as moving the decimal point three places to the left; $3.726^{m} = 372.6^{cm}$, because multiplying by 10 twice is the same as moving the decimal point two places to the right.

In reducing from a lower to a higher denomination, *move the decimal point to the left as many places as there are intervals in the table between the given denomination and the required denomination.*

In reducing from a higher to a lower denomination, *move the decimal point to the right as many places as there are intervals in the table between the given denomination and the required denomination.*

In measures of surface it takes 100 of one denomination to make one of the next higher; hence the decimal point must be moved two places for every interval.

In measures of volume it takes 1000 of one denomination to make one of the next higher; hence the decimal point must be moved three places for every interval.

EXAMPLES.

1. Reduce 6453^{m} to kilometers.
2. Reduce 4.15^{m} to centimeters.

3. Reduce 6.45^{l} to milliliters.

4. In 9780^{m} how many kilometers?

5. Write 4^{Km}, 5^{Hm}, 2^{m}, and 8^{cm} as meters.

6. Write 84^{Hl}, 8^{l}, and 92^{cl} as liters.

7. Write 1872.6^{m} as kilometers; as centimeters; as millimeters.

8. Write 67.43^{Kg} as tons; as grams; as milligrams.

9. Write 7529^{cl} as liters; as hektoliters.

10. Write 96547^{mg} as grams; as kilograms.

11. Write 7.653^{Hl} as liters; as centiliters.

12. 43720^{mm} equals how many meters? how many centimeters? what fraction of a kilometer?

13. Write 8^{Ha}, 6^{a}, and 72^{ca} as hektars.

14. Write $968.32^{sq\,m}$ as ars; as square centimeters.

15. Write 546.31^{Ha} as square kilometers; as centars.

16. Write $8915200^{sq\,cm}$ as square meters; as ars; as hektars.

17. How many cubic millimeters are there in a cubic dekameter?

18. In $2.15^{cu\,m}$ how many cubic millimeters?

19. Express $2328000^{cu\,cm}$ as sters; as dekasters; as decisters.

20. What is the value in cubic centimeters of 297^{l}? in cubic meters? in cubic kilometers?

21. Write $0.853^{cu\,m}$ as hektoliters; as liters; as centiliters; as cubic centimeters.

22. Write $81470^{cu\,cm}$ as liters; as hektoliters; as cubic meters.

23. Express 29.73^{Hl} as liters; as centiliters; as cubic meters; as cubic centimeters.

24. What is the weight of $276.5^{\text{cu cm}}$ of water?

25. Find the weight in kilograms of $0.0316^{\text{cu m}}$ of water.

26. How many decigrams does a dekaliter of water weigh?

27. What is the weight in kilograms of 12^{Hl} of water?

28. How much will a cubic hektometer of water weigh in kilograms? Express the same quantity of water in liters.

29. What is the amount of 34789.56^{l} of water in cubic centimeters? in cubic meters? in cubic kilometers? its weight in grams? in kilograms?

30. What is the amount of 294.7361^{Hl} of water in cubic meters? in liters? in cubic millimeters? its weight in tons? in grams? in milligrams?

89. All operations in the metric system are performed as in decimal fractions. If metric numbers are expressed in different denominations, they must be reduced to the same denomination before they can be added or subtracted.

EXAMPLES.

1. How many grams are there in 23.45^{Kg} and 15.8^{Dg}?

2. Add together 1.23^{dm}, 306.7^{mm}, 0.5219^{Dm}, and 2.91^{m}, and express the sum in centimeters.

3. Express the sum of 305^{mg}, 218^{Dg}, and 7^{T} in kilograms.

4. Express in square meters $1^{\text{Ha}} + 250^{\text{a}} - 150^{\text{ca}} - 1500^{\text{sq cm}}$.

5. Find the sum of $1871^{\text{cu cm}}$, 541^{l}, and 4.51^{Hl}, and give the answer in liters.

6. Express in cubic meters $7^{\text{cu Dm}} + 54^{\text{Kl}} + .03^{\text{Hl}} + 5400^{\text{cu dm}}$.

7. Multiply 17.28^{g} by 312500, and give the product in kilos.

8. Multiply the sum of 7^{Km}, 823^{m}, and 125^{mm} by 5.12.

9. Divide 3035.25^{cm} by 0.0375.

10. Divide 2700^{Hl} by 90^{cl}.

11. Find the value in cubic decimeters of $\frac{11}{18}$ of $87^{\text{cu m}}$ $62^{\text{cu dm}}$ $300^{\text{cu cm}}$.

12. What will 100^{l} of mercury weigh, mercury being 13.5 times as heavy as water?

13. What weight of mercury will a vessel contain whose capacity is $20^{\text{cu cm}}$?

14. What is the weight of water in a tank if it would take 98 minutes to empty it at the rate of 8.7^{l} a minute? If it were filled with oil at $18.75 a hektoliter, what would the contents be worth?

Rectangular Surfaces and Volumes.

90. For principles see sections 80 and 81.

EXAMPLES.

1. How many square decimeters are there in a board 4^{m} long and 0.4^{m} wide?

2. How many hektars are there in a strip of land 62^{dm} broad and 1.7^{Hm} long?

3. How many centars are there in a sidewalk 0.42^{Km} long and 2.8^{m} wide?

4. How many ars are there in a field 54^{m} long and 28.4^{m} wide?

5. Two rectangular fields are 6^{Hm} long and 19^{Dm} wide and 7^{Hm} long and 18^{Dm} wide respectively; how many more hektars are there in the second than in the first?

6. How many bricks, each 20^{cm} long and 10^{cm} wide, will it take to pave a sidewalk 3.3^{m} wide and 1.7^{Km} long?

7. If a person steps 0.8^{m} at each step, how many steps will he take in walking around a rectangular field, which contains 1080^{Ha}, and whose breadth is 1800^{m}?

8. A rectangular piece of ground is 32^{m} 7^{dm} long and 19^{m} 5^{cm} broad. Find the cost of enclosing it with a path 1^{m} 5^{dm} broad at 3 francs 5 centimes a square meter:

(i) when the path is outside the ground.
(ii) when the path is part of the ground.

9. How many liters are there in a vat 2.8^{m} long, 2^{m} wide, and 5^{dm} deep?

10. How many liters are there in a box 1.2^{m} long, 8^{cm} wide, and 50^{mm} deep?

11. A cistern is 4^{m} long, 24^{dm} wide, and 80^{cm} deep. How much water will it hold in cubic meters? in liters?

12. How many cubic meters of air will a room contain whose length is 5^{m} 2^{dm}, whose breadth is 4^{m}, and whose height is 35^{dm}? What is the amount in liters?

13. There is a bin 7.6^{m} long, 4.3^{m} wide, and 3.86^{m} deep. How many hektoliters of wheat will it contain?

14. How many hektoliters of oats can be put into a bin that is 2^{m} long, 1.3^{m} wide, and 1.5^{m} deep?

15. What is the cost of a pile of wood whose dimensions are 2^{m}, 1.9^{m}, and 42.5^{m}, at $2 per ster?

16. What is the cost of digging a cellar 3^{Dm} wide, 5^{Dm} 4^{m} long, and 2^{m} 6^{dm} deep, at the rate of 50 cts. a ster?

17. How many sters are there in a wall 24^{m} long, 8^{m} 5^{dm} high, and 52^{cm} thick? What would be the cost of building it at $4.25 a cubic meter?

18. What weight of water (in kilograms) may be contained in a cistern 1.75^{m} long, 1.3^{m} broad, and 0.8^{m} deep?

19. How many liters of water can be contained in a cistern 5^{m} long, 3^{m} wide, and 2^{m} deep? What would be the weight of the water in kilograms?

20. A bin is 3.4^{m} long, 1.36^{m} wide, and 0.84^{m} deep. How many kilograms of water will it hold? How many hektoliters of wheat will it contain?

21. How many hektoliters will a bin hold that is 3^{m} long, 22^{dm} wide, and 0.015^{Hm} deep? How many kilograms of water will it hold?

22. Required the weight in centigrams of the water in a vessel 1^{m} 2^{cm} long, 6^{dm} broad, and 5^{dm} 1^{mm} deep.

23. A cistern is 5^{m} long, 36^{dm} wide, and 90^{cm} deep. How much water will it hold in cubic meters? in liters? in cubic centimeters? in grams? in kilograms?

24. A box 2.3^{m} long, 196.7^{cm} broad, and 901.9^{mm} deep contains how many liters? If filled with water, how many grams would the water weigh?

25. A vat is 6.3^{m} long, 3^{m} wide, and 4.2^{m} deep. How long will it take a water-pipe to fill the vat, if the current flows at the rate of 3.6^{Dl} a minute?

26. A cubical cistern is 6^{m} in each dimension. If 1.725^{Hl} of water can flow out per minute, how much must flow in per minute to fill it in an hour?

27. How many grams of a liquid $1\frac{1}{2}$ times as heavy as water will fill a cube whose edge is 20^{cm}? How many liters?

28. Find the weight in grams of a bar of gold 1^{dm} long, $2\frac{1}{2}^{cm}$ wide, and 2^{cm} thick, assuming the bar to be 19 times as heavy as its own volume of water.

29. What must be the length of a pile of wood, 2^{m} high and 1.25^{m} wide, to contain 12^{st}?

30. What must be the length of a box, 1^{m} wide and 1^{m} deep, to contain 4500^{l}?

31. A cistern holding $10\frac{1}{8}^{cu\,m}$ is 25^{dm} wide and 5^{m} long; find its depth in centimeters.

32. A roof 10.5^{m} long by 5.4^{m} wide drains into a tank 1^{m} deep with a base 1.25^{m} by 2.5^{m}. What depth of water must fall on the roof to fill the tank?

33. How many hektars of land can be flooded to the depth of 5^{mm} from a tank holding 1000^{T} of water?

The Metric System Compared with the Common System.

91. The following table of equivalents should be committed to memory, since by its use weights and measures of one system can readily be converted into weights and measures of the other.

1 meter	= 39.37 inches.
1 kilometer	= 0.62138 mile.
1 square meter	= 1550 square inches.
1 hektar	= 2.471 acres.
1 cubic meter	= 1.308 cubic yards.
1 ster	= 0.2759 cord.
1 liter	= 1.0567 liquid quarts,
	= 0.908 dry quart.
1 gram	= 15.432 grains.
1 kilogram	= 2.2046 pounds Avoirdupois.

I. Reduce 34^{a} to square rods.

$$\begin{array}{r} 2.471 \text{ A.} \\ 0.34 \\ \hline 9884 \\ 7413 \\ \hline 0.84014 \text{ A.} \\ 160 \\ \hline 134.42240 \text{ sq. rd.} \end{array}$$

34^{a} must first be reduced to hektars, the denomination given in the equivalent. Since $1^{Ha} = 2.471$ A., 0.34^{Ha} equals 0.34 of 2.471 A., which equals 0.84014 A. Then reducing to square rods, we have 134.4224 sq. rd.

II. Reduce 5 mi. 3 fur. 10 rd. to kilometers.

$$\begin{array}{l} 40\underline{)10.00} \text{ rd.} \\ 8\underline{)3.25} \text{ fur.} \\ 5.40625 \text{ mi.} \end{array}$$

$$\begin{array}{r} 0.62138)5.40625(8.7004^{Km} \\ \underline{497104}\phantom{(8.7004^{Km}} \\ 435210\phantom{(8.7004^{Km}} \\ \underline{434966}\phantom{(8.7004^{Km}} \\ 244000\phantom{(8.70^{Km}} \end{array}$$

The compound number must first be reduced to miles, the denomination given in the equivalent. Since $1^{Km} = 0.62138$ mi., in 5.40625 mi. there are as many kilometers as 0.62138 is contained times in 5.40625, which equals 8.7004^{Km}.

In reducing from metric to common weights and measures, *multiply the number, expressed in the denomination of the equivalent, by the equivalent.*

In reducing from common to metric weights and measures, *divide the number, expressed in the denomination of the equivalent, by the equivalent.*

EXAMPLES.

1. Reduce 600^{Km} to miles.
2. Reduce 10^{m} to feet.
3. Reduce 42.5^{l} to gallons.
4. Reduce 16.75^{Hl} to bushels.
5. Reduce 18^{st} to cords.
6. Reduce 50^{g} to grains.

7. Reduce 126^{g} to ounces Avoirdupois.

8. Reduce 20^{Kg} to pounds Avoirdupois.

9. Find the length of a centimeter in inches.

10. Find the number of pints in a dekaliter.

11. Reduce 36^{Hl} to bushels, pecks, quarts, and pints.

12. Reduce 40.0973^{Km} to miles, rods, feet, and inches.

13. Reduce 12^{Kg} to Troy Weight.

14. Reduce 250^{Kg} to Avoirdupois Weight.

15. Reduce to pounds Avoirdupois and the decimal of a pound $2^{T}\ 4^{Kg}\ 165^{g}$.

16. How many kilometers make a mile?

17. How many hektars make a square mile?

18. Reduce 80 A. 40 sq. rd. to hektars.

19. Reduce 87 bu. 3 pk. 4 qt. to hektoliters.

20. Reduce $68\frac{3}{4}$ yd. to meters.

21. How many liters are there in 6 gal. of water?

22. How many meters are there in 25 ft.?

23. 2 gal. 3 qt. $1\frac{1}{3}$ pt. equal how many liters?

24. 2 mi. 17 ft. 16.2 in. equal how many kilometers?

25. 3 mi. 2 rd. 10 ft. equal how many meters?

26. How many kilometers are there in 2 mi. 6 fur. 39 rd. 5 yd.?

27. How many hektoliters are there in 57 gal. $3\frac{1}{2}$ pt.?

28. If the distance between two places is $1\frac{1}{8}$ miles, what is the number of kilometers? of centimeters?

29. Find the equivalent of the rod in the hektometers.

30. A lead pencil is $1\frac{3}{4}^{dm}$ long; 32186 of them arranged in a line would extend how many miles?

31. Find the weight in grams of a quart of water.

32. Find the weight in grams of a cubic yard of water.

33. Find the number of kilograms in a cubic foot of water.

34. Find the weight in kilos of 15 gal. of water.

35. A cubic foot is what part of a cubic meter?

36. Find the weight in kilograms of a block containing 12516 cu. in., assuming the weight of a cubic inch of the material to be 2 oz.

37. A train is 27 min. in passing through a tunnel, the length of which is 11220^{m}; find the speed of the train in miles per hour.

38. A platform bears a weight of 100 lb. per square foot; what is the weight in kilograms per square meter?

39. Find in acres the area of a plot 300^{m} long and 2^{Hm} wide.

40. How many square rods are there in a field 300^{m} long and $\frac{1}{10}$ of a kilometer wide?

41. How many cubic yards are there in a cistern, the dimensions of which are 64^{dm}, 225^{cm}, and 3.75^{m}?

42. How many hektoliters of grain will a bin hold whose interior length, width, and depth are each 6 ft. 6 in.?

43. Find the value of a pile of wood 4.5^{m} long, 1.4^{m} wide, and 1.8^{m} high, at $3.75 a cord.

44. Find the value of a pile of wood 15 ft. long, 4 ft. wide, and $4\frac{1}{2}$ ft. high, at $1.10 a ster.

45. Albany, N. Y., is in 42° 39′ 50″, and Montreal in 45° 31′ north latitude; find the distance between them in kilometers.

92. When an equivalent is given in a problem, that equivalent, and no other, should be used in the solution. In some cases the equivalent is one not given in the preceding section. Other problems are given to show that it is possible to make all such reductions by using merely the two equivalents, $1^{m} = 39.37$ in. and $1^{g} = 15.432$ gr.

EXAMPLES.

1. The distance from Boston to Albany is 320^{Km}; find the distance in miles, assuming the meter to equal $3\frac{7}{25}$ ft.

2. If one kilometer equals five eighths of a mile, how many turns will a wheel make in 20 mi., the circumference of the wheel being $4^{m}\ 5^{mm}$?

3. The deciliter is 0.026 of a gallon; what will be the weight in grams of a pint of water?

4. One dekagram is 0.3527 oz. Avoirdupois; how many pounds Avoirdupois are there in a quintal?

5. The kilogram equals 2 lb. 8 oz. 3 pwt.; how many centigrams equal one grain?

6. What fractional part of $\frac{1}{24}$ of the Avoirdupois ton is 12^{Kg}, the ounce being equal to 28.35^{g}?

7. A tunnel is 2 mi. 21 ch. 13.2 yd. long. Find its length in meters (1 mi. $= 1.61^{Km}$).

8. If a pound Avoirdupois equals 0.4536^{Kg}, how many grains are there in $3\frac{2}{3}^{g}$?

9. A dekaliter is 2.6 gal. What will be the weight in grams of a quart of distilled water at its greatest density?

10. If a quart is $\frac{35}{37}$ of a liter, how many quarts are there in a box 2^{m} long, 17^{dm} wide, and 80^{cm} deep?

11. Given that a meter equals 3.2809 ft.; find how many square meters there are in 1000 sq. yd.

12. Find the number of cubic inches (to the nearest tenth) in the British imperial gallon, which contains 10 lb. of water. (Given $1^{cu\,m} = 35.3$ cu. ft. and $1^{Kg} = 2.2$ lb.)

13. The gram contains 15.432 gr. How many pounds Avoirdupois make a myriagram?

14. The meter equals 39.37 in.; compare the kilometer with the mile.

15. The meter equals 39.37 in.; compute from this datum the value of 4 mi. in kilometers.

16. The meter equals 39.37 in.; express in the metric system 1 ft. and 1 sq. ft.

17. The meter equals 39.37 in.; find how many hektars make an acre.

18. One centimeter equals 0.3937 in.; find how many cubic meters there are in a cord of wood.

19. Find the weight in grams of a cubic yard of water ($1^m = 39.37$ in.).

20. How many liters are there in 10 gal. 3 qt. 1 pt. 3 gi., the gallon being 231 cu. in., and the meter 39.37 in.?

21. A cubical vat measures 9 ft. in each direction; what is its capacity in liters? (Given $1^m = 39.37$ in.)

22. How many liters are contained in a cubical box 13 in. long, 13 in. wide, and 13 in. deep on the inside? (Given $1^m = 39.37$ in.) How many grams of water will such a box hold?

MISCELLANEOUS EXAMPLES.

1. A tank can be emptied in 86 min. by a pipe flowing at the rate of 10.8^l a minute; find the value of the contents of the tank when filled with oil worth $15.25 a hektoliter.

2. If a ream of paper is 11.76^{cm} thick, find the thickness in millimeters of a single sheet.

3. A speculator bought 18.54^{a} of land for $2500, and sold it for $4.50 a square meter; find the amount he gained.

4. The distance between two places measured on a map is 156^{mm}. What is the distance in kilometers if the scale of the map is 1 to 80000?

5. The scale of a map is 4^{cm} to a hektometer. The distance between two points, measured on the map, is 354.2^{mm}; what is the actual distance between the points in kilometers?

6. The area of a court is $50.7^{sq\,m}$. How many square slabs of marble, each $150^{sq\,cm}$ on the surface, will pave it?

7. A man bought a piece of land 52.5^{m} square for $120; at what price per ar must it be sold to gain $56.40?

8. A man bought 30^{m} of cloth at $2.50 per meter; at what price per yard must it be sold to gain $25?

9. A man buys 454 bu. of wheat for $3 a bushel, and sells the wheat at $8.75 a hektoliter; how much does he gain?

10. A merchant buys $2\frac{2}{5}^{Hm}$ of silk for $480, and sells the silk at $1.95 a yard; does he gain or lose, and how much?

11. If a man pays 600 francs for 75^{l} of wine, what is the price per gallon in United States money?

12. A field is 5^{Km} long and 300^{m} wide; find its area in hektars and in acres.

13. Find how many cubic centimeters there are in a dekaliter of water. Find also how many pounds Avoirdupois there are in the same water.

14. The dimensions of a box are 3.1^{m}, 1.5^{m}, and 0.6^{m}; what is the contents in cubic yards? also in dekasters?

15. A rectangular vessel is 7^{m} long, 4.5^{dm} wide, and 20^{cm} deep; find its capacity in cubic meters, hektoliters, and gallons.

16. A vessel is 3^{dm} long, 20^{cm} wide, and 100^{mm} deep; how many liters of water will it contain? How many grams? How many cubic inches? How many pounds?

17. The water contained in a vessel 2^{dm} long, 30^{cm} wide, and 300^{mm} deep, would weigh how many kilograms? would measure how many cubic inches? how many gallons?

18. A cistern is 4^{m} long, 24^{dm} wide, and 80^{cm} deep.

(i) How much water will it hold in cubic meters?
(ii) How much in liters?
(iii) Find approximately the amount in gallons.
(iv) What would the contents weigh in kilograms?

CHAPTER VII.

SPECIAL PROBLEMS.

Carpeting Rooms.

93. Carpeting comes in rolls, and is sold by the yard or meter. In estimating the amount of carpeting necessary for a floor, we must ascertain the number of strips, and then multiply the length of a strip by the number of strips. In making a carpet, the strips may run either lengthwise or across the room; the former way is the more usual, and should be used in solving problems, except when stated to the contrary.

Oil-cloth and some other materials for floors are sold by the square yard or square meter.

I. How many yards of carpet $\frac{5}{8}$ yd. wide does it require to cover a floor 22 ft. long and 16 ft. wide? How much will be turned under?

$\frac{5}{8}$ yd. $= 1\frac{7}{8}$ ft.

16 ft. $\div 1\frac{7}{8}$ ft. $= 16 \times \frac{8}{15} = \frac{128}{15} = 8\frac{8}{15}$ strips.

22 ft. $\times 9 = 198$ ft. $= 66$ yd. of carpet.

$9 - 8\frac{8}{15} = \frac{7}{15}$.

$\frac{7}{15}$ of $1\frac{7}{8} = \frac{7}{\cancel{15}} \times \frac{\cancel{15}}{8} = \frac{7}{8}$ ft. $= 10\frac{1}{2}$ in. turned under.

It takes as many strips as $1\frac{7}{8}$ ft. is contained times in 16 ft., which equals $8\frac{8}{15}$ strips. But a strip is never split; hence it is necessary to buy 9 strips, and then turn under (or cut off) the excess. Since the length of the room is 22 ft., each strip is 22 ft. long; for 9 strips it takes 9 times 22 ft., which equals 198 ft., or 66 yd.

Since 9 strips are bought and only $8\frac{8}{15}$ strips are necessary to cover the floor, the remainder, $\frac{7}{15}$ of a strip, is turned under (or cut off).

Since the carpet is $1\frac{7}{8}$ ft. wide, $\frac{7}{15}$ of a strip is $\frac{7}{15}$ of $1\frac{7}{8}$ ft., which equals $\frac{7}{8}$ ft., or $10\frac{1}{2}$ in.

II. How many meters of carpet 80^{cm} wide will it require to cover a floor 5.48^{m} long and 4.6^{m} wide, if the strips are to run across the room?

$$.8^{m})\underline{5.48^{m}}$$
$$6.85 \text{ strips.}$$

$$\begin{array}{r} 4.6^{m} \\ \underline{7} \\ 32.2^{m} \end{array}$$

Since the strips run across the room, the width of the carpet runs in the same direction as the length of the room, and we must divide the length of the room by the width of the carpet to find the number of strips. Otherwise the work is the same as in the preceding example.

NOTE. In examples dealing with the common system of weights and measures, common fractions are used. In examples dealing with the metric system, use decimal fractions.

EXAMPLES.

1. How many yards of carpet $\frac{3}{4}$ of a yard wide does it require to cover a floor 17 ft. long and 16 ft. 6 in. wide?

2. How many yards of carpet 22 in. wide will it take to cover a floor 16 ft. by $17\frac{1}{2}$ ft.?

3. A person, purchasing a carpet for a room 21 ft. long and 15 ft. 9 in. wide, chooses a material which is $\frac{3}{4}$ of a yard wide, and the pattern of which is complete in each yard of length. How much carpet must he buy in order that the pattern may be unbroken?

4. How many yards of carpet $\frac{5}{8}$ of a yard wide will it require to cover a floor $18\frac{1}{2}$ ft. long and 14 ft. wide, if the strips are to run across the room?

5. A room is 24 ft. 3 in. long and 16 ft. 4 in. wide. Find the cost of covering it with a carpet $\frac{3}{4}$ of a yard wide at $1.25 per yard.

6. How many yards of carpet $\frac{7}{8}$ of a yard wide will it take to cover a floor 28 ft. long and 17 ft. 9 in. wide, if there is a waste of 4 in. in each strip in matching patterns? How much will be turned under?

7. How many yards of carpet $1\frac{1}{8}$ yd. wide does it require to cover a floor 22 ft. 8 in. long and 13 ft. 6 in. wide, if the strips run across the room? How much will be turned under? Find the cost of the carpet at 69 cts. a yard.

8. How many meters of carpet 90^{cm} wide does it require for a floor 7^{m} long and 5.4^{m} wide?

9. How many meters of carpet 70^{cm} wide will it take to cover a floor 5.56^{m} long and 4.7^{m} wide, if the strips are to run across the room?

10. How many meters of carpet 75^{cm} wide will it require to cover a floor 6^{m} long and 4.8^{m} wide? How much will be turned under?

11. How many meters of carpeting 75^{cm} wide will be needed for a room $5\frac{3}{5}^{m}$ square? How wide a strip must be turned under?

12. A floor 10^{m} by 6.5^{m} is to be covered with a carpet 90^{cm} wide; find the cost at $1.25 per meter.

13. The floor of a room is 5.25^{m} by 4.75^{m}; the carpet is 75^{cm} wide and is $4.25 a meter. Find the cost of the carpet if 3^{m} is wasted in matching the pattern.

14. How many meters of carpet 9^{dm} wide will cover a floor 6^{m} long and 5^{m} 4^{dm} wide? What would be the cost of the carpet at $2.50 a centar?

15. It takes 54 yd. $2\frac{1}{2}$ ft. of carpet to cover a floor $23\frac{1}{2}$ ft. long and $15\frac{3}{4}$ ft. wide; find the width of the carpet.

16. It takes 37.5^{m} of carpet to cover a floor 6.25^{m} by 5.1^{m}; find the width of the carpet.

17. What will it cost to floor a room $17\frac{1}{2}$ ft. long and 16 ft. wide at $1.10 per square yard?

18. Find the cost of covering with oil-cloth a floor 7.56^{m} long and 5.5^{m} wide at $62\frac{1}{2}$ cts. a square meter.

19. Find the cost of covering with linoleum a floor $19\frac{1}{2}$ ft. square at 75 cts. a square yard.

Plastering Rooms.

94. In estimating the amount of plastering for a room, we must take the area of the ceiling and the four walls, and from it subtract the area of the doors, windows, etc. The area of the four walls is the same as that of a rectangle whose dimensions are the perimeter and height of the room.

Note. The length of a base-board equals the perimeter of the room minus the width of the doors.

I. Find the cost of plastering a room 32 ft. long, 21 ft. wide, and $9\frac{1}{2}$ ft. high, if 21 sq. yd. be allowed for doors and windows, at 33 cts. per square yard.

64	32	$.33
42	21	$165\frac{5}{9}$
106	32	9)165
$9\frac{1}{2}$	64	$18\frac{1}{3}$
53	672 sq. ft.	165
954	1007	198
1007 sq. ft.	9)1679 sq. ft.	33
	$186\frac{5}{9}$ sq. yd.	$54.63
	21	
	$165\frac{5}{9}$ sq. yd.	

Twice the length added to twice the width equals the perimeter, and this multiplied by the height equals the number of square feet in the four walls. The area of the ceiling equals the product of the length and breadth. Adding these two results we have 1679 sq. ft., or $186\frac{5}{9}$ sq. yd. Subtracting from this 21 sq. yd., the allowance for doors and windows, we have $165\frac{5}{9}$ sq. yd. If one square yard costs 33 cts., $165\frac{5}{9}$ sq. yd. cost $165\frac{5}{9}$ times 33 cts., which equals $54.63.

EXAMPLES.

1. How many square yards of plastering are there in a room 18 ft. 8 in. long, 14 ft. 6 in. wide, and 9 ft. high, making no allowance for doors and windows?

2. How many square meters of plastering are there in a room 6.4^{m} long, 4.8^{m} wide, and 3^{m} high, making no allowance for doors and windows?

3. Find the cost of plastering a room 28 ft. 8 in. long, 18 ft. wide, and 10 ft. high, if 19 sq. yd. be allowed for doors and windows, at 30 cts. a square yard.

4. Find the cost of plastering a room 8.4^{m} long, 5.2^{m} wide, and 3.5^{m} high, if $17^{sq\,m}$ be allowed for doors and windows, at 38 cts. a square meter.

5. Find the cost of plastering the walls of a room 12 ft. 11 in. square and 9 ft. 3 in. high, allowing for two windows and one door each 6 ft. 2 in. by 2 ft. 4 in., at 28 cts. a square yard.

6. Find the cost of plastering a room 6.4^{m} square and 3.8^{m} high at 42 cts. a square meter. There is a base-board 30^{cm} high, and an allowance of $13.6^{sq\,m}$ is made for doors and windows.

7. Find the cost of plastering a room 17 ft. 4 in. long, 15 ft. 4 in. wide, and 10 ft. 6 in. high, at 35 cts. a square yard. Make allowance for a door 8 ft. by 3 ft. 6 in., three windows each 5 ft. 6 in. by 3 ft., and a base-board 1 ft. 4 in. high.

8. Find the cost of plastering the walls of a room 10.5^{m} long, 8.4^{m} wide, and 4.2^{m} high, at 45 cts. a square meter. Make allowance for a door 2.9^{m} by 1.2^{m}, two windows each 2.4^{m} by 1^{m}, and a base-board 32^{cm} high.

PAPERING ROOMS.

95. Paper is generally sold by the roll. To find the number of rolls necessary for a room, we must divide the area of the walls by the area of one roll. When the ceiling is to be papered, its area should be added to the area of the walls.

I. How many rolls of paper, 7.5^{m} long and 50^{cm} wide, will be required for a room 6.4^{m} long, 5.2^{m} wide, and 3.5^{m} high, deducting $14^{sq\,m}$ for doors and windows? Find the cost of papering the room at 90 cts. a roll and of putting on a border at 12 cts. a meter.

12.8	7.5	\$.90	\$.12
10.4	.50	18	23.2
23.2	3.750	16.20	\$2.784
3.5		2.78	
1160	3.75)67.20(17+	\$18.98	
696	375		
81.20	2970		
14.	2625		
$67.2^{sq\,m}$	345		
	18 rolls.		

The area of the walls is found to be $67.2^{sq\,m}$, and the area of one roll of paper is $3.75^{sq\,m}$. It will require as many rolls as $3.75^{sq\,m}$ is contained times in $67.2^{sq\,m}$, which equals 17 and a fraction. But a fraction of a roll is never sold; hence it is necessary to buy 18 rolls. If 1 roll costs 90 cts., 18 rolls cost 18 times 90 cts., which equals \$16.20. The border runs around the room, and its length equals the perimeter of the room. If 1 meter costs 12 cts., 23.2^{m} cost 23.2 times 12 cts., which equals \$2.78. The entire cost is the sum of \$16.20 and \$2.78, which equals \$18.98.

EXAMPLES.

1. How many rolls of paper, $7\frac{1}{2}$ yd. long and 18 in. wide, will be required for a room 26 ft. long, 20 ft. 6 in. wide, and 9 ft. 4 in. high, deducting 17 sq. yd. for doors and windows?

2. How many rolls of paper, 8^{m} long and 45^{cm} wide, will be required for a room 7.8^{m} long, 5.4^{m} wide, and 3^{m} high, deducting $21^{sq\,m}$ for doors and windows?

3. Find the cost of papering a room 23 ft. 8 in. long, 20 ft. 6 in. wide, and 10 ft. high, with paper, each roll of which is 8 yd. long and 18 in. wide, at $62\frac{1}{2}$ cts. a roll, allowing for a door 7 ft. 6 in. by 3 ft. 3 in., and two windows each 5 ft. 8 in. by 3 ft. 3 in.

4. Find the cost of papering a room 7.5^{m} square and 3.5^{m} high, with paper, each roll of which is 8^{m} long and 50^{cm} wide, at $1.10 a roll, and of putting on a border at 15 cts. a meter. Make allowance for two doors each 2.8^{m} by 1.2^{m}, two windows each 2.2^{m} by 1^{m}, and a base-board 30^{cm} high.

5. Find the cost of papering a room $19\frac{1}{2}$ ft. long, $16\frac{2}{3}$ ft. wide, and $10\frac{1}{2}$ ft. high, with paper, each roll of which is $7\frac{1}{2}$ yd. long and 20 in. wide, at 80 cts. a roll, and of putting on a border at $12\frac{1}{2}$ cts. a yard. Make allowance for a door 8 ft. by $3\frac{1}{4}$ ft., three windows each $5\frac{1}{2}$ ft. by 3 ft., and a base-board $1\frac{1}{2}$ ft. high.

6. Find the cost of papering the walls and ceiling of a room 11.2^{m} long, 9.6^{m} wide, and 5^{m} high, with paper, each roll of which is 7.5^{m} long and 50^{cm} wide, at $1.25 a roll. Make allowance for two doors each 3.5^{m} by 1.5^{m}, three windows each 2.5^{m} by 1.4^{m}, and a base-board 40^{cm} high.

7. How many yards of paper $1\frac{1}{8}$ yd. wide will be needed to paper the walls of a room 10 ft. high, 18 ft. long, and 12 ft. wide?

8. Find the cost of papering a room 11 yd. 2 ft. 4 in. long, 6 yd. 2 ft. wide, and 5 yd. 2 ft. 6 in. high, with paper 1 yd. 4 in. wide, at 6 cts. a yard.

9. Find the cost of papering a room 6.78^{m} long, 4.1^{m} wide, and 3.64^{m} high, with paper 96^{cm} wide, at 8 cts. a meter.

Board Measure.

96. The standard thickness for boards is one inch. A board one inch or less in thickness contains as many **board feet** as there are square feet in its surface. For boards more than one inch thick, and for other kinds of lumber, we must multiply the number of square feet in the surface by the number of inches in thickness in order to find the number of board feet. For example, a board 8 ft. long, 1 ft. wide, and 1 in. or less in thickness, contains 8 board feet; a plank 8 ft. long, 1 ft. wide, and 3 in. thick, contains 24 board feet.

When the metric system is used, the standard thickness is 25^{mm}. A board 25^{mm} or less in thickness contains as many **board meters** as there are square meters in its surface. For boards more than 25^{mm} thick, and for other kinds of lumber, we must multiply the number of square meters in the surface by the number of times 25^{mm} is contained in the thickness. For example, a board 4^{m} long, 50^{cm} wide, and 25^{mm} or less in thickness, contains 2 board meters; a plank 4^{m} long, 50^{cm} wide, and 10^{cm} thick, contains 8 board meters.

If a board is tapering, we must take the average width, which is one half the sum of its two end widths.

In buying and selling boards, it is customary to quote them by the hundred or thousand, meaning a hundred or thousand board feet, or a hundred or thousand board meters.

EXAMPLES.

1. How many board feet are there in a board 16 ft. long, 9 in. wide, and $\frac{7}{8}$ in. thick?

2. How many board feet are there in a board 17 ft. 6 in. long, 1 ft. 3 in. wide, and 1 in. thick?

3. How many feet, board measure, are there in a plank 12 ft. 4 in. long, 2 ft. 3 in. wide, and 4 in. thick?

4. How many feet, board measure, are there in a plank 16 ft. 4 in. long, 1 ft. 7 in. wide, and $4\frac{1}{2}$ in. thick?

5. How many feet, board measure, are there in a plank 12 ft. 4 in. long, 2 ft. 5 in. wide at one end, 2 ft. 1 in. wide at the other, and 4 in. thick?

6. How many feet of board are there in a plank 17 ft. long, 22 in. wide at one end, 13 in. wide at the other, and 3 in. thick?

7. Find the number of board feet in a stick of timber 18 ft. long and 8 in. square.

8. Find the cost of 72 boards, each 11 ft. long, 16 in. wide, and $\frac{3}{4}$ in. thick, at $16.50 per M.

9. Find the cost of 14 joists, each 4 in. by 3 in., and 10 ft. long, at $13.75 per M.

10. Find the cost of the flooring for two rooms, each 24 ft. by $20\frac{1}{2}$ ft., with boards $1\frac{1}{4}$ in. thick, at $27 per M.

11. How many board meters are there in a board 7^{m} long, 18^{cm} wide, and 20^{mm} thick?

12. How many board meters are there in a board 7.5^{m} long, 20^{cm} wide, and 30^{mm} thick?

13. How many meters, board measure, are there in a plank 5.8^{m} long, 30^{cm} wide, and 75^{mm} thick?

14. How many meters, board measure, are there in a plank 6^{m} long, 36^{cm} wide at one end, 30^{cm} wide at the other, and 11^{cm} thick?

15. Find the number of board meters in a stick of timber 9^{m} long and 30^{cm} square.

16. Find the cost of 50 boards, each 4^{m} long. 36^{cm} wide, and 25^{mm} thick, at $18 per C.

17. Find the cost of 24 planks, each 4.5^m long, 42^{cm} wide, and 8^{cm} thick, at $22.50 per C.

18. Find the cost of a stick of timber 10.5^m long, 30^{cm} wide at one end, 25^{cm} wide at the other, and 20^{cm} thick, at $18.50 per C.

Work Problems.

97. Problems concerning work should be solved by considering the fractional part of the work that can be done in a unit of time. For example, if a man can do a piece of work in 8 days, he can do $\frac{1}{8}$ of the work in one day; if a cistern can be filled by a pipe in $2\frac{2}{5}$ hours, $\frac{1}{2\frac{2}{5}}$, or $\frac{5}{12}$, can be filled in one hour.

I. A can do a piece of work in 8 days, working 10 hours a day, and B can do it in 6 days, working 12 hours a day; in how many days of 9 hours each can they together do it?

$\frac{1}{80}+\frac{1}{72}=\frac{9+10}{720}=\frac{19}{720}$.
$1\div\frac{19}{720}=1\times\frac{720}{19}=37\frac{17}{19}$ hr.
$37\frac{17}{19}\div 9=\frac{720}{19}\div 9=\frac{80}{19}=4\frac{4}{19}$ da.

Since A can do the work in 80 hr., he can do $\frac{1}{80}$ of it in 1 hr.; since B can do it in 72 hr., he can do $\frac{1}{72}$ of it in 1 hr.; hence both together can do $\frac{1}{80}+\frac{1}{72}$, or $\frac{19}{720}$, in 1 hr. Since they can do $\frac{19}{720}$ in 1 hr., it will take as many hours to do the whole as $\frac{19}{720}$ is contained times in 1, which equals $37\frac{17}{19}$ hr. Since they work 9 hr. a day, it will take as many days as 9 is contained times in $37\frac{17}{19}$, which equals $4\frac{4}{19}$ da.

II. A cistern can be filled by a pipe in $4\frac{1}{6}$ hours, and can be emptied by another pipe in $6\frac{2}{3}$ hours; if both pipes be opened, in what time will the cistern be filled?

$\frac{1}{4\frac{1}{6}}-\frac{1}{6\frac{2}{3}}=\frac{6}{25}-\frac{3}{20}=\frac{24-15}{100}=\frac{9}{100}$.
$1\div\frac{9}{100}=1\times\frac{100}{9}=11\frac{1}{9}$ hr.

By the first pipe $\frac{6}{25}$ of the cistern will be filled in 1 hr., and by the second pipe $\frac{3}{20}$ will be emptied in the same time; hence, when both pipes are open, $\frac{6}{25}-\frac{3}{20}$, or $\frac{9}{100}$, will

be filled in 1 hr. It will take as many hours to fill the cistern as $\frac{9}{100}$ is contained times in 1, which equals $11\frac{1}{9}$ hr.

EXAMPLES.

1. A can mow a field in $4\frac{1}{2}$ days, and B can mow it in 6 days; how long will it take them both to mow it?

2. A can build a wall in $18\frac{3}{4}$ days, and B in $31\frac{1}{4}$ days; how long will it take them both together to build it?

3. A cistern can be filled by a pipe in $3\frac{1}{2}$ hours, and by another in $2\frac{1}{4}$ hours; how long will it take both together to fill it?

4. A can do a certain piece of work in 6 days, B in 8 days, and C in 9 days. How long will it take them to do it together?

5. A can dig a ditch in 5 days, B in 7 days, and C in 9 days; how long will it take them all together to dig it?

6. A can do a piece of work in 12 days, B in 15 days, and C in 20 days; what fractional part of the work can they together do in 3 days?

7. A cistern has two pipes, one of which can fill it in 3 hours, and the other in 4 hours; a third pipe can empty it in 2 hours. If all three are opened when the cistern is empty, in what time will it be filled?

8. A cistern can be filled by a pipe in 75 min., and can be emptied by another pipe in 30 min.; if the cistern is full, and both pipes are open, in what time will the cistern be emptied?

9. Pipes A and B can fill a cistern in 3 min. and 5 min. respectively, and C can empty it in $7\frac{1}{2}$ min. In what time will the cistern be filled when A, B, and C are all open?

10. A and B can do a piece of work in 12 days. A, working alone, can do the same work in 20 days. How long would it take B to do it?

11. A can do a piece of work in 10 days; A and B can do the same work together in 7 days; in how many days can B, working alone, do the work?

12. A can do a piece of work in 10 days, A and C can do it in 7 days, and A and B can do it in 6 days; in how many days can B and C together do it?

13. A can do $\frac{1}{3}$ of a piece of work in 4 days, B $\frac{1}{4}$ in 5 days, C $\frac{1}{5}$ in 3 days, and D $\frac{1}{6}$ in $1\frac{1}{2}$ days; how long will it take them all to do it?

14. Three men can do a piece of work in 12 hours; A and B can do it in 16 hours, and A and C in 18 hours. What part can B and C do in $9\frac{1}{7}$ hours?

15. A can do a certain piece of work in 10 days, working 8 hours a day. B can do the same work in 9 days, working 12 hours a day. They decide to work together, and to finish the work in 6 days. How many hours a day must they work?

16. A does $\frac{4}{11}$ of a piece of work in 6 days, when B comes along and helps him, and they finish it in 5 days; how long would it take B alone to do the work?

17. A can do as much work in 4 hours as B in 6, and B in $3\frac{1}{2}$ as C in 5. A does half a certain piece of work in 12 hours; in what time can it be finished by B and C, working separately equal times?

18. A and B can do a piece of work in 6 days, A and C in 7 days, and B and C in 8 days; in what time can all three do it, working together, and in what time can each one do it alone?

Clock Problems.

98. In twelve hours the minute hand of a clock passes over the face twelve times, and if the hour hand were stationary, the two hands would be together twelve times. But in this interval of twelve hours, the hour hand, instead of remaining stationary, passes over the face once; hence the two hands are together once less than twelve times, or eleven times. Since the two hands progress with a regular movement, there always is the same interval between two successive times when the hands are together, and this interval is $\frac{1}{11}$ of 12 hours, which equals 1 hr. $5\frac{5}{11}$ min. It is also true of any other position of the hands, that there is an interval of 1 hr. $5\frac{5}{11}$ min. between two successive times when the hands have the same relative position.

I. At what time between 7 and 8 o'clock are the hands of a clock together?

$$\begin{array}{l}
1 \text{ hr. } 5\frac{5}{11} \text{ min.} \\
\quad\quad 7 \\
\hline
7 \text{ hr. } 38\frac{2}{11} \text{ min.}
\end{array}$$

Ans. $38\frac{2}{11}$ min. past 7 o'clock.

The hands are together at 12 o'clock, and there are 7 intervals from 12 o'clock to the required time. Since each interval is 1 hr. $5\frac{5}{11}$ min., 7 intervals equal 7 times 1 hr. $5\frac{5}{11}$ min., which equals 7 hr. $38\frac{2}{11}$ min. Hence the required time is $38\frac{2}{11}$ min. past 7 o'clock.

II. At what time between 5 and 6 o'clock are the hands of a clock at right angles?

$$\begin{array}{l}
1 \text{ hr. } 5\frac{5}{11} \text{ min.} \\
\quad\quad 2 \\
\hline
2 \text{ hr. } 10\frac{10}{11} \text{ min.} \\
3 \\
\hline
5 \text{ hr. } 10\frac{10}{11} \text{ min.}
\end{array}
\qquad
\begin{array}{l}
1 \text{ hr. } 5\frac{5}{11} \text{ min.} \\
\quad\quad 8 \\
\hline
8 \text{ hr. } 43\frac{7}{11} \text{ min.} \\
9 \\
\hline
5 \text{ hr. } 43\frac{7}{11} \text{ min.}
\end{array}$$

Ans. $10\frac{10}{11}$ min. past 5 o'clock, or $43\frac{7}{11}$ min. past 5 o'clock.

The hands are at right angles at 3 o'clock and at 9 o'clock. There are 2 intervals from 3 o'clock to the required time; hence we add 2 hr. $10\frac{10}{11}$ min. to 3 o'clock to obtain one answer. There are 8 intervals from 9 o'clock to the required time; hence we add 8 hr. $43\frac{7}{11}$ min. to 9 o'clock to obtain the other answer, writing 5 hr. instead of 17 hr., because 17 is 5 more than 12.

III. Find when first after 1 o'clock the hands of a clock make an angle of 60° with each other.

$$\begin{array}{l} 1 \text{ hr. } \ 5\tfrac{5}{11} \text{ min.} \\ \qquad\quad 3 \\ \hline 3 \text{ hr. } 16\tfrac{4}{11} \text{ min.} \\ 10 \\ \hline 1 \text{ hr. } 16\tfrac{4}{11} \text{ min.} \end{array}$$

Ans. $16\frac{4}{11}$ min. past 1 o'clock.

The hands of a clock make an angle of 60° with each other at 2 o'clock and at 10 o'clock. When first after 1 o'clock they make an angle of 60° with each other, they have the same relative position as at 10 o'clock. There are 3 intervals from 10 o'clock to the required time; hence we add 3 hr. $16\frac{4}{11}$ min. to 10 o'clock to obtain the answer.

EXAMPLES.

1. At what time between 4 and 5 o'clock are the hands of a clock together?

2. At what time between 6 and 7 o'clock are the hands of a clock together?

3. At what time between 8 and 9 o'clock are the hands of a clock together?

4. At what time between 12 and 1 o'clock are the hands of a clock opposite each other?

5. At what time between 3 and 4 o'clock are the hands of a clock opposite each other?

6. At what time between 9 and 10 o'clock are the hands of a clock opposite each other?

7. At what time between 2 and 3 o'clock are the hands of a clock at right angles?

8. At what time between 4 and 5 o'clock are the hands of a clock at right angles?

9. At what time between 9 and 10 o'clock are the hands of a clock at right angles?

10. At what time between 11 and 12 o'clock are the hands of a clock at right angles?

11. Find when first after 11 o'clock the hands of a clock make an angle of 30° with each other.

12. Find when first after 2 o'clock the hands of a clock make an angle of 60° with each other.

13. Find when first after 6 o'clock the hands of a clock make an angle of 120° with each other.

14. Find when first after 4 o'clock the hands of a clock make an angle of 150° with each other.

Comparison of Thermometers.

99. There are two important points to be determined in the graduation of a thermometer, — the freezing point and the boiling point of water.

In the Fahrenheit scale the freezing point is marked 32°, and the boiling point 212°; the intervening space is divided into 180 equal parts called degrees.

In the Centigrade scale the freezing point is marked 0°, and the boiling point 100°; the intervening space is divided into 100 degrees.

In the Réaumur scale the freezing point is marked 0°, and the boiling point 80°; the intervening space is divided into 80 degrees.

In expressing temperatures it is customary to indicate the scale referred to by the initial letters F., C., and R.

Temperatures below 0° are indicated by the minus sign. For example, $-15°$ C. indicates 15° below 0°; $-10°$ F. indicates 10° below 0° or 42° below the freezing point.

Since 180 Fahrenheit degrees = 100 Centigrade degrees = 80 Réaumur degrees, 9 Fahrenheit degrees = 5 Centigrade degrees = 4 Réaumur degrees

I. Express 95° F. in Centigrade scale.

$$\begin{array}{l} 95 \\ \underline{32} \\ 63 \end{array} \qquad \frac{5}{\cancel{9}} \text{ of } \overset{7}{\cancel{63}} = 35° \text{ C.}$$

Subtracting 32 from 95, we find that 95° F. is 63 degrees above the freezing point. Since 9 Fahrenheit degrees = 5 Centigrade degrees, there are $\frac{5}{9}$ as many Centigrade degrees as Fahrenheit degrees; hence 95° F. is 35° above the freezing point in the Centigrade scale, or 35° C.

II. Express 45° C. in the Fahrenheit scale.

$$\frac{9}{\cancel{5}} \text{ of } \overset{9}{\cancel{45}} = 81.$$

$$81 + 32 = 113° \text{ F.}$$

Since 9 Fahrenheit degrees = 5 Centigrade degrees, there are $\frac{9}{5}$ as many Fahrenheit degrees as Centigrade degrees; hence 45° C. is 81° above the freezing point in the Fahrenheit scale, or 113° F.

EXAMPLES.

1. Express 86° F. in the Centigrade scale.
2. Express 68° F. in the Centigrade scale.
3. Express 23° F. in the Centigrade scale.
4. Express −4° F. in the Centigrade scale.
5. Express 60° C. in the Fahrenheit scale.
6. Express 15° C. in the Fahrenheit scale.
7. Express −10° C. in the Fahrenheit scale.
8. Express −30° C. in the Fahrenheit scale.
9. Express 50° F. in the Réaumur scale.
10. Express 14° F. in the Réaumur scale.
11. Express 24° R. in the Fahrenheit scale.
12. Express −16° R. in the Fahrenheit scale.
13. Express 60° C. in the Réaumur scale.
14. Express −25° C. in the Réaumur scale.
15. Express 36° R. in the Centigrade scale.
16. Express −16° R. in the Centigrade scale.

SPECIFIC GRAVITY.

100. The **specific gravity** (sp. gr.) of any substance is its weight compared with the weight of an equal bulk of water. Since water is the standard, its specific gravity is 1. The specific gravity of any other substance denotes the number of times it is heavier than water. For example, if a bar of silver has a specific gravity of 10.5, it is 10.5 times as heavy as an equal bulk of water.

In the metric system of weights and measures, the weight of any bulk of water can readily be found by remembering that 1 cubic meter of water weighs 1 metric ton, 1 liter weighs 1 kilogram, and 1 cubic centimeter weighs 1 gram. In the common system of weights and measures, 1 cubic foot of water weighs 1000 ounces Avoirdupois.

When the bulk and specific gravity of a substance are known, the weight of the substance can be found by multiplying the weight of an equal bulk of water by the specific gravity.

When the bulk and weight of a substance are known, the specific gravity of the substance can be found by dividing the weight of the substance by the weight of an equal bulk of water.

When the weight and specific gravity of a substance are known, the weight divided by the specific gravity equals the weight of an equal bulk of water, and from this the bulk of the substance can readily be found.

All bodies weigh less in water than in air. It can be proved by experiment that the difference between the weight of a body in air and its weight in water equals the weight of the water displaced. Hence, if the weight of a body in air be divided by the difference between its weight in air and its weight in water, the result is the specific gravity.

I. Find the weight of a bar of copper (sp. gr. 8.79) 2 ft. long and 3 in. square.

$$\cancel{2}\times\frac{1}{\underset{2}{\cancel{4}}}\times\frac{1}{4}=\frac{1}{8}\text{ cu. ft.}$$

$$\frac{1}{\cancel{8}}\text{ of }\overset{125}{\cancel{1000}}=125\text{ oz.}$$

$$\begin{array}{r} 125 \\ \underline{8.79} \\ 1125 \\ 875 \\ \underline{1000} \\ 16)\overline{1098.75}\text{ oz.} \\ \overline{68.672}\text{ lb.} \end{array}$$

The dimensions expressed in feet are 2 ft., $\frac{1}{4}$ ft., and $\frac{1}{4}$ ft., and the product of these dimensions gives $\frac{1}{8}$ cu. ft. as the cubic contents. The weight of an equal bulk of water is $\frac{1}{8}$ of 1000 oz., or 125 oz. Multiplying this result by 8.79, we find the weight of the copper to be 1098.75 oz., which equals 68.672 lb.

II. If 650 $^{\text{cu cm}}$ of ether weigh 468$^{\text{g}}$, what is its specific gravity?

$$\begin{array}{l} 650)468.00(0.72 \\ \underline{4550} \\ 1300 \\ \underline{1300} \end{array}$$

650 $^{\text{cu cm}}$ of water weigh 650$^{\text{g}}$; hence the specific gravity of ether is as much as 650 is contained in 468, which equals 0.72.

III. Find the bulk of a piece of coal (sp. gr. 1.8) which weighs 56.88$^{\text{Kg}}$.

$$\begin{array}{l} 1.8)56.88(31.6 \\ \underline{54} \\ 28 \\ \underline{18} \\ 108 \\ \underline{108} \end{array}$$

Ans. 31600$^{\text{cu cm}}$.

Since the specific gravity of coal is 1.8, the weight of an equal bulk of water is as much as 1.8 is contained in 56.88$^{\text{Kg}}$, which equals 31.6$^{\text{Kg}}$, or 31600$^{\text{g}}$. The bulk of 31600$^{\text{g}}$ of water is 31600$^{\text{cu cm}}$, which is also the bulk of the coal.

EXAMPLES.

1. Find the weight of a cubic foot of ice (sp. gr. 0.92).

2. Find the weight of a gallon of milk (sp. gr. 1.03).

3. Find the weight in grains of a cubic inch of iron (sp. gr. 7.21).

4. Find the weight of a bar of platinum 10 in. long, 4 in. wide, and $1\frac{1}{2}$ in. thick, if its specific gravity is 22.07.

5. A tank is 6 ft. long, 4 ft. wide, and 3 ft. deep. How many pounds of sulphuric acid (sp. gr. 1.84) will it contain?

6. Find the specific gravity of a stone, a cubic foot of which weighs 185 lb.

7. Find the specific gravity of a liquid weighing 10 lb. per gallon.

8. A bar of gold 3 in. long, $1\frac{1}{2}$ in. wide, and $\frac{1}{2}$ in. thick weighs 25 oz. Avoirdupois; find its specific gravity.

9. A piece of iron weighs 12 lb. in air and $10\frac{14}{31}$ lb. in water; find its specific gravity.

10. Find the number of cubic inches in a pound of aluminium (sp. gr. 2.64).

11. Find the number of bushels in a ton of salt (sp. gr. 2.15).

12. A piece of glass weighs 4320 gr. in air and 3195 gr. in water; what is its specific gravity? its volume?

13. Find the weight of 58^{l} of sand (sp. gr. 1.65).

14. Find the weight of 6.32^{Hl} of olive oil (sp. gr. 0.915).

15. A plank is 5^{m} long, 3^{dm} wide, and 3^{cm} thick; find its weight in grams, if the specific gravity of the wood is 0.8.

16. A tank is 1.5^{m} wide, 3.2^{m} long, and 80^{cm} deep. How many kilograms of alcohol (sp. gr. 0.8) will be required to fill it one third full?

17. What is the weight in metric tons of a block of stone (sp. gr. 2.5) measuring 12.37^{m} by 7.14^{m} by 83^{cm}?

18. Find the specific gravity of an acid weighing 1.58^{Kg} per liter.

19. If 27^{l} of alcohol weigh 22.14^{Kg}, what is its specific gravity?

20. A brick 20^{cm} long, 11^{cm} wide, and 5.5^{cm} thick weighs 2.904^{Kg}; find its specific gravity.

21. A plate of iron 137^{cm} long, 643^{mm} wide, and 43^{mm} thick weighs 277.54^{Kg}. What is its specific gravity?

22. A stone weighs 8.42^{Kg} in air and 5.32^{Kg} in water; find its specific gravity.

23. A body weighs 460^{g} in air and 401.16^{g} in water; what is its specific gravity?

24. Find the number of cubic centimeters in a piece of brass (sp. gr. 8.38) weighing 86.733^{g}.

25. If alcohol (sp. gr. 0.81) costs $1.45 a kilogram, what is the price of a liter?

26. If salt (sp. gr. 2.15) costs $7.50 a metric ton, what is the price of a hektoliter?

27. If cork (sp. gr. 0.24) is worth $2\frac{1}{2}$ cts. a cubic decimeter, find the value of 10^{Kg}.

28. If marble (sp. gr. 2.83) is worth $28.50 a cubic meter, find the value of a block weighing 764^{Kg}.

Longitude and Time.

101. The **longitude** of a place is the arc or portion of the equator between a standard meridian and the meridian of the given place. A place is in east or west longitude, according as it is east or west of the standard meridian, and the longitude is reckoned in degrees, minutes, and seconds up to 180°, or half way round the earth. For example, long. 32° 25′ W. indicates a place situated on the meridian which is 32° 25′ west of the standard meridian. The meridian of

Greenwich, England, is usually taken as the standard by English-speaking people.

When two places are on the same side of the standard meridian, the difference of longitude is found by subtracting their longitudes; when two places are on opposite sides of the standard meridian, the difference of longitude is found by adding their longitudes. If, however, in the latter case, the sum exceeds 180°, it must be subtracted from 360° to obtain the correct difference of longitude.

The earth revolves on its axis once in 24 hours, thus making 360° of longitude pass under the sun in that time. In 1 hr. $\frac{1}{24}$ of 360°, or 15°, pass under the sun; in 1 min. $\frac{1}{60}$ of 15°, or 15′; in 1 sec. $\frac{1}{60}$ of 15′, or 15″. Hence a difference of 15° of longitude causes a difference of 1 hr. of time; a difference of 15′ of longitude causes a difference of 1 min. of time; a difference of 15″ of longitude causes a difference of 1 sec. of time.

I. The difference of time between two places is 2 hr. 15 min. 27 sec.; what is the difference of longitude?

2 hr.	15 min.	27 sec.
		15
33°	51′	45″

Since 1 hr. of time corresponds to 15° of longitude, 1 min. of time to 15′ of longitude, and 1 sec. of time to 15″ of longitude, 15 times the number of hours, minutes, and seconds equals the number of degrees, minutes, and seconds.

II. The difference of longitude between two places is 48° 24′ 36″; what is the difference of time?

15)48°	24′	36″
3 hr.	13 min.	$38\frac{2}{5}$ sec.

Since 15° of longitude corresponds to 1 hr. of time, 15′ of longitude to 1 min. of time, and 15″ of longitude to 1 sec. of time, $\frac{1}{15}$ of the number of degrees, minutes, and seconds equals the number of hours, minutes, and seconds.

EXAMPLES.

1. The difference of time between two places is 5 hr. 42 min. 22 sec.; what is the difference of longitude?

2. The difference of time between St. Petersburg and St. Paul is 8 hr. 13 min. 36 sec.; what is the difference of longitude?

3. The difference of time between Boston and St. Louis is 1 hr. 16 min. 47 sec.; what is the difference of longitude?

4. The time in Montreal is 4 hr. 53 min. $56\frac{4}{5}$ sec. earlier than in London; what is the difference of longitude between the two places?

5. The time in Berlin is 44 min. $14\frac{1}{15}$ sec. later than in Paris; what is the difference of longitude between the two places?

6. The difference of longitude between two places is 71° 4′; what is the difference of time?

7. Find the difference of time between New York (long. 74° 0′ 3″ W.) and San Francisco (long. 122° 25′ 40″ W.).

8. Find the difference of time between Ottawa (long. 75° 42′ 4″ W.) and Washington (long. 77° 2′ 48″ W.).

9. Find the difference of time between Bombay (long. 72° 54′ E.) and Cape of Good Hope (long. 18° 29′ E.).

10. Find the difference of time between Constantinople (long. 28° 59′ 14″ E.) and Quebec (long. 71° 13′ 45″ W.).

11. Find the difference of time between Canton (long. 113° 14′ E.) and Chicago (long. 87° 37′ 30″ W.).

12. Find the difference of time between Boston (long. 71° 3′ 30″ W.) and St. Paul (long. 93° 5′ W.).

13. Find the difference of time between Pekin (long. 116° 27′ E.) and New York (long. 74° 0′ 3″ W.)

14. Find the difference of time between Rome (long. 12° 28′ 40″ E.) and Paris (long. 2° 20′ 14″ E.).

102. The earth revolves on its axis from west to east, and the sun seems to move from east to west. Of any two places, the sun rises earlier at the place farther east, and since the sun rises earlier, the clock-time is later. Hence, to find the clock-time of a given place when the clock-time of another place and their difference of time are known, *add the difference of time to the given time, when the place whose time is to be found is farther east; subtract the difference of time from the given time, when the place whose time is to be found is farther west.*

I. When it is 12 min. 30 sec. past 2 P.M. at Berlin (long. 13° 23′ 45″ E.), what is the time at New York (long. 74° 0′ 3″ W.)?

13° 23′ 45″
74° 0′ 3″
15)87° 23′ 48″
5 hr. 49 min. $35\frac{1}{5}$ sec.

2 hr. 12 min. 30 sec. P.M.
5 49 $35\frac{1}{5}$
8 hr. 22 min. $54\frac{4}{5}$ sec. A.M.

Ans. 22 min. $54\frac{4}{5}$ sec. past 8 A.M.

The difference of time is found to be 5 hr. 49 min. $35\frac{1}{5}$ sec. Since New York is west of Berlin, the time is earlier; hence we subtract 5 hr. 49 min. $35\frac{1}{5}$ sec. from 2 hr. 12 min. 30 sec. P.M. 2 hr. after noon is the same as 14 hr. after midnight, and as we cannot subtract 5 hr. from 2 hr., we subtract it from 14 hr., and write A.M. instead of P.M.

II. What is the longitude of a place whose time is 48 min. past 8 P.M., when it is half past 6 P.M. at Rome (long. 12° 28′ 40″ E.)?

8 hr. 48 min.
6 30
2 hr. 18 min.
15
34° 30′
12° 28′ 40″
46° 58′ 40″ E.

The difference of longitude is found to be 34° 30′. Since the time is later, the place is east of Rome; hence we add 34° 30′ to 12° 28′ 40″.

EXAMPLES.

1. Bangor is 15° 39′ east of Cincinnati; what time is it at Bangor when it is 5 o'clock P.M. at Cincinnati?

2. The longitude of Berlin is 13° 23′ 45″ E.; what time is it at Greenwich when it is midnight at Berlin?

3. What is the time at Canton (long. 113° 14′ E.) when it is noon at Greenwich?

4. The longitude of Boston is 71° 3′ 30″ W., and of Paris 2° 20′ 14″ E.; when it is 10 o'clock A.M. at Boston, what time is it at Paris?

5. The longitude of St. Petersburg is 30° 19′ E., and of New York 74° 0′ 3″ W.; when it is 1 o'clock P.M. at St. Petersburg, what time is it at New York?

6. When it is 10 o'clock at Boston, what time is it at Amherst, the longitude of Boston being 71° 3′ 30″ W., and that of Amherst being 72° 31′ 50″ W.?

7. The longitude of Boston is 71° 3′ 30″ W., and that of San Francisco is 122° 25′ 40″ W. When it is noon at Boston, what is the time at San Francisco?

8. Find what time it is at Cape of Good Hope (long. 18° 29′ E.) when it is noon at St. Paul (long. 93° 5′ W.).

9. When it is 6 min. 15 sec. past 4 A.M. at Pekin (long. 116° 27′ E.), what is the time at London (long. 5′ 48″ W.).

10. What is the longitude of a place whose time is 42 min. 42 sec. past 8 P.M. when it is midnight at Greenwich?

11. What is the longitude of a place whose time is 6 o'clock A.M. when it is quarter past 4 A.M. at Washington (long. 77° 2′ 48″ W.)?

12. What is the longitude of a place whose time is 35 min. past 10 A.M. when it is 5 o'clock P.M. at Paris (long. 2° 20′ 14″ E.)?

13. When it is noon at St. Paul (long. 93° 5′ W.), it is 37 min. 12 sec. past 1 P.M. at Bangor; what is the longitude of Bangor?

14. When it is 9 o'clock P.M. at Calcutta (long. 88° 20′ E.), it is 27 min. 19$\frac{2}{5}$ sec. past 5 P.M. at Jerusalem; what is the longitude of Jerusalem?

NOTE. The time considered in the preceding problems is the actual local time, but nearly all railroads, cities, and towns of the United States now use *standard time*, which is the time of some particular meridian. The meridians selected are those which are respectively 75°, 90°, 105°, and 120° west of Greenwich. The time of the meridian 75° W. is known as Eastern standard time; that of 90° W. is Central standard time; that of 105° W. is Mountain standard time; and that of 120° W. is Pacific standard time. By this method, when there is any difference of time between two places, the difference is one, two, or three hours, and all confusion arising from different local times is thereby avoided.

CHAPTER VIII.

RATIO AND PROPORTION.

Ratio.

103. The relation between two numbers is called their **ratio**, and it is determined by dividing the first by the second. The sign of ratio is the colon (:), which is the sign of division with the line omitted. For example, $6:4$ is read the *ratio of 6 to 4*, or *6 is to 4*, and its value is $6 \div 4$, or $\frac{6}{4}$.

The two numbers whose values are compared are called the **terms** of the ratio, and together they form a **couplet**. The first term is called the **antecedent**, and the second term is called the **consequent**.

A ratio can exist between two concrete numbers only when they are expressed in terms of the same unit, and the ratio is equal to the ratio of the corresponding abstract numbers. For example, 8 pt. : 15 pt. equals $8:15$.

When each term of a ratio is a single number, it is called a **simple ratio**. The product of two or more simple ratios is called a **compound ratio**. A simple ratio having a fraction in either term is also called a **complex ratio**. For example, $8:13$ and $2\frac{1}{2}:\frac{5}{8}$ are simple ratios, the latter of which is complex; $\left.\begin{matrix}6:10\\9:12\end{matrix}\right\}$ is a compound ratio.

Since antecedent and consequent bear the same relation to each other as dividend and divisor, both terms of a ratio may be multiplied or divided by the same number without affecting the value of the ratio. A complex ratio can be

simplified by multiplying both terms by their least common denominator.

A compound ratio can be simplified by taking the product of the antecedents for a new antecedent, and the product of the consequents for a new consequent.

When the antecedent and consequent of a ratio are interchanged, the resulting ratio is called the **inverse** of the given ratio.

I. Reduce $2\frac{1}{3} : 5\frac{1}{4}$ to a simple ratio.

$2\frac{1}{3} : 5\frac{1}{4}$.
$28 : 63$.
$4 : 9$.

Multiplying both terms by 12, we obtain $28:63$; and then dividing both terms by 7, we obtain $4:9$ as the simplest value.

II. Which is the greater ratio, $7:8$ or $8:9$?

$7:8 = \frac{7}{8} = \frac{63}{72}$.
$8:9 = \frac{8}{9} = \frac{64}{72}$.
$8:9$ is the larger.

Expressing the ratios in a fractional form, we have $\frac{7}{8}$ and $\frac{8}{9}$, which, after reducing to their least common denominator, equal $\frac{63}{72}$ and $\frac{64}{72}$. $\frac{64}{72}$ is greater than $\frac{63}{72}$; hence $8:9$ is greater than $7:8$.

EXAMPLES.

1. Reduce $3\frac{3}{8} : 5\frac{1}{7}$ to a simple ratio.

2. Reduce $7\frac{1}{2} : \dfrac{12}{7\frac{1}{3}}$ to a simple ratio.

3. Reduce $\left.\begin{matrix} 3:7 \\ 8:9 \end{matrix}\right\}$ to a simple ratio.

4. Reduce $\left.\begin{matrix} \frac{2}{3}:1\frac{1}{2} \\ 8\frac{1}{3}:6\frac{1}{4} \end{matrix}\right\}$ to a simple ratio.

5. Find the ratio of 2 pk. to 3 qt.

6. Find the ratio of 4 gal. to 1 cu. ft.

7. Find the ratio of a field 15 rd. long and 11 rd. wide to a field 14 rd. long and 12 rd. wide.

8. Which is the greater ratio, $7:11$ or $8:12$?

9. Which is the greater ratio, $\frac{2}{7} : \frac{3}{8}$ or $4\frac{2}{7} : 4\frac{3}{8}$?

10. Which is the greater ratio, $2.50 : $3.75 or 8 ft. : 12 ft.?

Simple Proportion.

104. An expression of equality between the two ratios is called a **proportion**, and the four terms are called **proportionals**. When a proportion consists of two simple ratios, it is called a **simple proportion**.

A proportion is indicated by putting a double colon (: :) or a sign of equality (=) between the two ratios. For example, 4 : 6 : : 10 : 15 is read *4 is to 6 as 10 is to 15;* 4 : 6 = 10 : 15 is read *the ratio of 4 to 6 equals the ratio of 10 to 15.*

The first and fourth terms of a proportion are called the **extremes**, and the second and third terms are called the **means**.

Three numbers are said to be in proportion when the ratio of the first to the second equals the ratio of the second to the third. The second number is called a **mean proportional** between the other two. For example, in the proportion 2 : 6 : : 6 : 18, 6 is a mean proportional between 2 and 18.

The solution of problems in proportion depends on the following principle: — *In any proportion the product of the extremes equals the product of the means.* This can be proved in any proportion by expressing the ratios in fractional form, and then multiplying both of them by the product of the denominators. As an illustration, take the proportion 2 : 3 : : 4 : 6.

2 : 3 : : 4 : 6.

$$\frac{2}{3} = \frac{4}{6}.$$

$$\frac{2 \times \cancel{3} \times 6}{\cancel{3}} = \frac{4 \times 3 \times \cancel{6}}{\cancel{6}}.$$

$$2 \times 6 = 4 \times 3.$$

The proportion written in fractional form becomes $\frac{2}{3} = \frac{4}{6}$. Multiplying both fractions by 3×6, the results are still equal. Hence $2 \times 6 = 4 \times 3$.

From the above principle it follows that *either extreme equals the product of the means divided by the other extreme; and either mean equals the product of the extremes divided by the other mean.*

I. Find a fourth proportional to 8, 10, and 12.

$$8 : 10 :: 12 : x.$$

$$x = \frac{\overset{5}{\cancel{10}} \times \overset{3}{\cancel{12}}}{\underset{2}{\cancel{8}}} = 15.$$

Let the required term be represented by x. Then $8 : 10 :: 12 : x$. Since either extreme equals the product of the means divided by the other extreme, $x = \frac{10 \times 12}{8} = 15$.

II. Find the second term of a proportion of which the first, third, and fourth terms are respectively $2\frac{2}{5}$, $3\frac{3}{7}$, and $7\frac{1}{2}$.

$$2\tfrac{2}{5} : x :: 3\tfrac{3}{7} : 7\tfrac{1}{2}.$$

$$x = 2\tfrac{2}{5} \times 7\tfrac{1}{2} \div 3\tfrac{3}{7}.$$

$$= \frac{\cancel{12}}{\cancel{5}} \times \frac{\overset{3}{\cancel{15}}}{2} \times \frac{7}{\underset{2}{\cancel{24}}} = \frac{21}{4} = 5\tfrac{1}{4}.$$

Let the required term be represented by x. Then $2\frac{2}{5} : x :: 3\frac{3}{7} : 7\frac{1}{2}$. Since either mean equals the product of the extremes divided by the other mean, $x = 2\frac{2}{5} \times 7\frac{1}{2} \div 3\frac{3}{7} = 5\frac{1}{4}$.

EXAMPLES.

1. Find a fourth proportional to 9, 51, and 75.

2. Find the third term of a proportion of which the first, second, and fourth terms are respectively 18, 15, and 100.

3. Find the number which has to $6\frac{3}{5}$ the same ratio which $11\frac{2}{3}$ has to $3\frac{1}{7}$.

4. Find the number to which $8\frac{1}{3}$ has the same ratio which 25 has to $37\frac{1}{2}$.

5. Find the third term of a proportion of which the first, second, and fourth terms are respectively $\frac{3}{10}$, $\frac{1}{3}$, and $\frac{7}{10}$.

6. Find a fourth proportional to 3.75, 0.23, and 0.16.

7. Find the number which has to 0.649 the same ratio which 58 has to 634.

8. Find the fourth term of a proportion of which the first, second, and third terms are respectively 3.81, 0.056, and 1.67.

105. The method of finding either term of a proportion when the other three are known is often called the **rule of three**. It is customary to represent the required term by x, and then arrange the terms so that x will be the fourth term. The number in the problem which corresponds to the answer must be the third term.

I. If 15 yards of silk cost $36, what will 25 yards cost?

$$15 : 25 :: 36 : x$$

$$x = \frac{\overset{5}{\cancel{25}} \times \overset{12}{\cancel{36}}}{\underset{\cancel{3}}{\cancel{15}}} = \$60.$$

Since the answer is to be dollars, make $36 the third term. 25 yards will cost more than 15 yards, and the fourth term will be greater than the third; hence the second term must be greater than the first, and the first couplet is 15 : 25. The answer is then found as in the preceding section.

II. If 8 men can do a piece of work in 5 days, how long will it take 10 men to do the same work?

$$10 : 8 :: 5 : x$$

$$x = \frac{\overset{4}{\cancel{8}} \times \cancel{5}}{\underset{\cancel{2}}{\cancel{10}}} = 4 \text{ days.}$$

Since the answer is to be days, make 5 days the third term. 10 men can do the work in less time than 8 men, and the fourth term will be smaller than the third; hence the second term must be smaller than the first, and the first couplet is 10 : 8. The answer is then found as in the preceding section.

In the solution of problems in simple proportion, *make that number the third term which is of the same kind as the required answer. If from the nature of the question the answer is to be greater than the third term, make the greater of the other two numbers the second term, and the smaller the first; if the answer is to be smaller than the third term, make*

the second term smaller than the first. Divide the product of the means by the first term, and the quotient is the fourth term, or answer.

EXAMPLES.

1. If 18 barrels of flour last a garrison 8 weeks, how long will 63 barrels last?

2. If the rent of 36^{Ha} of land is $48, how many hektars can be rented for $84?

3. If a stock of provisions will supply a garrison of 240 men 96 days, how long will the same stock supply 384 men?

4. If 36 men can do a piece of work in 22 days, how many men can do the same work in 8 days?

5. If a clock ticks 120 times in a minute, how many times will it tick in $2\frac{1}{3}$ hours?

6. If 3 lb. 7 oz. of butter cost $1.10, what will $14\frac{3}{4}$ lb. cost?

7. If a train runs 160^{Km} in 3 hours, how long will it take it to run 70^{Km}?

8. If a man earns $16 in 5 days, how much will he earn in 14 days?

9. If 22 yd. of silk 18 in. wide are required for a dress, how many yards of cloth 30 in. wide would be required for a similar dress?

10. A field can be mowed in 4 days of 11 hours each; how many days of 9 hours each will it take?

11. If 12 men can build a wall 19 rods long in a day, how long a wall will 32 men build in the same time?

12. If 14 yd. of cloth 32 in. wide will make a dress, how many yards of cambric 24 in. wide will be required to line it?

13. If 10.5^{st} of wood cost \$12.25, how many sters can be bought for \$50?

14. If $\frac{5}{9}$ of a warehouse is worth \$7000, what is $\frac{7}{12}$ of it worth?

15. If a post 5 ft. 4 in. high casts a shadow 6 ft. 4 in. long, how long a shadow will be cast by a steeple 176 ft. high?

16. At the time when a man 5 ft. 9 in. in height casts a shadow 4 ft. 6 in. long, what is the height of a tree that casts a shadow 52 ft. 6 in. long?

17. If a cistern can be filled in 2 hr. 27 min. by 3 pipes, in what time can it be filled by 7 pipes of the same size?

18. If $9\frac{1}{4}$ yards of cloth cost $\$23\frac{1}{8}$, how many yards can be bought for $\$38\frac{7}{16}$?

19. What is the cost of 60.5 tons of coal when 0.9 of a ton costs \$6.66?

20. A merchant failed and paid 60 cents on a dollar; how much would a creditor receive whose bill was \$1426?

21. If 108^{Hl} of oats last 100 horses 9 days, how long will 192^{Hl} last them?

22. If a man travels 64 rods in 0.05 of an hour, how many minutes will it take him to go a mile?

23. If 18 men can perform a piece of work in 42 days, in how many days can they perform the same work with the assistance of 9 more men?

24. A piece of work can be done in 50 days by 35 men. After 12 days 16 men strike. In how many days will the rest finish the work?

25. If $6\frac{1}{4}$ T. of coal cost £6 15 s. 5 d., what will be the price of 5 T. 3 cwt.?

26. By a pipe of a certain capacity a cistern can be emptied in $3\frac{7}{15}$ hours; in what time can it be emptied by a pipe, the capacity of which is $\frac{2}{5}$ greater?

Compound Proportion.

106. An expression of equality between a compound ratio and a simple ratio, or between two compound ratios, is called a **compound proportion.**

I. If 4 men dig a trench 84 feet long and 5 feet wide in 3 days of 8 hours each, how many men can dig a trench 420 feet long and 3 feet wide in 4 days of 9 hours each?

$$\left.\begin{array}{r}84:420\\5:3\\4:3\\9:8\end{array}\right\}::4:x.$$

$$x=\frac{\overset{\cancel{84}}{\cancel{420}}\times\cancel{3}\times\cancel{3}\times 8\times\cancel{4}}{\cancel{84}\times\cancel{5}\times\cancel{4}\times\underset{\cancel{3}}{\cancel{9}}}=8\text{ men.}$$

Since the answer is to be men, make 4 men the third term. The number of men required depends upon four conditions — the length of the trench, the width of the trench, the number of days, and the number of hours per day; all of these must be considered in stating the problem. A trench 420 ft. long will require more men than a trench 84 ft. long, and the first ratio is 84 : 420; a trench 3 ft. wide will require less men than a trench 5 ft. wide, and the second ratio is 5 : 3; to complete the work in 4 da. will require less men than to complete it in 3 da., and the third ratio is 4 : 3; days of 9 hr. each will require less men than days of 8 hr. each, and the fourth ratio is 9 : 8. Dividing the product of the means by the product of the given extremes, we have 8 men as the answer.

In the solution of problems in compound proportion, *make that number the third term which is of the same kind as the required answer. Take the other terms in pairs of the same kind, and form a ratio of each pair as in simple proportion. Divide the product of the means by the product of the given extremes, and the quotient is the fourth term, or answer.*

EXAMPLES.

1. If 6 men in 15 days earn \$135, how much will 9 men earn in 18 days?

2. If 6 men can dig 6 rods of a ditch in 6 hours, how many rods will 12 men dig in 12 hours?

3. If 16 men build 18 rods of wall in 12 days, how many men will be needed to build 72 rods in 8 days?

4. If the wages of 12 men for 8 days of 8 hours each are \$135, what will be the wages of 25 men for 12 days of 10 hours each?

5. If a man travels 117 miles in 15 days, travelling 9 hours a day, how far would he go in 20 days, travelling 12 hours a day?

6. If 5 men can do a piece of work in 7 days of 10 hours each, in how many days can 12 men do the same, working 8 hours per day?

7. If $2\frac{1}{2}$ acres of pasturage can support 5 oxen for $3\frac{1}{2}$ days, how many acres would be required to support 26 oxen for $17\frac{1}{2}$ days?

8. If 14 horses eat 70 bushels of grain in 20 days, how many bushels will suffice 30 horses 50 days?

9. If 8 horses consume $3\frac{1}{2}$ tons of hay in 30 days, how long will $4\frac{9}{10}$ tons last 10 horses?

10. If 9 men build $247\frac{2}{13}$ rods of wall in 28 days, in how many days will 8 men build 51 rods?

11. If 2 men, working 8 hours, can carry 12000 bricks to the height of 50 feet, how many bricks can 1 man, working 10 hours, carry to the height of 30 feet?

12. If a six cent loaf weighs 8 ounces when wheat is \$1.25 per bushel, how much bread may be bought for 50 cents when wheat is \$1.00 per bushel?

13. If 49 men can empty a reservoir in 65 days, pumping 8 hours a day, how many hours a day must 196 men pump to empty it in 26 days?

14. If 5 horses will consume 8 bu. 1 pk. 6 qt. of oats in 6 days, what quantity of oats will 7 horses consume in 11 days?

15. If it take 35^{Kg} of wool to make a piece of cloth 25^{m} long and $\frac{3}{5}^{m}$ wide, how long a piece of cloth, $\frac{1}{5}^{m}$ wide, can be made from 112^{Kg}?

16. If a family of 9 persons spends \$305 in 4 months, how many dollars will maintain it 8 months, if 5 persons were added to the family?

17. If a man travels 1440 miles in 36 days, travelling 10 hours a day at the rate of 4 miles an hour, in what time will he travel 576 miles, going 8 hours a day at the rate of 3 miles per hour?

18. If 3 men can build a wall 60 feet long, 8 feet high, and 3 feet thick, in 64 days of 9 hours, how many days of 8 hours will 20 men require to build a wall 400 feet long, 9 feet high, and 5 feet thick?

19. If a slab of marble, 8 feet long, 3 feet wide, and 3 inches thick, weighs 1050 pounds, how much will another slab of the same marble weigh which is 6 feet long, 2 feet wide, and 2 inches thick?

20. If 6 iron bars, 4 feet long, 3 inches broad, and 2 inches thick, weigh 288 pounds, find the weight of 15 bars, each $6\frac{1}{2}$ feet long, $2\frac{1}{4}$ inches broad, and $1\frac{1}{2}$ inches thick.

21. If 25 men, working 8 hours a day, do $\frac{3}{5}$ of a piece of work in 24 days, in how many days of 10 hours each will 30 men finish the piece of work?

22. If 12 pipes, each delivering 12 gallons a minute, fill a cistern in 3 hr. 24 min., how many pipes, each delivering 16 gallons a minute, will fill a cistern 6 times as large in 6 hr. 48 min.?

23. A man has a bin 7 ft. long, $2\frac{1}{2}$ ft. wide, and 2 ft. deep, which contains 28 bushels of corn; how deep must he build another, which is to be 18 ft. long, 1 ft. $10\frac{1}{2}$ in. wide, in order to contain 120 bushels?

24. If 496 men, in 5 days of 12 hr. 6 min. each, dig a trench of 9 degrees of hardness, 465 ft. long, $3\frac{3}{5}$ ft. wide, and $4\frac{2}{3}$ ft. deep, how many men will be required to dig a trench of 2 degrees of hardness, $168\frac{3}{4}$ ft. long, $7\frac{1}{2}$ ft. wide, and $2\frac{4}{5}$ ft. deep, in 22 days of 9 hr. each?

Cause and Effect.

107. Problems in proportion can also be solved by the application of the following principle: *Like causes produce like effects, and the ratio between any two causes equals the ratio between the effects produced.*

NOTE. As examples of causes may be mentioned men at work, time, and goods bought or sold; as examples of effects, work done, wages, and cost of goods.

I. If 15 yards of silk cost $36, what will 25 yards cost?

$$15 : 25 :: 36 : x$$

$$x = \frac{\overset{5}{\cancel{25}} \times \overset{12}{\cancel{36}}}{\underset{\cancel{3}}{\cancel{15}}} = \$60.$$

Let the required number of dollars be represented by x. The first and second causes are respectively 15 yd. and 25 yd. The first and second effects are respectively $36 and x dollars. Hence the proportion is $15 : 25 :: 36 : x$

II. If 8 men can do a piece of work in 5 days, how long will it take 10 men to do the same work?

$$\left.\begin{matrix}8:10\\5:x\end{matrix}\right\}::1:1$$

$$x=\frac{\overset{4}{\cancel{8}}\times\cancel{5}\times 1}{\underset{\cancel{2}}{\cancel{10}}\times 1}=4 \text{ days.}$$

Let the required number of days be represented by x. The first causes are 8 men and 5 days, and the second causes are 10 men and x days. The effects are the same, and can each be represented by 1. Hence the proportion is $\left.\begin{matrix}8:10\\5:x\end{matrix}\right\}::1:1$. Since x is a mean, its value is found by dividing the product of the extremes by the product of the given means.

III. If 4 men dig a trench 84 feet long and 5 feet wide in 3 days of 8 hours each, how many men can dig a trench 420 feet long and 3 feet wide in 4 days of 9 hours each?

$$\left.\begin{matrix}4:x\\3:4\\8:9\end{matrix}\right\}::\left\{\begin{matrix}84:420\\5:3\end{matrix}\right.$$

$$x=\frac{\cancel{4}\times\cancel{3}\times 8\times\overset{\cancel{84}}{\cancel{420}}\times\cancel{3}}{\cancel{4}\times\underset{\cancel{3}}{\cancel{9}}\times\cancel{84}\times\cancel{5}}=8 \text{ men.}$$

Let the required number of men be represented by x. The first causes are 4 men, 3 da., and 8 hr., and the second causes are x men, 4 da., and 9 hr. The first effect is a trench 84 ft. long and 5 ft. wide, and the second effect is a trench 420 ft. long and 3 ft. wide. Hence the proportion is $\left.\begin{matrix}4:x\\3:4\\8:9\end{matrix}\right\}::\left\{\begin{matrix}84:420\\5:3\end{matrix}\right.$.

NOTE. The illustrative problems are the same three that were explained by the rule of three in the two preceding sections. All problems given under either head can be solved by either method.

EXAMPLES.

1. How many hektars of land can be bought for \$84, when 3^{Ha} can be bought for \$26.25?

2. If a horse-car goes 4 miles in 35 minutes, how far will it go in 3 hours?

3. If a tree 24 feet high casts a shadow 30 feet long, what must be the height of a building to cast a shadow 55 feet long?

4. If 16^{m} of silk cost 120 francs, what will 25^{m} cost?

5. If 14 men can build a wall in 10 days, how many men will it take to build the same wall in 7 days?

6. If 67.5^{m} of carpeting 80^{cm} wide will cover a floor, how many meters 90^{cm} wide will it take to cover it?

7. If a pasture of 18 acres will feed 8 cows 5 months, how many months will a pasture of 27 acres feed 12 cows?

8. A man receives $18 for 6 days' work of 8 hours each; what should he receive for 5 days' work of 9 hours each?

9. If 8 men spend $32 in 13 weeks, what will 24 men spend in 52 weeks?

10. If the wages of 72 men for 5 days is $450, how many men may be hired for 12 days for $540?

11. A man, travelling 9 hours a day, goes 234 miles in 15 days; how far can he go in 30 days, travelling 8 hours a day?

12. If a man travelling uniformly, 7 hours per day, goes 455 miles in 26 days, how far can he go in 20 days, travelling 9 hours per day at the same rate per hour as before?

13. If 6 men can build 20 feet of a stone wall in 10 days, how many men can build 360 feet of the same wall in 90 days?

14. If 17 men can reap a field in 9 days, how long would it take to reap half of it when 5 men refuse to work?

15. If 3 men can reap 8 acres in 5 days, working 8 hours a day, in how many days can 8 men, working 12 hours a day, reap 192 acres?

16. If it costs $7.20 to transport $18\frac{1}{2}$ cwt. $5\frac{1}{2}$ miles, what will it cost to transport $112\frac{3}{4}$ tons $62\frac{1}{2}$ miles?

17. If 27 men, working 10 hours a day, do a piece of work in 14 days, how many hours a day must 12 men work to do the same amount of work in 45 days ?

18. If 24 men can saw 90 cords of wood in 6 days, when the days are 9 hours long, how many cords can 8 men saw in 36 days, when they are 12 hours long ?

19. If a block of granite 8 ft. long, 2 ft. wide, and 1 ft. 6 in. thick, weighs 920 lb., how much will a block of the same kind of granite weigh which is 12 ft. long, 3 ft. wide, and 2 ft. thick ?

20. If 6 men do a certain piece of work in 17 days of 9 hours each, how many days of $8\frac{1}{2}$ hours each will 24 men, working at the same rate, require to do 20 such pieces ?

21. A wall which was to be 36 ft. high was raised 9 ft. in 16 days by 16 men ; how many men will be needed to finish the work in 4 days ?

22. If 8 ounces of bread can be bought for 10 cents when wheat is $1.00 per bushel, what weight of it may be bought for 18 cents when the price of wheat is $1.12 per bushel ?

23. If 30 lb. of cotton will make 3 pieces of muslin 42 yd. long and $\frac{5}{8}$ yd. wide, how many pounds will it take to make 50 pieces, each containing 35 yd., $1\frac{1}{8}$ yd. wide ?

24. If 6 men can build a wall 80 ft. long, 10 ft. high, and 9 ft. thick in 100 days of 9 hours, how many days of 10 hours will be required by 15 men to build a wall 200 ft. long, 9 ft. high, and 5 ft. thick ?

25. If 5 compositors in 16 days, 11 hours long, can compose 25 sheets of 24 pages in each sheet, 44 lines in a page, and 40 letters in a line, in how many days, 10 hours long, can 9 compositors compose a volume (to be printed in the same kind of type), consisting of 36 sheets, 16 pages to a sheet, 50 lines to a page, and 45 letters to a line ?

Partitive Proportion.

108. The process of dividing a number into parts which are proportional to given numbers is called **partitive proportion,** and the parts are called **proportional parts.**

I. Divide 168 into four parts which shall be to each other as 3, 5, 7, and 9.

$3+5+7+9=24$

$\frac{3}{\cancel{24}}$ of $\overset{7}{\cancel{168}}=21$;

$\frac{5}{\cancel{24}}$ of $\overset{7}{\cancel{168}}=35$;

$\frac{7}{\cancel{24}}$ of $\overset{7}{\cancel{168}}=49$;

$\frac{9}{\cancel{24}}$ of $\overset{7}{\cancel{168}}=63$.

The number 168 may be conceived as divided into a number of equal parts, 3 of which make up the first part, 5 the second, 7 the third, and 9 the fourth; thus the number of equal parts is 24. Hence the first part equals $\frac{3}{24}$ of 168, or 21; the second part equals $\frac{5}{24}$ of 168, or 35; the third part equals $\frac{7}{24}$ of 168, or 49; and the fourth part equals $\frac{9}{24}$ of 168, or 63.

Ans. 21, 35, 49, and 63.

II. Divide \$580 into three parts which shall be to each other as $\frac{1}{2}$, $\frac{2}{3}$, and $1\frac{1}{4}$.

$\frac{1}{2}=\frac{6}{12}$ $\quad 6+8+15=29$

$\frac{2}{3}=\frac{8}{12}$ $\quad \frac{6}{\cancel{29}}$ of $\overset{20}{\cancel{580}}=120$;

$1\frac{1}{4}=\frac{15}{12}$

$\frac{8}{\cancel{29}}$ of $\overset{20}{\cancel{580}}=160$;

$\frac{15}{\cancel{29}}$ of $\overset{20}{\cancel{580}}=300$.

Reducing the fractions to their L.C.D., we have $\frac{6}{12}$, $\frac{8}{12}$, and $\frac{15}{12}$. Since the fractions now have the same denominator, the parts which are proportional to the fractions are proportional to the numerators. Hence we divide \$580 into three parts proportional to 6, 8, and 15.

Ans. \$120, \$160, and \$300.

EXAMPLES.

1. Divide 324 into two parts which shall be to each other as 19 to 8.

2. Divide 90 into five parts which shall be to each other as 1, 2, 3, 4, and 5.

3. Divide 968 into three parts which shall be to each other as $2\frac{1}{2}$, $3\frac{1}{3}$, and $4\frac{1}{4}$.

4. Divide 420 into three parts, such that they shall be proportional to $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

5. Divide the reciprocal of 8 into two parts which shall be to each other as the reciprocals of 4 and $2\frac{2}{3}$.

6. Coffee is mixed in the ratio of 2 lb. of Java to 1 lb. of Mocha; how much of each kind is there in a mixture weighing 75 lb.?

7. The cost of a horse and harness was $384, and the horse cost seven times as much as the harness; find the cost of each.

8. An alloy contains 325 parts of copper to 175 parts of zinc; how much of each metal is contained in 43^{Kg} 850^{g} of this alloy?

9. If bell metal is made of 25 parts of copper to 11 parts of tin, find the weight of each metal in a bell weighing 1044 pounds.

10. A father divided $1550 among three sons in parts proportional to their ages, which were respectively 17, 20, and 25 years; how much did each receive?

11. A man said, "I will spend half my income, save a third of it, and devote a fourth to business." His income was $780. Point out his blunder, and divide his income rightly in the proportion intended by him.

12. Gunpowder is composed of nitre, charcoal, and sulphur, in the proportion of 15, 3, and 2. A certain quantity of gunpowder is known to contain 20 cwt. of charcoal; find its weight, and also the weight of nitre and sulphur it contains.

Simple Partnership.

109. An association of two or more persons for the transaction of business is called a **partnership**. Such a partnership association is called a **firm, company,** or **house,** and the persons associated together are called **partners.** The money and property invested in the business is called **capital** or **stock.** The **resources** or **assets** of a firm are its property of all kinds together with the amounts due it; the **liabilities** of a firm are its debts.

When the capital of several partners is invested for the same time, the partnership is called **simple partnership.** The profits and losses are shared in proportion to the amount of capital each partner has invested in the business, except when some other special agreement has been made.

I. A, B, and C formed a partnership; A put in \$700, B \$800, and C \$1000; what was each partner's share of a profit amounting to \$950?

$$\begin{array}{r} 700 \\ 800 \\ \underline{1000} \\ 2500 \end{array} \qquad \begin{array}{l} \dfrac{7\cancel{00}}{\cancel{2}\cancel{5}\cancel{00}} \text{ of } \overset{38}{\cancel{950}} = 266; \\[2ex] \dfrac{8\cancel{00}}{\cancel{2}\cancel{5}\cancel{00}} \text{ of } \overset{38}{\cancel{950}} = 304; \\[2ex] \dfrac{10\cancel{00}}{\cancel{2}\cancel{5}\cancel{00}} \text{ of } \overset{38}{\cancel{950}} = 380. \end{array}$$

The method of partitive proportion is used, dividing the profit into parts proportional to 700, 800, and 1000.

A, \$266; B, \$304; C, \$380.

EXAMPLES.

1. A and B form a partnership, A putting in \$3000 and B \$2500; what is each partner's share of a profit amounting to \$2200?

2. A, B, and C invested in trade as follows: A \$800, B \$600, and C \$900. What was each partner's share of a profit amounting to \$1350?

3. A and B formed a partnership, and A's capital was equal to $\frac{7}{9}$ of B's; what was each partner's share of a profit amounting to \$3600?

4. A, B, C, and D traded in company. A put in \$7500, B \$7000, C \$9500, and D \$8000; what was each partner's share of a profit amounting to \$9280?

5. A, B, and C formed a partnership, A putting in \$1500, B \$1800, and C \$1400. On closing business they found they had lost \$800. What was the loss of each?

6. A, B, and C hired a pasture for \$100; A put in 12 cows, B 8 cows, and C 5 cows; how much should each pay?

7. A bankrupt owed \$550 to A, \$675 to B, and \$875 to C. His entire property was sold for \$1043.28; what was each creditor's share?

8. A, B, and C engaged in trade with a joint capital of \$9000. At the end of a year A's gain was \$1250, B's \$1000, and C's \$1500. What was each partner's share of the capital?

Compound Partnership.

110. When the capital of the several partners is invested for unequal times, the partnership is called **compound partnership**. The division of the profits and losses depends both

on the amount of each partner's capital and the time for which it is invested.

I. A, B, and C invested in trade as follows: A \$300 for 10 months, B \$400 for 8 months, and C \$600 for 6 months. What was each partner's share of a profit amounting to \$1960?

$$\begin{aligned} 300 \times 10 &= 3000 \\ 400 \times \ \ 8 &= 3200 \\ 600 \times \ \ 6 &= \underline{3600} \\ &\ 9800 \end{aligned}$$

$$\frac{\cancel{3000}}{\cancel{9800}} \text{ of } \overset{20}{\cancel{1960}} = 600; \quad \frac{\cancel{3200}}{\cancel{9800}} \text{ of } \overset{20}{\cancel{1960}} = 640; \quad \frac{\cancel{3600}}{\cancel{9800}} \text{ of } \overset{20}{\cancel{1960}} = 720.$$

A, \$600; B, \$640; C, \$720.

The use of \$300 for 10 mo. is equivalent to the use of 10 times \$300, or \$3000, for 1 mo.; the use of \$400 for 8 mo. is equivalent to the use of 8 times \$400, or \$3200, for 1 mo.; the use of \$600 for 6 mo. is equivalent to the use of 6 times \$600, or \$3600, for 1 mo. The amounts invested are thus reduced to the same standard, and the profit is divided into parts proportional to 3000, 3200, and 3600.

EXAMPLES.

1. A and B enter into partnership. A contributes \$1200 for 13 months, and B \$1600 for 10 months. What is the share of each in a gain of \$1300?

2. Three partners, A, B, and C, furnish capital as follows: A \$500 for 2 months, B \$400 for 3 months, and C \$200 for 4 months. They gain \$600; what is each partner's share?

3. Two men hire a pasture for \$50; one puts in 20 horses for 12 weeks, and the other 25 horses for 10 weeks. How much should each pay?

4. A, B, and C hire a pasture for \$92. A pastures 6 horses for 8 weeks, B 12 oxen for 10 weeks, and C 50 cows for 12 weeks. If 5 cows are reckoned as 3 oxen, and 3 oxen as 2 horses, how much shall each man pay?

5. Three men harvested and thrashed a field of grain on shares, A furnishing 4 hands 5 days, B 6 hands 4 days, and C 5 hands 8 days. The whole crop was 630 bushels, of which they had one fifth; how much did each receive?

6. Three men contract to do a piece of work for $8775. The first man employs 20 men, 24 days, 10 hours a day; the second 25 men, 20 days, 12 hours a day; the third 30 men, 25 days, 9 hours a day. How much should each of the contractors receive?

7. A, B, and C contract to build a piece of railroad for $7500. A employs 30 men 50 days; B employs 50 men 36 days; and C employs 48 men and 10 horses 45 days (each horse to be reckoned equal to 1 man), and is to have $115.50 for overseeing the work. How much is each man to receive?

8. A and B rent a pasture for $690 per annum. A puts in 200 sheep, and B 160; at the end of 6 months they dispose of half their stock and allow C to put in 120; what should A, B, and C pay severally towards the rent at the year's end?

9. A and B entered into partnership for one year. A had $800 in the business during the first 4 months, and $400 more during the remainder of the year; B had $500 during the first 7 months, and $1300 during the last 5 months. At the end of the year they found they had lost $3800; what was each partner's loss?

10. A, B, and C formed a partnership and cleared $1200. A put in $8000 for 4 months, and then added $2000 for 6 months; B put in $16000 for 3 months, and then withdrawing half his capital, continued the remainder 5 months longer; C put in $13500 for 7 months. How should the profit be divided?

11. A and B entered into partnership for 3 years, A putting in \$5000, and B \$6000. At the end of a year A put in \$3000, and B put in \$1000. At the end of the second year A took out \$4000. At the end of the third year they divided a profit of \$8140. What was each partner's share?

12. A and B engaged in trade for 1 year. Jan. 1st A advanced \$2400 and B \$3600; May 1st C was admitted to the firm with \$4000; July 1st B withdrew \$1000; and Oct. 1st C withdrew \$1500. Their profits for the year were \$6800; what was each partner's share?

13. A and B began business Jan. 1st, each with a capital of \$2500. Apr. 1st A added \$500, and Aug. 1st he added \$800 more. June 1st B added \$1000. What was the share of each, at the year's end, of a profit of \$5425?

14. A's gain is \$840, B's gain is \$1125, and C's gain is \$1820. A's capital was in trade 7 months, B's 9 months, and C's 14 months. How much of the capital \$13875 did each own?

AVERAGES OR ALLIGATION.

111. The process of finding the average or mean value of several quantities of different values is called **alligation medial.**

I. A grocer mixed 12 lb. of tea worth 40 cents a pound, 10 lb. worth 65 cents a pound, and 8 lb. worth 75 cents a pound; what was the mixture worth a pound?

$$\begin{array}{r} 40 \times 12 = 480 \\ 65 \times 10 = 650 \\ 75 \times \underline{\ \ 8 = 600} \\ 30\,)\,\underline{1730\ \ } \\ 57\tfrac{2}{3}\text{ cts.} \end{array}$$

12 lb. at 40 cts. a pound are worth 480 cts.; 10 lb. at 65 cts. a pound are worth 650 cts.; 8 lb. at 75 cts. a pound are worth 600 cts. Adding, we find the value of 30 lb. to be 1730 cts.; hence 1 lb. is worth $\frac{1}{30}$ of 1730 cts., or $57\frac{2}{3}$ cts.

EXAMPLES.

1. Four children weigh respectively 62 lb., 77 lb., 89 lb., and 102 lb.; find their average weight.

2. In a certain school there are 15 pupils 10 years old, 6 pupils 9 years old, 10 pupils 8 years old, 8 pupils 7 years old, and 3 pupils 6 years old; find their average age.

3. Find the average daily expenses of a travelling salesman whose expenses for the week were as follows: Monday \$10.50, Tuesday \$3.84, Wednesday \$5.25, Thursday \$4.33, Friday \$6.78, and Saturday \$9.44.

4. A grocer mixed 16 lb. of coffee worth 25 cents a pound, 24 lb. worth 30 cents a pound, and 10 lb. worth 33 cents a pound; what was the mixture worth a pound?

5. Teas are mixed as follows: 40 lb. worth 70 cents a pound, 60 lb. worth 60 cents a pound, 100 lb. worth 50 cents a pound, and 80 lb. worth 40 cents a pound; for what should the mixture be sold a pound?

6. Find the value per gallon of the following mixture: 6 gal. of wine worth \$1.10 per gallon, 14 gal. of wine worth \$1.35 per gallon, 7 gal. of wine worth \$1.50 per gallon, and 5 gal. of water.

7. A merchant sold 75 bbl. of flour at \$5.60 per barrel, 45 bbl. at \$5.95 per barrel, 30 bbl. at \$6.10 per barrel, and 25 bbl. at \$6.50 per barrel; what was the average price per barrel?

8. A goldsmith combined 7 oz. of gold 22 carats fine, 12 oz. 20 carats fine, 10 oz. 15 carats fine, and 5 oz. of alloy; how many carats fine was the composition?

9. A miller mixes 18 bu. of wheat at \$1.44 with 6 bu. at \$1.32, 6 bu. at \$1.08, and 12 bu. at \$0.84. What will be his gain per bushel if he sells the mixture at \$1.50?

10. Some sugar is adulterated as follows: $\frac{3}{10}$ is worth 8 cents per pound, $\frac{4}{9}$ is worth 10 cents per pound, $\frac{2}{15}$ is worth 12 cents per pound, and the remainder, 33 pounds, is sand. What is the mixture worth per pound?

112. The process of finding the proportion of several quantities that may be used to form a mixture of given average value is called **alligation alternate**.

I. Find the proportion in which teas worth respectively 65 and 80 cents a pound must be taken to form a mixture worth 70 cents a pound.

$$70\left|\begin{array}{l}65+\ \ 5\ ..\ 10\ ..\ 2\\ 80-10\ ..\ \ \ 5\ ..\ 1\end{array}\right.$$

Ans. 2 lb. at 65 cts. and 1 lb. at 80 cts.

Write the given prices in a column with the price of the mixture at the left. If tea worth 65 cts. be sold for 70 cts., there is a gain of 5 cts., which is indicated by + 5 annexed to 65; if tea worth 80 cts. be sold at 70 cts., there is a loss of 10 cts., which is indicated by − 10 annexed to 80. A gain of 5 cts. a pound on 10 lb. will exactly balance a loss of 10 cts. a pound on 5 lb., and we write 10 opposite 65 + 5, and 5 opposite 80 − 10. This means that the two kinds must be mixed in the ratio of 10 to 5; dividing both numbers by 5, we obtain 2 lb. of the first and 1 lb. of the second. We can take any number of pounds that are in the ratio of 2 to 1.

Notice that the number of pounds taken at first of either kind is the same as the number of cents gained or lost on the other kind.

II. Find the proportion in which two kinds of vinegar worth respectively 12 and 15 cents a quart must be taken to form a mixture worth $13\frac{1}{3}$ cents a quart.

$$13\tfrac{1}{3}\left|\begin{array}{l}12+1\tfrac{1}{3}\ ..\ 1\tfrac{2}{3}\ ..\ 5\\ 15-1\tfrac{2}{3}\ ..\ 1\tfrac{1}{3}\ ..\ 4\end{array}\right.$$

Ans. 5 qt. at 12 cts. and 4 qt. at 15 cts.

Using the same process as in the preceding problem, we find that the two kinds are to be mixed in the ratio of $1\frac{2}{3}$ to $1\frac{1}{3}$. This ratio can be simplified by multiplying both numbers by 3, thus obtaining 5 qt. of the first kind, and 4 qt. of the second kind.

III. A grocer wishes to mix coffees worth respectively 22, 30, 35, and 40 cents a pound so as to make a mixture worth 32 cents a pound; how many pounds of each kind shall he take?

$$32\left|\begin{array}{llll} 22+10 & 8\,.\,.\,4 \\ 30+\ \ 2 & 3 \\ 35-\ \ 3 & 2 \\ 40-\ \ 8 & 10\,.\,.\,5 \end{array}\right.$$

Ans. 4 lb. at 22 cts.,
3 lb. at 30 cts.,
2 lb. at 35 cts.,
5 lb. at 40 cts.

$$32\left|\begin{array}{llll} 22+10 & 3 \\ 30+\ \ 2 & 8\,.\,.\,4 \\ 35-\ \ 3 & 10 \\ 40-\ \ 8 & 2\,.\,.\,1 \end{array}\right.$$

Ans. 3 lb. at 22 cts.,
4 lb. at 30 cts.,
10 lb. at 35 cts.,
1 lb. at 40 cts.

We begin as before, and link them together two by two, always linking one on which there is a gain with one on which there is a loss. Comparing the first and fourth, we find that they must be taken in the ratio of 4 to 5; comparing the second and third, we find that they must be taken in the ratio of 3 to 2. We can take any quantities provided that the first and fourth are in the ratio of 4 to 5, and the second and third in the ratio of 3 to 2.

By linking the first with the third, and the second with the fourth, we obtain an entirely different answer.

When more than two kinds are to be mixed, they can be taken in an infinite number of ways, for it is only necessary to combine them two by two, so as to make mixtures of the required value, and these mixtures may be combined in any proportions whatever. In solving problems it is generally best to take the combinations that involve the smallest numbers.

IV. How much sugar worth respectively 6 and 10 cents a pound must be mixed with 20 lb. worth 9 cents a pound in order that the mixture may be worth 8 cents a pound?

$$8\left|\begin{array}{lll} 6+2 & 1\,.\,.\,10, & 2\,.\,.\,1;\ 10+1=11. \\ 9-1 & 2\,.\,.\,20 & \\ 10-2 & & 2\,.\,.\,1 \end{array}\right.$$

Ans. 11 lb. at 6 cts. and 1 lb. at 10 cts.

Link the first with the second and the first with the third. Comparing the first and second, we find that they must be taken in the ratio of 1 to 2; hence 10 lb. of the first must be taken with 20 lb. of the second. Comparing the first and third, we find that they must be taken in the ratio of 1 to 1. Thus we must take 10 lb. of the first, and in addition equal quantities of the first and third.

V. A trader mixed oats worth respectively 32, 35, 40, and 42 cents a bushel in order to make a mixture of 45 bu. worth 36 cents a bushel; how many bushels of each kind did he take?

$$36\left|\begin{array}{l|l} 32+4 & 6\ .\ .\ 3 \\ 35+1 & 4 \\ 40-4 & 1 \\ 42-6 & 4\ .\ .\ 2 \end{array}\right.$$

$3+4+1+2=10;$

$$\frac{3}{\cancel{10}_{2}} \text{ of } \overset{9}{\cancel{45}} = \frac{27}{2} = 13\tfrac{1}{2};$$

$$\frac{\overset{2}{\cancel{4}}}{\cancel{10}_{\cancel{2}}} \text{ of } \overset{9}{\cancel{45}} = 18;$$

$$\frac{1}{\cancel{10}_{2}} \text{ of } \overset{9}{\cancel{45}} = \frac{9}{2} = 4\tfrac{1}{2};$$

$$\frac{\cancel{2}}{\cancel{10}_{\cancel{2}}} \text{ of } \overset{9}{\cancel{45}} = 9.$$

Ans. $13\frac{1}{2}$ bu. at 32 cts., 18 bu. at 35 cts., $4\frac{1}{2}$ bu. at 40 cts., and 9 bu. at 42 cts.

We find that the different kinds may be mixed in quantities proportional to 3, 4, 1, and 2. Dividing 45 into parts proportional to these numbers, we obtain $13\frac{1}{2}$ bu. of the first kind, 18 bu. of the second, $4\frac{1}{2}$ bu. of the third, and 9 bu. of the fourth.

If integral answers are desired, the ratios must be so taken that the sum of the numbers representing them is a divisor of the entire quantity.

EXAMPLES.

1. How shall corn at 45 cts. a bushel be mixed with oats at 36 cts. a bushel that the mixture may be worth 40 cts. a bushel?

2. Find the proportion in which oils worth respectively $1.30 and 85 cts. a gallon must be taken to form a mixture worth $1.10 a gallon.

3. In what proportion shall sugars worth respectively 7 and 12 cents a pound be taken to form a mixture worth $9\frac{1}{3}$ cents a pound?

4. Find the proportion in which sugars worth respectively 5 and 8 cents a pound must be taken to form a mixture worth $6\frac{3}{4}$ cents a pound.

5. In what proportion must alcohol (sp. gr. 0.82) be mixed with water to make a mixture having a specific gravity of 0.9?

6. A grocer wishes to mix teas worth respectively 40, 55, and 65 cents a pound so as to make a mixture worth 50 cents a pound; how many pounds of each kind shall he take?

7. Find the proportion in which three kinds of rice worth respectively $8\frac{1}{3}$, 11, and $12\frac{1}{2}$ cents a pound must be taken to form a mixture worth $10\frac{1}{2}$ cents a pound.

8. A merchant has coats worth \$12 each, vests worth \$6 each, and hats worth $\$4\frac{1}{2}$ each; how many of each must he sell in order that the average price may be $\$7\frac{1}{2}$?

9. How much water must be mixed with wine worth 90 cts. per gallon to make a mixture worth 60 cts. per gallon?

10. A grocer wishes to mix syrups worth respectively 42, 56, 64, and 75 cents a gallon so as to make a mixture worth 60 cents a gallon; how many gallons of each kind shall he take?

11. In what proportion shall sugars worth respectively 6, $7\frac{1}{2}$, $8\frac{1}{2}$, and 9 cents a pound be taken to form a mixture worth 7 cents a pound?

12. A grocer has wines worth respectively \$1.10, \$1.30, \$1.35, and \$1.50 per gallon, which he wishes to mix with water so as to form a mixture worth \$1.25 per gallon; how many gallons of each shall he take?

13. Teas at 3 s. 6 d., 4 s., and 6 s. a pound are mixed to produce a tea worth 5 s. a pound; what is the least integral number of pounds that the mixture can contain?

14. How many acres of land worth \$70 an acre must be added to a farm of 75 acres worth \$100 an acre in order that the whole may average \$80 per acre?

15. How many bushels of corn at 50 cents a bushel must be mixed with 100 bu. of oats at 80 cents a bushel that the mixture may be worth 75 cents a bushel?

16. How many pounds of chicory at 6 cts. a pound and coffee at 28 cts. a pound must be mixed with 30 lb. of coffee worth 35 cts. a pound in order that the mixture may be worth 20 cts. a pound?

17. A goldsmith has 4 oz. of gold 20 carats fine and 6 oz. 22 carats fine; how many ounces of alloy must be combined with it to make a mixture 16 carats fine?

18. A farmer wishes to mix 20 bu. of oats worth 33 cents a bushel with oats worth respectively 35, 38, and 40 cents a bushel, making a mixture worth 36 cents a bushel; how many bushels of each kind must he take?

19. A grocer wishes to mix 15 lb. of coffee at 40 cents a pound and 25 lb. at 35 cents a pound with two kinds worth respectively 25 and 28 cents a pound so that the mixture may be worth 30 cents a pound; how much of the latter kinds must he take?

20. A wholesale dealer has an order for 1000 bu. of wheat at 75 cents a bushel; how shall he mix three kinds of wheat, valued respectively at 72, 76, and 80 cents a bushel, to fill the order?

21. How many pounds of tea worth respectively 50, 60, and 75 cents a pound must be taken to make a mixture of 70 lb. worth 65 cents a pound?

22. A man has 100 three-cent pieces, which he wishes to exchange for dimes, half-dimes, two-cent pieces, and cents, and still have the same number of coins; how many of each kind will he receive?

23. A lady bought 100 yd. of cloth for \$10, some at 3 cents a yard, some at 8 cents, some at 12 cents, and some at 15 cents; how many yards of each kind did she buy?

24. A dealer paid \$182 for 20 barrels of flour, giving \$10 for first quality and \$7 for second; how many barrels were there of each?

25. An alloy, formed from two metals whose specific gravities are 8.29 and 10.35, has a specific gravity of 9.87; how many grams of each metal are there in a kilogram of the alloy?

26. Find how much gold 15, 17, and 22 carats fine must be mixed with 5 oz. 18 carats fine so as to make 12 oz. 20 carats fine.

27. A man paid \$70 to three men for 35 days' labor; to the first he paid \$5 per day, to the second \$1 per day, and to the third \$0.50 per day. How many days did each work?

CHAPTER IX.

PERCENTAGE.

113. The process of computing by hundredths is called **percentage.** **Per cent,** a contraction of the Latin *per centum,* means *by the hundred.* For example, 4 per cent of 25 means 4 hundredths of 25.

The sign % is used in place of the words *per cent.* For example, 6% means 6 per cent. 6% has the same value as 0.06 or $\frac{6}{100}$.

The number on which the percentage is reckoned is called the **base,** the number of hundredths taken is called the **rate,** the fraction denoting the number of hundredths is called the **rate per cent,** and the part of the base corresponding to the rate per cent is called the **per cent** or **percentage.** For example, 5% of 200 = 0.05 of 200 = 10; 200 is the base, 5 is the rate, 5% or 0.05 is the rate per cent, and 10 is the per cent or percentage.

To Express a Rate Per Cent as a Common Fraction.

114. I. Express $31\frac{1}{4}\%$ as a common fraction.

$$31\tfrac{1}{4}\% = \frac{31\frac{1}{4}}{100} = \frac{\overset{5}{\cancel{125}}}{4} \times \frac{1}{\underset{4}{\cancel{100}}} = \frac{5}{16}.$$

Since any per cent equals the same number of hundredths, $31\frac{1}{4}\%$ equals $\frac{31\frac{1}{4}}{100}$, which equals $\frac{5}{16}$.

EXAMPLES.

Express as common fractions the following rates per cent:

1. $6\frac{1}{4}\%$.	3. $8\frac{1}{3}\%$.	5. $11\frac{1}{9}\%$.	7. $14\frac{2}{7}\%$.
2. $6\frac{2}{3}\%$.	4. 10%.	6. $12\frac{1}{2}\%$.	8. $16\frac{2}{3}\%$.

9. 20%.	19. 80%.	29. 140%.
10. 25%.	20. $83\frac{1}{3}$%.	30. $162\frac{1}{2}$%.
11. $33\frac{1}{3}$%.	21. $87\frac{1}{2}$%.	31. $266\frac{2}{3}$%.
12. $37\frac{1}{2}$%.	22. 6%.	32. 340%.
13. 40%.	23. $7\frac{1}{2}$%.	33. $\frac{1}{2}$%.
14. 50%.	24. 28%.	34. $\frac{3}{10}$%.
15. 60%.	25. 35%.	35. $\frac{3}{8}$%.
16. $62\frac{1}{2}$%.	26. $68\frac{3}{4}$%.	36. $\frac{5}{6}$%.
17. $66\frac{2}{3}$%.	27. 125%.	37. $\frac{4}{15}$%.
18. 75%.	28. $133\frac{1}{3}$%.	38. $\frac{18}{25}$%.

NOTE. The fractional equivalents of the first twenty-one examples given above should be committed to memory by the student, as they are very often made use of.

TO EXPRESS A COMMON FRACTION AS A RATE PER CENT.

115. I. Express $\frac{2}{7}$ as a rate per cent.

$\frac{2}{7} = .28\frac{4}{7} = 28\frac{4}{7}\%$. Since we are to obtain the rate per cent, or number of hundredths, to which $\frac{2}{7}$ is equal, we divide the numerator by the denominator, carrying the division to two decimal places and retaining the remainder as a fraction.

EXAMPLES.

Express as rates per cent the following common fractions:

1. $\frac{1}{3}$.	6. $\frac{7}{10}$.	11. $\frac{7}{16}$.	16. $\frac{3}{200}$.
2. $\frac{1}{4}$.	7. $\frac{5}{9}$.	12. $\frac{13}{20}$.	17. $\frac{1}{300}$.
3. $\frac{1}{6}$.	8. $\frac{3}{11}$.	13. $\frac{9}{40}$.	18. $\frac{5}{4}$.
4. $\frac{3}{8}$.	9. $\frac{11}{12}$.	14. $\frac{11}{50}$.	19. $\frac{7}{3}$.
5. $\frac{5}{7}$.	10. $\frac{2}{15}$.	15. $\frac{1}{160}$.	20. $\frac{13}{4}$.

To Find any Per Cent of a Number.

116. I. What is $62\frac{1}{2}\%$ of 72?

$62\frac{1}{2}\% = \frac{5}{8}$; $\frac{5}{\cancel{8}}$ of $\overset{9}{\cancel{72}} = 45$.

Since $62\frac{1}{2}\%$ equals $\frac{5}{8}$, $62\frac{1}{2}\%$ of 72 equals $\frac{5}{8}$ of 72, which is 45.

II. What is 12% of \$210?

$$\begin{array}{r} 210 \\ 0.12 \\ \hline \$25.20 \end{array}$$

Since 12% equals 0.12, 12% of \$210 equals 0.12 of \$210, which is \$25.20.

Note 1. When a problem can be done mentally, use the method of example I.; otherwise use the method of example II.

Note 2. When a number is said to be a certain per cent more or less than another number, the second number should be taken as the base; never use the sum of two numbers thus spoken of as the base.

EXAMPLES.

1. What is 8% of 1200?
2. What is 76% of \$26.25?
3. What is $7\frac{1}{2}\%$ of \$2450?
4. What is $1\frac{3}{4}\%$ of \$264?
5. What is 45% of $\frac{5}{9}$?
6. What is 75% of $12\frac{1}{2}$?
7. What is $33\frac{1}{3}\%$ of 16 gal. 2 qt.?
8. What is $37\frac{1}{2}\%$ of 6 ft. 8 in.?
9. What is 100% of 1760 bu.?
10. What is 120% of 250 mi.?
11. What is $1\frac{1}{2}\%$ of 1280 men?
12. What is $\frac{3}{4}\%$ of 140 books?
13. How many pounds are there in $\frac{4}{5}\%$ of 6 cwt. 1 qr.?
14. What is $62\frac{1}{2}\%$ of 175% of 20% of \$576?

15. A man bought a house for \$6750, paying 25% cash; how much remained due?

16. A farmer had 320 sheep, and lost $6\frac{1}{4}\%$ of them; how many had he left?

17. A man has a yearly income of \$1400, and pays $17\frac{6}{7}\%$ of it for house rent; what rent does he pay?

18. If an ore yields 62% of pure iron, how many pounds of iron can be obtained from a ton of ore?

19. A miller charges 6% for grinding; how many quarts will he take when he grinds 25 bu.?

20. A merchant failed and paid 60% of his debts; how much was received by a creditor to whom he owed \$2180?

21. If a yard of cloth shrinks $4\frac{1}{2}\%$ in length in sponging, what fraction of a yard will it measure after sponging?

22. The amount of sunshine recorded in a certain city in the month of April was 33% of the possible amount, and the average length of the nights in that month is 10 hr. 30 min.; find the number of hours of sunshine in the month.

23. A man has a capital of \$12500; he puts 15% of it in stocks, $33\frac{1}{3}\%$ in land, and 25% in mortgages; how many dollars has he left?

24. A man had \$3250 in the bank; he drew out 12% of it at one time, and later 15% of the remainder; how much was left in the bank?

25. A's salary is \$1350, and B's salary is $11\frac{1}{9}\%$ more than A's; what is B's salary?

26. A boy's age is 60% less than his father's age, and his father is 45 years old; what is the boy's age?

To Find the Base when any Per Cent of it is Known.

117. I. 54 is $12\frac{1}{2}\%$ of what number?

$$\begin{array}{r} 54 \\ 8 \\ \hline 432 \end{array}$$

Since $12\frac{1}{2}\%$ equals $\frac{1}{8}$, 54 is $\frac{1}{8}$ of the required number; hence the number equals 8 times 54, or 432.

II. 55 is $83\frac{1}{3}\%$ of what number?

$$55 \div \frac{5}{6} = \overset{11}{\cancel{55}} \times \frac{6}{\cancel{5}} = 66.$$

Since $83\frac{1}{3}\%$ equals $\frac{5}{6}$, 55 is $\frac{5}{6}$ of the required number, and the number is as much as $\frac{5}{6}$ is contained in 55, which equals 66.

Note. The methods of examples I. and II. are to be used only when the rate per cent equals a small common fraction.

III. 455 is 52% of what number?

```
.52)455.00(875
    416
     390
     364
      260
      260
```

Since 52% equals 0.52, 455 is 0.52 of the required number, and the number is as much as 0.52 is contained in 455, which equals 875.

IV. What number increased by 28% of itself equals 320?

```
1.28)320.00(250
     256
      640
      640
        0
```

A number increased by 28% of itself equals 128% of itself, which is the same as 1.28 times the required number. If 320 is 1.28 times some number, the number is as much as 1.28 is contained in 320, which equals 250.

V. Having sold 36% of my land, I have 224 acres left; how much had I at first?

```
.64)224.00(350 A.
    192
     320
     320
       0
```

Since 36% was sold, there remained the difference between 100% and 36%, or 64%. Using the same method as in the preceding examples, we find that 224 A. is 64% of 350 A.

EXAMPLES.

1. 204 is 24% of what number?

2. 9.24 is 11% of what number?

3. 7465 is $33\frac{1}{3}$% of what number?

4. 126 is 175% of what number?

5. $5\frac{1}{16}$ is $37\frac{1}{2}$% of what number?

6. 15 is $\frac{5}{6}$% of what number?

7. $5674.83 is 105% of what sum?

8. 0.024 T. is $2\frac{1}{2}$% of how many pounds?

9. 261 is 16% more than what number?

10. What is the number to which if 2% of itself be added the sum is 516?

11. What fraction increased by $16\frac{2}{3}$% of itself equals $\frac{11}{12}$?

12. 96 is $14\frac{2}{7}$% less than what number?

13. What number diminished by 36% of itself equals 336?

14. What fraction diminished by 28% of itself equals $\frac{1}{2}$?

15. A clerk spends $672 a year, which is 56% of his salary; what is his salary?

16. A bankrupt has $5760, and with that sum can pay 40% of his debts; find the entire amount of his indebtedness.

17. The average daily attendance in a certain school is 95, which is $83\frac{1}{3}$% of the entire number; how many pupils are there in the school?

18. For collecting a bill an attorney received $2.52, which was $1\frac{1}{8}$% of the bill; what was the amount of the bill?

19. A man drew out $37\frac{1}{2}$% of his bank deposit to pay a bill of $675; how much had he remaining in the bank?

20. A milkman sold milk at 7 cents a quart, which was $233\frac{1}{3}\%$ of the cost; what did it cost a gallon?

21. The population of a certain town has gained 25% within the last five years. It is now 6575; what was it five years ago?

22. A man who weighs 80^{Kg} is 220% heavier than his son; what is the weight of the son?

23. A regiment lost $6\frac{1}{4}\%$ of its men and had 675 remaining; how many were there at first?

24. Having sold 30% of my land, I have 17 acres remaining; how much had I at first?

25. The expenses of a concert were $37\frac{1}{2}\%$ of the receipts, and the profits amounted to \$375; what were the expenses?

26. A man owning $33\frac{1}{3}\%$ of a factory sold 60% of his share for \$9000; what was the value of the entire property at the same rate?

27. A merchant bought some flour and sold 30% of it to one customer and $33\frac{1}{3}\%$ of the remainder to another, and then had 1120 barrels remaining; how many barrels did he buy?

28. The ages of two brothers together amount to 42 years, and the age of the younger is 75 % of the age of the older; what is the age of each?

29. The number of pupils in a school is 378, and the number of boys is 25 % more than the number of girls; how many boys are there? how many girls?

30. Two farmers together own 540^{Ha} of land, and one farm is 20 % smaller than the other; what is the size of each farm?

To Find what Per Cent one Number is of Another.

118. I. What per cent of 90 is 60?

$\frac{60}{90}=\frac{2}{3}=66\frac{2}{3}\%$. 60 is $\frac{60}{90}$ of 90. Reducing $\frac{60}{90}$ to its lowest terms, we obtain $\frac{2}{3}$, which equals $66\frac{2}{3}\%$.

II. What per cent of \$378 is \$135?

$$
\begin{array}{r l}
378)135.00(.35\frac{5}{7}=35\frac{5}{7}\%. \\
\underline{1134} \\
2160 \\
\underline{1890} \\
\frac{270}{378}=\frac{30}{42}=\frac{5}{7}
\end{array}
$$

\$135 is $\frac{135}{378}$ of \$378. To reduce $\frac{135}{378}$ to a rate per cent, we divide the numerator by the denominator, carrying the division to two decimal places.

EXAMPLES.

1. What per cent of 6 is 3?
2. What per cent of 5 is 1?
3. What per cent of 16 is 6?
4. What per cent of 25 is 18?
5. What per cent of 92 is 23?
6. What per cent of 100 is $22\frac{1}{4}$?
7. What per cent of 3 is $\frac{1}{3}$?
8. What per cent of $\frac{4}{9}$ is $\frac{5}{6}$?
9. What per cent of 1 score is 1 dozen?
10. What per cent of \$40 is \$45?
11. What per cent of 30000 bu. is 50 bu.?
12. What per cent of \$25 is 10 cts.?
13. What per cent of 30 ft. is 25 in.?
14. What per cent of 1 rd. 3 yd. 2 ft. 5 in. is 7 ft.?

15. What per cent of $105 is 30% of $250?

16. What per cent of $2412 is 25% of $2856?

17. From a hogshead of molasses containing 72 gal., 6 gal. leaked out; what per cent was lost?

18. If a house worth $1500 rents for $175 a year, what per cent of its value is the rent?

19. If $33\frac{1}{4}$ tons of iron are obtained from 165 tons of ore, what per cent of the ore is iron?

20. A gold ring is 18 carats fine; what per cent of it is pure gold?

21. If a city in five years increases in population from 23525 to 29171, what is the gain per cent?

22. A clerk has a salary of $1600, and his expenses for a year amounted to $1252; what per cent of his salary did he save?

23. A pedlar bought 30 doz. pencils; after selling 160, what per cent of the original number remained?

24. A man bought a horse and carriage for $525, paying $275 for the horse; what per cent of the value of the horse was the value of the carriage?

25. If carpet, which should be one yard wide, is only $34\frac{1}{2}$ inches wide, what per cent should be deducted from the price?

26. If a merchant uses a false weight of $15\frac{1}{2}$ oz. instead of a pound, what per cent does he gain by his dishonesty?

27. Of an alloy containing 21 parts copper and 4 parts nickel, what per cent is copper? what per cent is nickel?

28. In a mixture of copper and zinc, the copper is to the zinc as $3\frac{1}{2}$ is to $2\frac{1}{3}$; express the percentage of each ingredient in the mixture.

Profit and Loss.

119. Problems in percentage dealing with gains and losses in business transactions are grouped together under the subject of **Profit and Loss.** The cost price is always the base in determining the gain or loss per cent.

EXAMPLES.

1. A piece of cloth containing 40 yards cost $5; for what must it be sold a yard to gain $12\frac{1}{2}\%$?

2. A dealer bought lamps at $6.50 a dozen, and sold them at a loss of 4%; what did he receive apiece for them?

3. A grocer bought a barrel of sugar for $14.75, and sold it at a profit of 8%; what was his gain?

4. A grocer bought 500 bags of coffee, each bag containing $49\frac{1}{4}$ pounds, at 12 cents a pound, and sold it at a profit of $16\frac{2}{3}\%$; what did he receive for the entire amount?

5. I buy one fifth of an acre of land for $2178. For how much a square foot must I sell it in order to gain 20% of the cost?

6. What was the cost when $17\frac{1}{2}\%$ was gained by selling an article for $253.80?

7. Find the cost of a carriage if $16\frac{2}{3}\%$ was lost by selling it for $250.

8. If a boy sells a book for $1.10, and thereby loses 12%, what did the book cost him?

9. If I sell coffee at 2s. 3d. per pound, and thereby gain 35%, what did I give per pound?

10. What per cent is gained in buying oil at 80 cents a gallon, and selling it at 12 cents a pint?

11. What per cent would be lost in buying a field for \$115, and selling it for \$100 ?

12. A grocer buys sugar at 18 cents a kilo, and sells it at 1 cent per 50^g; how much per cent does he gain ?

13. A and B barter; A makes of 10 cents $12\frac{1}{2}$ cents, and B makes of 15 cents 19 cents; which makes the greater per cent, and how much ?

14. If I sell 22 things for what 36 cost me, what per cent is gained ?

15. What per cent is gained by buying coal by the long ton, and selling it by the short ton at the same price ?

16. Having purchased an acre of land, I sell from it a rectangular lot, 121 yards long and 25 yards wide, for what the whole acre cost me; what per cent do I gain on the land thus sold ?

17. If the cost is three fourths of the selling price, what is the gain per cent ?

18. If a merchant sells goods for three fourths of their cost, what per cent does he lose ?

19. If 25% of the amount received for an article is gain, what is the gain per cent ?

20. A horse that cost $6\frac{1}{3}$% of \$25000 was sold for \$1000; what was the loss per cent ?

21. I gained $33\frac{1}{3}$% in selling a horse, and with the proceeds bought another horse which I afterwards sold for \$120, thereby losing 25% ; did I gain or lose by the transactions ?

22. If a wagon is purchased at 20% less than \$50, and is afterwards sold at 25% more than it cost, at what price is it sold ?

23. Four coats were sold for \$15 apiece; a profit of 25% was made on two of them, and on the other two 25% was lost; what was the total gain or loss?

24. A merchant selling goods at a certain price loses 5%; but if he had sold them for \$20 more, he would have gained 3%; what did the goods cost him?

25. A man purchased three horses for \$500. The first horse cost $37\frac{1}{2}$% less than the second, and the third horse cost 60% less than the first; what was the price of each?

26. If 25 pounds of tea at 60 cents per pound be mixed with 30 pounds at 49 cents, find the price of the mixture per pound in order that there may be a profit of $16\frac{2}{3}$%.

27. A merchant sold one half of a certain lot of goods at a gain of 18%, one third at a loss of 5%, and the remainder at half cost; did he gain or lose, and what per cent?

28. A man buys 150 pounds of sugar, and, after selling 100 pounds, finds he has been selling it at a loss of 5%; at what rate per cent advance on the cost must he sell the remaining 50 pounds that he may gain 10% on the entire transaction?

29. A speculator had 5000 barrels of flour that cost him \$8 a barrel; he sold 30% of the lot at an advance of 10% on the cost, and 50% of the remainder at a further advance of $2\frac{1}{2}$% on the cost; he closed out the lot at \$8.50 a barrel. Find how much he made, and what per cent of the cost.

30. A merchant sold a lot of flour at \$8.40 a barrel, and thereby gained 20%. He afterwards sold another lot of the same flour for \$203, and thereby gained 16%. How many barrels were there in the last lot?

120. In certain problems the principles of proportion can be used to good advantage.

I. If I sell a horse for \$175 and gain 5%, what would have been my gain per cent if I had sold it for \$200?

$175 : 200 :: 105 : x.$

$$x = \frac{\overset{8}{\cancel{200}} \times \overset{15}{\cancel{105}}}{\underset{7}{\cancel{175}}} = 120.$$

Ans. 20 %.

A gain of 5% is the same as 105% of the cost. Let x represent the per cent when sold for \$200. By proportion the first price is to the second price as the first per cent is to the second per cent. We thus obtain 120%, which is the same as a gain of 20%.

EXAMPLES.

1. If there is a gain of $12\frac{1}{2}$% on tea at 90 cents a pound, what would be the gain per cent at 84 cents a pound?

2. A farmer sold a pair of oxen for \$200, and gained 20%; what per cent would he have gained if he had sold them for \$225?

3. By selling a horse for \$162, a man lost 10%; for how much should he have sold it to gain 10%?

4. If 5% be lost by selling an article for \$2.47, find the per cent gain or loss by selling it at \$2.99.

5. A man lost $13\frac{1}{3}$% by selling a lot of land for \$850; what would it have brought if it had been sold at a loss of $8\frac{1}{3}$%?

6. A man sells flour at \$6.50 a barrel, and gains 10%; what per cent would he gain if he sold the flour for \$8.25 a barrel?

7. By selling potatoes at $62\frac{1}{2}$ cents a bushel, 10% was lost; at how much should they be sold to gain 25%?

8. A man sold a house for \$5000, and thereby gained 20%; would he have gained or lost, and how much per cent, if he had sold it for \$4000?

9. If I lose 10% by selling goods at 28 cents a yard, for what should they be sold to gain 20%?

10. A village lot was sold for $230, which was 8% less than it cost; had it been sold for $300, what would have been the gain per cent?

11. A firm sold a fire engine for $7050, and lost 6% on its cost; for how much ought they to have sold it to gain 12%?

12. If a house had been sold for $7992, there would have been a gain of 8%; how much per cent is gained or lost by selling it for $7511?

13. If 50% is gained by selling goods at 3s. 9d. per pound, what per cent would be gained or lost by selling them at 2s. 11d. per pound?

Commercial Discount.

121. When articles are sold for less than the regular price, the difference between the regular price and the price at which they are sold is called the **discount**, and the price at which they are sold is called the **net price**. The discount is generally a certain per cent of the regular price.

Manufacturers and wholesale dealers issue price lists subject to various discounts.

I. What is the net price of a carriage billed at $275, 8% off for cash?

$$\begin{array}{r} 275 \\ .08 \\ \hline 22.00 \end{array} \qquad \begin{array}{r} 275 \\ 22 \\ \hline \$253 \end{array}$$

The discount is 8% of $275, which is $22; subtracting this from $275, we find the net price to be $253.

II. Find the net amount of a bill for $762 subject to the following discounts: 40, 5, and 10.

$$\begin{array}{r} 762 \\ .60 \\ \hline 20)\overline{457.20} \\ 22.86 \\ \hline 10)\overline{434.34} \\ 43.43 \\ \hline \$390.91 \end{array}$$

Allowing a discount of 40% is the same as taking 60% of the amount of the bill; hence 60% of \$762, or \$457.20, is the amount after the first discount. Dividing by 20 gives 5% of \$457.20, and subtracting this result, we find \$434.34 to be the amount after the second discount. Dividing by 10 gives 10% of \$434.34, and subtracting this result, we find the net amount to be \$390.91.

III. At what per cent above cost must a merchant mark his goods in order to allow a discount of 25%, and still make a profit of 15%?

$$\begin{array}{l} .75)1.15(1.53\tfrac{1}{3} \\ \quad\;\; 75 \\ \quad\;\; \overline{400} \\ \quad\;\; 375 \\ \quad\;\; \overline{\;\;250} \\ \quad\;\;\;\; 225 \\ \quad\;\;\;\; \overline{\;\;\frac{25}{75} = \frac{1}{3}} \end{array}$$

If he makes a profit of 15%, he sells his goods for 115% of the cost. If he allows a discount of 25% from the marked price, 115% of the cost is 75% of the marked price. Then the marked price is as much as 0.75 is contained in 115%, which equals $153\frac{1}{3}$%; that is, the goods must be marked $53\frac{1}{3}$% above cost.

Ans. $53\frac{1}{3}$%.

EXAMPLES.

1. What is the net price of a set of books, the list price of which is \$24, subject to a discount of 20%?

2. What is the net price of a set of furniture, the list price of which is \$225, subject to a discount of $8\frac{1}{3}$%?

3. When a piano, whose list price is \$850, is sold at 45% off, what is the discount?

4. What is the discount on a bill of groceries amounting to \$48.92, 7% off for cash?

5. After a discount of 20% had been given, a refrigerator was sold for \$14.40; what was the list price?

6. After a discount of $6\frac{2}{3}$% had been given, a barrel of flour was sold for \$7; what was the list price?

7. What is the net price of a bill of goods amounting to \$170, 35% discount and $2\frac{1}{2}$% off for cash?

8. What is the net price of a barrel of oil, the list price of which is \$18, subject to a discount of $12\frac{1}{2}$% and 4% off for cash?

9. Find the difference between the net value of a bill of goods for \$1200, less a discount of 20% and 5% off for cash, and the net value of the same bill, less a discount of 25%.

10. Find the net amount of a bill for \$234.60 subject to the following discounts: 50, 10, and $7\frac{1}{2}$.

11. Find the net amount of a bill for \$1036 subject to the following discounts: 30, $12\frac{1}{2}$, and 5.

12. Find the net amount of a bill for \$856.25 subject to the following discounts: 40, 5, 15, and $2\frac{1}{2}$.

13. A book must be marked at what per cent on the original cost in order to allow a discount of 20% of the market price, and still give 10% profit?

14. At what per cent above cost must a merchant mark his goods in order to allow a discount of $33\frac{1}{3}$%, and still make a profit of 25%?

15. At what per cent above cost must a merchant mark his goods in order to allow a discount of $16\frac{2}{3}$%, and still make a profit of $16\frac{2}{3}$%?

16. A merchant on opening a case of goods found them slightly damaged; at what per cent above cost must he mark them in order to allow a discount of 20%, and yet lose only 8%?

17. A house was bought for \$500; what must be the asking price in order to fall on it 10%, and still make 10% on the purchase?

18. A merchant bought cloth at \$3.60 a yard; at what price must it be marked that $12\frac{1}{2}\%$ may be abated from the asking price, and still a profit made of $16\frac{2}{3}\%$?

19. A merchant bought carpeting at 60 cents a yard. At what price must it be marked in order that in selling it he may abate 20% on the marked price, and yet make a profit of $33\frac{1}{3}\%$?

20. A tradesman marks his goods at 25% above cost, and deducts 12% of the amount of any customer's bill for cash; what per cent does he make?

21. A merchant marked some goods 30% above cost, and then sold them at a discount of 25%; did he gain or lose, and what per cent?

22. If from the retail price of a book 20% is deducted, and a discount of 10% is made on the balance, and then the book sells for \$1.33, what is the retail price?

23. A manufacturer, who allows a discount of 20% from the list price with 5% off for cash, receives \$262.20 as the net price of a bill of goods; find the list price.

24. A merchant sold a quantity of goods for \$29900. He deducts 5% from the amount of the bill for cash, and finds that he has made 15% on the investment; what did he pay for the goods?

Commission.

122. A person who buys or sells goods for another is known as a **commission merchant** or **agent**. The compensation received is called **commission**, and it is usually a certain per cent of the amount expended or collected.

After the commission and other charges have been deducted from the amount of a business transaction, the remainder is called the **net proceeds**.

I. Find the net proceeds on the sale of 500 bbl. of flour at \$4.25 per barrel; the commission for selling was $2\frac{1}{4}\%$, and the charges for freight and storage were 32 cts. per barrel.

$$\begin{array}{r} 4.25 \\ 500 \\ \hline 2125.00 \\ .02\frac{1}{4} \\ \hline 53125 \\ 425000 \\ \hline 47.8125 \\ 160. \quad\quad \\ \hline 207.81 \quad \end{array} \qquad \begin{array}{r} 0.32 \\ 500 \\ \hline 160.00 \\ \\ \\ 2125.00 \\ 207.81 \\ \hline \$1917.19 \end{array}$$

At \$4.25 per barrel, the price of 500 bbl. of flour is \$2125. $2\frac{1}{4}\%$ of \$2125 is \$47.81, the amount of the commission; at 32 cts. per barrel the charges for freight and storage amount to \$160. The sum of these expenses is \$207.81, which subtracted from \$2125 gives \$1917.19 as the net proceeds.

II. \$1000 includes a sum to be invested and a commission of 5% of the sum to be invested; what is the sum to be invested?

$$\begin{array}{l} 1.05)1000.00(\$952.38 \\ \quad\;\; \underline{945} \\ \quad\;\;\; 550 \\ \quad\;\;\; \underline{525} \\ \quad\;\;\;\; 250 \\ \quad\;\;\;\; \underline{210} \\ \quad\;\;\;\;\; 400 \\ \quad\;\;\;\;\; \underline{315} \\ \quad\;\;\;\;\;\; 850 \\ \quad\;\;\;\;\;\; \underline{840} \end{array}$$

Since \$1000 includes both the sum to be invested and 5% of that sum, it is 105% of the investment. If \$1000 is 105% of the investment, the investment is as much as 1.05 is contained in \$1000, which equals \$952.38.

EXAMPLES.

1. At an auction sale \$542.68 was received for a lot of furniture; what was the auctioneer's commission at 5% ?

2. Find the commission at $3\frac{1}{4}\%$ on the sale of 952 yards of carpeting at 62 cents a yard.

3. An architect charged $2\frac{1}{2}\%$ for the plans and specifications of a building costing \$60000, and $2\frac{1}{2}\%$ for superintending the work of erection; what was his fee?

4. A real estate agent sold a house for \$7500, and charged $\frac{3}{4}\%$ commission; find the net proceeds of the sale.

5. A commission merchant sold a consignment of sugar for \$3420, and charged $2\frac{3}{4}\%$ commission; the expenses for freight and storage were \$24.70; find the net proceeds.

6. A commission merchant sold 1350 baskets of peaches at \$1.20 a basket; the commission for selling was $3\frac{3}{4}\%$, the charge for guaranteeing payment was $1\frac{1}{4}\%$, and the charge for cartage was 4 cents a basket; find the net proceeds.

7. An agent's commission at 5% amounted to \$362.25; find the amount of his sales.

8. If I sell goods at 4% commission, and receive \$60, what amount have I sold?

9. A travelling agent, who received 5% commission, remitted \$893 as the net proceeds for a week's sales; find the amount of his sales.

10. A real estate agent received $8\frac{1}{3}\%$ commission for selling some land, and forwarded \$4400 to the owner as net proceeds; for how much was the land sold?

11. An attorney received \$58.50 as his commission for collecting a bill of \$975; what was the rate of commission?

12. A book agent sold 84 books at \$3.50 apiece, and after deducting his commission, remitted \$176.40 to the publishers; what was the rate of his commission?

13. An agent invested \$15000 in cotton for a manufacturing company, charging a commission of $1\frac{3}{4}\%$; what was the entire bill for cotton and commission?

14. A collector, who charges 8% commission on what he collects, pays me \$534.75 for a bill of \$775; what fractional part of the bill does he collect?

15. An agent received a sum of money to expend, after deducting his commission of $3\frac{1}{2}\%$; he expended \$523.67; what was the sum he received?

16. \$1200 includes a sum to be invested and a commission of 4% of the sum to be invested; what is the sum to be invested?

17. A commission merchant receives \$625 to invest in goods, after deducting his commission of $4\frac{1}{6}\%$; find the amount of his commission.

18. At \$6.25 per ton, how many tons of coal can I buy for \$1000 and allow $2\frac{1}{2}\%$ commission?

19. An agent receives \$6150 to invest in cotton at $10\frac{1}{4}$ cents per pound; his commission is $2\frac{1}{2}\%$; how many pounds of cotton can he buy?

20. A commission merchant received \$1356.60 to be expended in flour at \$4.75 a barrel, after deducting his commission of 2%; how many barrels did he buy?

21. A sends B \$5000, with which B is to purchase lumber, after providing for his commission of 2%; how much will B have to expend for lumber, and what will be the amount of his commission?

22. An agent received \$1250, with which to purchase goods, after deducting his commission of $2\frac{1}{2}\%$, and paying \$11.50 for cartage; what sum did he pay for the goods?

23. A commission merchant sold goods for \$7125, and received 4% commission; he invested the net proceeds in lumber, charging 2% commission for buying; find the value of the lumber bought.

24. A commission merchant sold 1050 pounds of butter at 24 cents a pound, and received a commission of $2\frac{1}{2}\%$; he invested the net proceeds in corn at 45 cents a bushel, charging $1\frac{1}{2}\%$ commission; how many bushels of corn did he buy?

25. A real estate agent sold a lot of land for \$8000, and with the proceeds bought a house, after deducting his commission of $2\frac{1}{2}$ % for selling and 4 % for buying; what was his entire commission?

INSURANCE.

123. **Insurance** is a contract whereby one party agrees for a specified consideration to pay a specified sum of money in case of loss by certain risks.

In **fire insurance** the agreement is to pay for all losses by fire not exceeding a certain sum, and in **marine insurance** the same is true for all losses arising from accidents of navigation. **Life insurance** secures a certain sum of money to a person's heirs in case of death.

The written contract is called the **policy**, and the sum paid for insurance is called the **premium**; the premium is computed at a certain per cent on the amount insured, or at so much on the \$100.

I. A store worth \$30000 was insured for $\frac{5}{6}$ of its value at $1\frac{1}{4}$ %; find the annual premium.

$$\begin{array}{r} 6\overline{)30000} \\ 5000 \\ \underline{5} \\ 25000 \\ \underline{.01\frac{1}{4}} \\ 6250 \\ \underline{25000} \\ \$312.50 \end{array}$$

$\frac{5}{6}$ of \$30000 is \$25000; $1\frac{1}{4}$ % of \$25000 equals \$312.50.

II. For how much must a cargo of lumber worth \$18750 be insured at $3\frac{1}{2}$ %, so that in case of loss the owner may recover both the value of the lumber and the premium?

```
.965)18750.000($19430.05
     965
     9100
     8685
      4150
      3860
       2900
       2895
          5000
          4825
```

Since the premium is $3\frac{1}{2}\%$ of the amount insured, the value of the lumber is $100\% - 3\frac{1}{2}\%$, or $96\frac{1}{2}\%$, of the amount insured. Hence the entire amount is as much as .965 is contained in $18750, which equals $19430.05.

EXAMPLES.

1. What premium must be paid for an insurance of $4500 on a house at $1\frac{3}{4}\%$?

2. What is the expense of insuring a mill worth $12000 for $\frac{1}{2}$ of its value at $4\frac{1}{2}\%$?

3. If the premium for insurance at $\frac{3}{4}\%$ is $16.50, what is the amount insured?

4. A stable was insured for $\frac{4}{5}$ of its value at $2\frac{3}{4}\%$, and the premium was $55; what was the value of the stable?

5. If a premium of $60 is paid for an insurance of $4800 on a house, what is the rate of insurance?

6. A merchant paid $20 for an insurance of $2500 on a stock of goods; find the rate of insurance.

7. A cargo worth $8150 was insured at $3\frac{3}{4}\%$ for $\frac{9}{10}$ of its value; in case of shipwreck what would be the actual loss of the owner?

8. A house was insured for $5500 at $2\frac{1}{4}\%$; during the fourth year of the insurance the house was burned; find the actual loss of the insurance company, making no allowance for interest.

9. A house, which had been insured for \$2500 for 12 years at $1\frac{3}{8}\%$, was wholly destroyed by fire; how much did the amount received from the company exceed the sum of the premiums paid?

10. For what sum ought a ship worth \$15200 to be insured at 5%, so that in case of loss the owner may recover both the value of the ship and the premium?

11. A merchant had a cargo of sugar worth \$3200 insured at $2\frac{1}{4}\%$, so as to cover both the value of the sugar and the premium; for how much was the sugar insured?

12. Find the premium paid when goods worth \$9500 are insured at 3% so as to cover both the value of the goods and the premium.

13. A ship was insured for \$25600 to cover both the value of the ship and the premium of $6\frac{1}{4}\%$; find the value of the ship.

14. A mill is insured for \$8000 in one company and for \$5000 in another company; a fire causes a loss of \$6500; what amount should each company pay?

15. A building worth \$12500 is insured for $\frac{4}{5}$ of its value in three companies; the first takes $\frac{1}{4}$ of the risk at $\frac{7}{8}\%$, the second takes $\frac{2}{5}$ of it at 1%, and the third takes the remainder at $1\frac{1}{8}\%$; what is the total premium?

16. If the building in the preceding problem should be damaged to the amount of \$2025, what ought each company to pay?

17. A man 30 years old had his life insured for \$5000 at \$25.25 per \$1000; what was the annual premium?

18. If a man pays a yearly premium of \$74.20 for life insurance at \$21.20 on \$1000, what is the sum insured?

19. A man had his life insured for $10000 at $32.40 per $1000; should he die after paying eight yearly premiums, how much more would his heirs receive than had been paid in premiums?

Taxes.

124. A **tax** is a sum of money assessed on the person or property of an individual for public purposes.

Taxes are apportioned among the tax-payers according to the estimated value of their property.

In some states each male citizen pays a certain sum without regard to his property; this is called a **poll tax.**

In computing taxes the amount of the poll taxes, if any, is deducted from the entire sum to be raised; the remainder divided by the value of the taxable property gives the rate of taxation, which may be expressed as a per cent, or as so much on $1000. For example, if $3000 is to be raised by taxing property worth $600000, the rate is $\frac{1}{2}\%$, or $5 on $1000.

I. A tax of $15377.42 is to be raised in a town, the taxable property of which is valued at $1672316; there are 528 polls, each taxed $2; find the rate of taxation.

```
 528      15377.42
   2       1056
----     --------
1056      14321.42

1672316)14321.420(.008564
         13378528
         --------
           9428920
           8361580
          --------
          10673400
          10033896
          --------
            6395040
```

Ans. $8.564 on $1000.

The tax on 528 polls at $2 each amounts to $1056. Subtracting this amount from $15377.42, we find $14321.42 to be the amount to be raised by taxing the property. If $14321.42 is to be raised on $1672316, the amount to be raised on $1 is as much as 1672316 is contained in $14321.42, which equals $0.008564; this is the same as $8.564 on $1000.

II. Using the rate of taxation found in the preceding problem, find the tax paid by a man who pays a poll tax and owns property valued at $7132.

```
   7132
.008564
  28528
 42792
35660
57056
61.078448
2
$63.08
```

If the tax on $1 is $0.008564, on $7132 the tax is 7132 times $0.008564, which equals $61.08. Adding the poll tax, we find the entire tax to be $63.08.

EXAMPLES.

1. Find the rate of taxation when $8647.29 is to be raised by taxing property valued at $768648.

2. If the assessed valuation of a town is $2362724, and the town has 637 polls, paying $1.50 each, what must be the rate of taxation in order to raise $24516?

3. A tax of $9426.88 is to be assessed in a town; the real estate is valued at $442300, and the personal property at $496720; there are 486 polls, paying $1.25 each; what is the rate of taxation?

4. A tax of $7137 is to be assessed in a town having 610 polls and a valuation of $817326; if one sixth of the tax be laid on the polls, what will be the amount of each poll tax, and what will be the rate of taxation?

5. At the rate of $13.70 on $1000, find the tax paid by a man who pays a poll tax of $1.80, and owns property valued at $72540.

6. Find the tax to be paid by a man who pays a poll tax of $1.40, and owns property valued at $18732, when the rate of taxation is $14.32 on $1000.

7. A man owns property valued at \$25420, of which \$2400 is exempt from taxation; find his tax at the rate of \$10.26 on \$1000.

8. A tax of \$9426 is to be raised in a town, the taxable property of which is valued at \$921495; find the tax on property valued at \$2340.

9. Find the rate of taxation when a man, who owns property valued at \$28000, pays a tax of \$370.16.

10. A man pays a tax of \$799.07, including a poll tax of \$2; the rate of taxation is \$16.30 on \$1000; find the value of his property.

11. Find the value of the taxable property of a town when a tax of \$11131.02 is raised at the rate of \$11.25 on \$1000.

12. A collector receives 8% for collecting taxes, and pays into the treasury \$94625.64 after deducting his commission; how much did he collect?

13. The taxes assessed in a town amounted to \$34271.60; the collector received $1\frac{1}{2}$% for collecting, and 7% of the taxes could not be collected; find the net proceeds.

14. Find the entire tax that must be assessed in order that a town may receive \$12134 after the collector deducts his commission of $2\frac{1}{2}$%.

15. The net proceeds of the taxes assessed in a town, after deducting the collector's commission of 2%, amounted to \$50638.95, and 5% of the taxes could not be collected; find the amount of taxes assessed.

16. In a certain city the cost of public schools for the next school year is estimated at \$36848. Find what amount of school tax must be assessed, the cost of collecting being 2% of the assessed tax, and allowing 6% of the assessed tax to be not collectible.

Duties.

125. **Duties** are taxes levied by the government on imported goods.

An **ad valorem duty** is a certain per cent of the cost of the goods in the country from which they were imported.

A **specific duty** is a tax levied upon goods according to the quantity, without reference to the value. In calculating specific duties, allowances are made (1) for the weight of the box, cask, or bag containing the goods, called **tare**; (2) for the loss of liquids in barrels or casks, called **leakage**; (3) for the loss of liquids in bottles, called **breakage**. The weight of goods before any allowances have been made is called the **gross weight**, and the weight after all allowances have been made is called the **net weight**.

I. Find the duty, at 35% ad valorem, on linen goods invoiced at £128 7s. 6d.

```
12)6. d.
20)7.5 s.
£128.375
  4.8665
  641875
 770250
770250
1027000
513500
$624.7369375
```

```
 624.74
    .35
 312370
187422
218.6590
```

Ans. $218.66.

The value of the goods in United States money is $624.74, and 35% of $624.74 is $218.66.

II. Find the duty at 4 cents a gallon, on 150 hhd. of molasses, 63 gal. in a hogshead, leakage 2%.

```
  63        9450
 150         189
9450        9261
 .02         .04
189.00   $370.44
```

150 hhd. equals 9450 gal. The leakage is 2% of this quantity, which equals 189 gal. This leaves 9261 gal. on which duty must be paid. At 4 cts. a gallon, the duty on 9261 gal. is 9261 times 4 cts., which equals $370.44.

EXAMPLES.

1. Find the duty, at 50% ad valorem, on 1650 yd. of silk valued at $1.85 per yard.

2. Find the duty, at 50 cts. per gallon, on 40 casks of sherry wine, each containing 30 gal.

3. What is the duty, at $2\frac{1}{4}$ cts. per pound, on 3500 lb. of rice, tare 5%?

4. What is the duty, at 3 cts. per pound, on 80 hhd. of sugar, each weighing 500 lb., tare $12\frac{1}{2}$%?

5. A merchant imported a lot of china, invoiced at £422 10 s.; what was the duty at 55% ad valorem?

6. Find the duty, at $2.50 per pound and 25% ad valorem, on 500 boxes of cigars, each containing 50 cigars, invoiced at $4.50 per box, and weighing $12\frac{1}{2}$ lb. per 1000.

7. A jeweller imported from Geneva 40 doz. watches, invoiced at 312.50 francs per dozen; find the duty at 25% ad valorem.

8. A merchant imported 1800 yd. of Brussels carpeting $\frac{3}{4}$ yd. wide, invoiced at 4 s. 6 d. per yard; find the duty at 30 cts. per square yard, and in addition 30% ad valorem.

9. Find the cost per dozen if the duty on 30 doz. clocks at 30% ad valorem is $135.

10. A merchant imported a lot of needles valued at $600, and paid $210 duty; what was the rate of duty?

11. A man paid $51.25, including a duty of 25%, for a watch; how much was the duty?

12. Find the entire cost of a lot of glassware on which a duty of $623.70 was paid; the duty was 45% ad valorem, and damages of 16% were allowed for breakage.

MISCELLANEOUS EXAMPLES.

1. The difference between $12\frac{1}{2}\%$ and $16\frac{2}{3}\%$ of a number equals 60; what is the number?

2. What per cent of a pound Troy is a pound Avoirdupois?

3. A horse costs $\frac{3}{4}$ as much as a carriage; what per cent of the price of the horse was the price of the carriage?

4. A gain of 25% was made by selling flour at $6 per barrel; at what price ought it to be sold to gain $33\frac{1}{3}\%$?

5. If 10% is lost by selling boards at $7.20 per M, what per cent would be gained by selling them for 90 cents per C?

6. When eggs were sold at $83\frac{1}{3}\%$ of the cost, there was a loss of 5 cents a dozen; how much would have been lost by selling them for 80% of the cost?

7. If the cost be three fifths of the selling price, what is the gain per cent?

8. A man sold 160 acres of land for $4563.20, which was 8% less than it cost. What was the cost per acre?

9. A man bought a yacht for $3500, and sold it at a loss of 20%; the buyer sold it at a gain of 25%; what did the latter receive for it?

10. A grocer bought a barrel ($31\frac{1}{2}$ gal.) of vinegar for $5.95, and $5\frac{5}{9}\%$ of it leaked out; for how much a gallon must the remainder be sold in order to gain 20%?

11. A jeweller sold two watches for $60 each; on one he gained 25%, and on the other he lost 25%; did he gain or lose, and how much?

12. A clock, marked 40% above cost, was sold at a reduction of 40%; what was the gain or loss per cent?

13. A man sold two horses, which he had previously purchased, for \$200 each. On one of them he made a profit of $33\frac{1}{3}\%$, and on the other he incurred a loss of 20%. Did he gain or lose in the transaction? What did each horse cost him?

14. A horse worth \$250 was bought for \$25 less, and sold for \$25 more than its real value; what was the gain per cent?

15. If a piece of land is bought for \$3500, and a man who owns $\frac{2}{5}$ of it, sells one half of his share for \$800, how much per cent does he gain by the transaction?

16. 75% of the area of a farm is arable; of the remainder 85% is pasture, and the rest is waste; if the area of the waste is 3 A. 20 sq. rd., what is the area of the farm?

17. For each of three successive years the population of a town rose 50%, and at the end of the third year it was 2700; what was the population at the beginning of the time?

18. The population of a city in 1871 increased 4% on that of 1870, in 1872 it increased 5% on that of 1871, and in 1873 it increased 6% on that of 1872 and amounted to 1389024; find the population in 1870.

19. A man invests a sum of money in such a way that it gains 10%; he adds the gain to the original sum, and invests the whole in such a way as to gain 15%; the entire amount is then \$50600; what is the amount originally invested?

20. A man put \$780 in the bank, which was 15% of all his money; he afterwards deposited 25% of the remainder of his money; how much money had he then in the bank, and what per cent was this of all his money?

21. A rectangular field contains 110 acres, and $37\frac{1}{2}\%$ of the length is 381.183 yards; what is the breadth in rods?

22. A and B gain in business \$5040, of which A is to have 10% more than B; what is the share of each?

23. A commission merchant sold 350 barrels of flour, charging $2\frac{1}{3}\%$ commission and 2% for guaranteeing payment; the net proceeds amounted to \$2511.25; find the price of the flour per barrel.

24. My agent sells for me 2000 yards of cloth at 24 cents a yard; he allows the purchaser 5% discount for cash, and charges me $2\frac{1}{2}\%$ on the cash receipts; how much money does he pay over to me?

25. At \$4 a ton, how many tons of coal can be bought for \$8526, after paying a commission of $1\frac{1}{2}\%$?

26. For what sum must a building valued at \$31200 be insured so that, in case of fire, the owner may recover both the value of the building and the premium of $2\frac{1}{2}\%$?

27. A jeweller pays \$14.80 duty on an imported watch, which is 20% ad valorem; what must he sell the watch for in order to gain 25%?

28. A town is taxed \$3600. The real estate of the town is valued at \$560000 and the personal property at \$152500. There are 600 polls, each of which is taxed \$1.25. What is the tax on \$1000? What is A's tax, who pays for four polls, and owns real estate valued at \$4100 and personal property valued at \$1800?

CHAPTER X.

INTEREST AND DISCOUNT.

126. Money paid for the use of money is called **interest.** The sum of money for the use of which interest is paid is called the **principal,** and the sum of the principal and interest is called the **amount.** Interest reckoned on the principal alone is called **simple interest.**

The **rate of interest** is the number of hundredths of the principal taken as the interest for one year. The legal rate of interest varies in different states, but 6% is the most common. In this book 6% is to be understood when no rate of interest is mentioned.

In computing interest it is customary to consider a year as consisting of 12 months of 30 days each. At 6% the interest on $1 for one year is 6 cents; for one month it is $\frac{1}{12}$ of 6 cents, or 5 mills; for one day it is $\frac{1}{30}$ of 5 mills, or $\frac{1}{6}$ of a mill. Hence, to find the interest on $1 for any given time at 6%, *take six times as many cents as there are years, five times as many mills as there are months, and one sixth as many mills as there are days, and find their sum.*

I. Find the interest of $237.72 for 3 yr. 9 mo. 23 da. at 6%.

3 yr. 18	237.72	
9 mo. . . .045	$.228\frac{5}{6}$	
23 da. . . $.003\frac{5}{6}$	11886	
$.228\frac{5}{6}$	7924	
	190176	
	47544	
	47544	
Ans. $54.40.	54.39826	

The interest on $1 for 3 yr. 9 mo. 23 da. is found by the method just given to be $0.228\frac{5}{6}$. The interest on $237.72 is 237.72 times $0.228\frac{5}{6}$, which equals $54.40.

NOTE. In computing interest the only fractions that can occur are $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$. The best way to multiply by $\frac{2}{3}$ is to set down $\frac{1}{3}$ of the multiplicand twice; to multiply by $\frac{5}{6}$, set down $\frac{1}{2}$ and $\frac{1}{3}$ of the multiplicand.

After finding the interest at 6%, the interest at any other rate can easily be found by dividing by 6, and then multiplying by the number expressing the rate. The interest at certain rates can be found more briefly as follows:

3% . . . divide the interest at 6% by 2.
4% . . . subtract from the interest at 6% $\frac{1}{3}$ of itself.
$4\frac{1}{2}$% . . subtract from the interest at 6% $\frac{1}{4}$ of itself.
5% . . . subtract from the interest at 6% $\frac{1}{6}$ of itself.
$5\frac{1}{2}$% . . subtract from the interest at 6% $\frac{1}{12}$ of itself.
$6\frac{1}{2}$% . . add to the interest at 6% $\frac{1}{12}$ of itself.
7% . . . add to the interest at 6% $\frac{1}{6}$ of itself.
$7\frac{1}{2}$% . . add to the interest at 6% $\frac{1}{4}$ of itself.
8% . . . add to the interest at 6% $\frac{1}{3}$ of itself.
9% . . . add to the interest at 6% $\frac{1}{2}$ of itself.

II. Find the amount of \$1345.50 from Oct. 16th, 1883 to May 14th, 1888 at $4\frac{1}{2}$%.

1888— 5—14
1883—10—16
4— 6—28

4 yr. . . .24
6 mo. . . .03
28 da. . . $.004\frac{2}{3}$
$.274\frac{2}{3}$

1345.50
$.274\frac{2}{3}$
44850
44850
538200
941850
269100
4)369.56400
92.391
277.17
1345.50
\$1622.67

By the method shown in § 68 we find the difference between the dates to be 4 yr. 6 mo. 28 da. The interest of \$1345.50 for this time at 6% is \$369.56. Subtracting from this result $\frac{1}{4}$ of itself, we obtain \$277.17 as the interest at $4\frac{1}{2}$%. The amount equals the sum of the principal and interest, which is \$1622.67.

NOTE. In computing interest on English money, the principal should be expressed in pounds and decimal of a pound; the process then is exactly the same as with United States money. The decimal of the result should be reduced to lower denominations.

EXAMPLES.

Find the interest of

1. \$1000 for 1 yr. 2 mo. 12 da. at 6%.
2. \$257 for 3 yr. 7 mo. 24 da. at 6%.
3. \$237.28 for 7 yr. 2 mo. 9 da. at 6%.
4. \$178.99 for 3 yr. 11 mo. 14 da. at 6%.
5. \$1563.45 for 4 yr. 11 mo. 1 da. at 6%.
6. \$1000 for 6 yr. 4 mo. 15 da. at 8%.
7. \$1000 for 5 yr. 4 mo. 15 da. at. 20%.
8. \$1385.50 for 23 da. at 7%.
9. \$1461.75 for 4 yr. 9 mo. at 8%.
10. \$240 for 5 yr. 4 mo. at 7%.
11. \$850 for 2 yr. 5 mo. 20 da. at 4%.
12. \$1584 for 1 yr. 2 mo. 20 da. at 7%.
13. \$375.75 for 4 yr. 5 mo. 25 da. at $4\frac{1}{2}$%.
14. \$175 for 2 yr. 7 mo. 17 da. at 5%.
15. \$312.17 for 5 yr. 5 mo. 5 da. at 7%.
16. \$206264.80 for 7 mo. 7 da. at 3%.
17. \$90.25 from Feb. 7th, 1884 to May 11th, 1887 at 6%.
18. \$76.72 from Apr. 18th, 1882 to Jan. 26th, 1885 at 6%.
19. \$196.54 from Aug. 15th, 1872 to May 12th, 1880 at 6%.

20. \$15.82 from Dec. 26th, 1881 to July 2nd, 1882 at $7\frac{1}{2}\%$.

21. \$25 from Nov. 10th, 1884 to July 1st, 1887 at 5%.

22. \$64.50 from June 25th, 1885 to Aug. 10, 1887 at $6\frac{1}{4}\%$.

23. \$257.81 from Jan. 3rd, 1883 to Apr. 6th, 1883 at 8%.

24. \$580 from May 16th, 1882 to Oct. 8th, 1883 at 5%.

25. \$875.26 from Oct. 10th, 1876 to July 10th, 1878 at $7\frac{3}{4}\%$.

26. £1 for 1 mo. at 5%.

27. £17 8s. 3d. for 2 yr. 6 mo. 17 da. at $7\frac{1}{2}\%$.

28. £757 17s. 6d. for 1 yr. 3 mo. 10 da. at $4\frac{1}{2}\%$.

Find the amount of

29. \$333.33 for 3 yr. 3 mo. 3 da. at 3%.

30. \$369.29 for 2 yr. 3 mo. 1 da. at 9%.

31. \$547.63 for 5 yr. 5 mo. 25 da. at $4\frac{1}{2}\%$.

32. \$762 for 3 yr. 4 mo. 26 da. at $4\frac{1}{4}\%$.

33. \$647.21 for 4 yr. 11 mo. 11 da. at $3\frac{1}{2}\%$.

34. \$392.10 for 6 yr. 9 mo. 15 da. at $3\frac{3}{4}\%$.

35. \$175 from Apr. 1st, 1882 to Jan. 15th, 1883 at $5\frac{1}{2}\%$.

36. \$2368 from Dec. 19th, 1879 to June 18th, 1885 at $6\frac{1}{2}\%$.

37. \$96.52 from Nov. 25th, 1887 to Mar. 1st, 1888 at 4%.

38. \$842.70 from Mar. 5th, 1880 to Aug. 2nd, 1887 at $5\frac{1}{4}\%$.

39. £22 10s. for 3 yr. 10 mo. at $3\frac{1}{2}\%$.

40. £50 12s. 5d. for 5 yr. 2 mo. 3 da. at 8%.

41. £46 6s. 8d. from June 20th, 1868 to May 5th, 1875 at 4%.

127. When the time is short, bankers and business men generally compute interest for the exact number of days, allowing 360 days to the year.

I. Find the interest of $175.50 from Apr. 28th to Aug. 10th at 7%.

$$\begin{array}{r} 2 \\ 31 \\ 30 \\ 31 \\ \underline{10} \\ 104 \text{ da.} \end{array} \qquad \begin{array}{r} 175.50 \\ \underline{104} \\ 70200 \\ \underline{17550} \\ 6000)\overline{18252.00} \\ 6)\overline{3.042} \\ \underline{.507} \\ 3.549 \end{array}$$

Ans. $3.55.

By the method shown in § 68 we find the exact number of days to be 104. The interest of $1 for 1 day at 6% is $\frac{1}{6}$ of a mill, or $\frac{1}{6000}$ of a dollar; the interest of any principal is $\frac{1}{6000}$ of the principal. Hence the interest for 104 days is $\frac{104}{6000}$ of the principal, which is found by multiplying by 104 and dividing by 6000. We thus find the interest at 6% to be $3.04. Adding to this result $\frac{1}{6}$ of itself, we obtain $3.55 as the interest at 7%.

This method may be stated as follows: To find the interest on any principal at 6%, *multiply the principal by the number of days, and divide by 6000.*

EXAMPLES.

Find the interest of

1. $421 for 62 da. at 6%.
2. $67.42 for 37 da. at 6%.
3. $104.25 for 90 da. at $7\frac{1}{2}$%.
4. $532.50 for 47 da. at $5\frac{1}{2}$%.
5. $1332 from June 15th to Oct. 24th at $4\frac{1}{2}$%.
6. $2375 from Jan. 1st to Mar. 20th at 8%.
7. $542.36 from Mar. 29th to Nov. 9th at 5%.
8. $36.75 from Apr. 12th to Aug. 2nd at $6\frac{1}{2}$%.

9. \$212.91 from June 30th to Dec. 1st at $5\frac{1}{4}\%$.

10. \$371.14 from Nov. 8th, 1886 to Mar. 16th, 1887 at $4\frac{3}{4}\%$.

11. \$614.25 from Dec. 16th, 1887 to Apr. 25th, 1888 at $7\frac{3}{10}\%$.

12. \$74.16 from Sept. 7th, 1888 to Jan. 10th, 1889 at $6\frac{1}{4}\%$.

Exact Interest.

128. The common methods used in computing interest are based upon the supposition that there are 360 days in a year; this, however, does not give strictly accurate results. To obtain the **exact interest**, we must reckon 365 days in a year, a method used by the United States Government and by some bankers and business men.

I. Find the exact interest of \$834.32 from Aug. 7th, 1884 to Jan. 12th, 1888 at 5%.

24	834.32	41.716	125.148
30	.05	158	18.057
31	41.7160	333728	\$143.21
30	3	208580	
31	125.148	41716	
12		365)6591.128(18.057	
158		365	
3 yr. 158 da.		2941	
		2920	
		2112	
		1825	
		2878	
		2555	

The exact time from Aug. 7th, 1884 to Jan. 12th, 1888 is 3 yr. 158 da. The interest of \$834.32 for 1 yr. at 5% is \$41.716; for 3 yr. it is 3 times \$41.716, or \$125.148. For 158 da. the interest is $\frac{158}{365}$ of \$41.716, which equals \$18.057. Adding these two results, we find the interest for the entire time to be \$143.21.

NOTE. The exact interest for 360 da. is $\frac{360}{365}$, or $\frac{72}{73}$, of the interest for a year, whereas by the common methods the interest for 360 da. equals the interest for a year; hence, for any number of days less than a year, the exact interest is $\frac{72}{73}$ of the common interest, and the exact interest can be found by deducting from the common interest $\frac{1}{73}$ of itself.

EXAMPLES.

Find the exact interest of

1. \$1642 for 95 da. at 6%.
2. \$805 for 163 da. at 7%.
3. \$222.50 for 208 da. at $4\frac{1}{2}$%.
4. \$347.75 from Jan. 9th to May 26th at $7\frac{1}{2}$%.
5. \$973.96 from Mar. 18th to Aug. 30th at $3\frac{1}{2}$%.
6. \$335.46 from June 15th to Oct. 31st at $7\frac{3}{10}$%.
7. \$581.38 from Apr. 19th to Dec. 4th at 3%.
8. \$767.25 from Nov. 17th, 1885 to Feb. 10th, 1887 at 6%.
9. \$91.80 from June 1st, 1886 to Sept. 4th, 1888 at 4%.
10. \$504.42 from Dec. 17th, 1883 to May 9th, 1887 at $3\frac{1}{2}$%.

TO FIND THE RATE PER CENT WHEN THE PRINCIPAL, INTEREST (OR AMOUNT), AND TIME ARE GIVEN.

129. I. Find the rate per cent when the interest of \$160 for 2 yr. 3 mo. 9 da. is \$19.11.

2 yr. . . .12
3 mo. . . .015
9 da. . . .$001\frac{1}{2}$
$.136\frac{1}{2}$

160
$.136\frac{1}{2}$
6)21.840
3.64

3.64)19.11($5\frac{1}{4}$%
1820
$\frac{91}{364} = \frac{1}{4}$

The interest of \$160 for 2 yr. 3 mo. 9 da. at 1% is \$3.64. To produce an interest of \$19.11, the rate per cent is as many times 1% as \$3.64 is contained times in \$19.11, which equals $5\frac{1}{4}$%.

NOTE. When the amount is given, the interest can be found by subtracting the principal from the amount.

EXAMPLES.

1. At what rate per cent must $370 be put on interest to gain $55.50 in 3 yr.?

2. If $50 gain $5.60 in 3 yr. 6 mo., what is the rate per cent?

3. At what rate per cent will $350 amount to $423.50 in 5 yr. 3 mo.?

4. At what per cent must $750 be loaned to amount to $876 in 2 yr. 4 mo. 24 da.?

5. At what rate per cent will $240 in 5 yr. give $64 interest?

6. The interest on $325.72 for 2 yr. 4 mo. is $34.20; what is the rate per cent?

7. At what rate per cent will $2500 amount in 3 yr. to $4320?

8. The interest on $437.21 for 9 yr. 9 mo. is $127.884; what is the rate per cent?

9. At what rate per cent will $850 earn $97.18 in 1 yr. 7 mo. 18 da.?

10. What is the rate per cent when $320 gains $5.28 from Aug. 24th to Oct. 30th?

11. The interest of $720 from Feb. 1st, 1886 to Apr. 13th, 1887 was $69.12; what was the rate per cent?

12. The interest on $127.50 from June 26th, 1798 to May 8th, 1802 was $36.975; calculate the rate per cent.

13. At what rate per cent will a sum of money double itself in 6 yr.?

14. At what rate per cent will a sum of money double itself in 12 yr. 6 mo.?

15. What is the rate per cent when a sum of money earns a sum $\frac{3}{5}$ as large in 7 yr. 2 mo.?

16. A person borrows \$500 on Apr. 10th, and on June 22nd pays his debt with \$510.20. At what rate per cent was he charged interest?

17. If I buy a house for \$5620 and receive \$1803 for rent in 2 yr. 3 mo. 15 da., what rate of interest do I get for my money?

18. The interest on £50 12 s. 6 d. for a year is £1 15 s. $5\frac{1}{4}$ d.; what is the rate per cent?

To Find the Time when the Principal, Interest (or Amount), and Rate Per Cent are Given.

130. I. Find the time in which the interest of \$446 at 5% will amount to \$46.75.

```
  446                     2.096 yr.
  .05                        12
22.30)46.75(2.096         1.152 mo.
       4460                  30
       21500              4.56 da.
       20070
        14300
        13380
```

The interest of \$446 for 1 yr. at 5% is \$22.30. To produce an interest of \$46.75 requires as many years as \$22.30 is contained times in \$46.75, which equals 2.096 yr., or 2 yr. 1 mo. 5 da.

Ans. 2 yr. 1 mo. 5 da.

Note. When there is a fraction of a day, less than half a day should be rejected; half a day or more should be called another day.

EXAMPLES.

1. How long must \$180 be on interest to gain \$99 at $5\frac{1}{2}\%$?

2. How long must \$133 be on interest at 7% to gain \$32.585?

3. In what time will \$680 at 4% amount to \$727.60?

4. How long must \$350 be on interest at 6% to amount to \$404.25?

5. In what time will \$260 gain \$53.30 at 6%?

6. How long must \$125 be on interest at $7\frac{1}{2}$% to gain \$15?

7. In what time at 6% will \$240 amount to \$720?

8. How long must \$1800 be on interest at $3\frac{1}{2}$% to gain \$11.55?

9. In what time will \$340 produce \$111.35 interest at 5%?

10. In what time will \$4500 at 5% gain \$181.25?

11. How long must \$360 be at interest at 6% to amount to \$386.70?

12. How long must \$350 be at interest at 6% to amount to \$395.50?

13. In what time will \$2275 amount to \$2673.125 at 5%?

14. In how many days will \$3245 gain \$80 at 7%?

15. A man received \$136.75 for the use of \$1820, which was 6% for the time; what was the time?

16. Calculate the date at which a sum of \$450, which was put at interest at 8% Dec. 30th, 1797, amounted to \$642.30.

17. Calculate the date at which a sum of \$234, which was put at interest at 9% Oct. 25th, 1798, amounted to \$351.

18. In what time will any principal double itself at 6%?

19. In what time will the interest on a sum of money be $\frac{2}{3}$ of the principal at $4\frac{1}{2}$%?

20. A certain sum of money was put at interest at 9% Dec. 21st, 1790; at what date did it become tripled?

21. A certain sum of money was put at interest at $7\frac{3}{10}\%$ Oct. 30th, 1836; at what date did it become tripled?

22. In what time will £1225 amount to £1417 18s. 9d. at 3%?

To Find the Principal when the Interest (or Amount), Rate Per Cent, and Time are Given.

131. I. Find the principal on which the interest for 3 yr. 9 mo. 18 da. at 6% is \$210.90.

```
 3 yr. . . .18         .228)210.900($925
 9 mo. . . .045              2052
18 da. . . .003              ----
           ----               570
           .228               456
                              ----
                              1140
                              1140
                              ----
```

The interest of \$1 for 3 yr. 9 mo. 18 da. at 6% is \$0.228. To produce an interest of \$210.90 will require as many dollars as \$0.228 is contained times in \$210.90, which equals \$925.

II. Find the principal which will amount to \$1500 in 1 yr. 7 mo. 20 da. at 4%.

```
 1 yr. . . .06            1.065 5/9)1500.
 7 mo. . . .035                 9      9
20 da. . . .003 1/3       -----------------
           ---------      9.59 ) 13500.00($1407.72
         3).098 1/3               959
           ---------              ----
            .032 7/9              3910
           ---------              3836
            .065 5/9              -----
                                   7400
                                   6713
                                   -----
                                    6870
                                    6713
                                    -----
                                    1570
```

The amount of \$1 for 1 yr. 7 mo. 20 da. at 4% is $\$1.065\frac{5}{9}$. It will require as many dollars to amount to \$1500 as $\$1.065\frac{5}{9}$ is contained times in \$1500, which equals \$1407.72.

EXAMPLES.

1. What sum invested at 4% will yield an annual income of $100?

2. What principal at 5% will gain $15 in 6 mo.?

3. What principal will yield an interest of $339.20 in 5 yr. 4 mo. at 6%?

4. What principal will yield an interest of $15.40 in 1 yr. 3 mo. at 7%?

5. What principal at 6% will amount to $3605.85 in 16 mo.?

6. What sum of money at 5% will amount to $2375.38 in 3 yr. 6 mo.?

7. What is the sum of money which if put at interest for 2 yr. 3 mo. at 4% will amount to $230?

8. What sum of money will produce $3437.74 interest in 7 mo. at 3%?

9. What principal will gain $176.25 in 2 yr. 4 mo. 6 da. at 5%?

10. At 4½% what principal will yield an interest of $360 in 7 mo. 15 da.?

11. What principal will gain $30 in 60 days at 2% a month?

12. What principal will yield $1.70 interest in 25 days at 6%?

13. What principal will in 5 yr. 8 mo. 15 da. at 5% give $287.70 interest?

14. What sum of money put at interest at 8% for 2 yr. 1 mo. 15 da. will amount to $842.40?

15. What sum at 4% will amount to \$578.88 in 1 yr. 9 mo. 18 da.?

16. Find the sum on which the interest at 9% for 5 yr. 1 mo. 18 da. is \$947.10.

17. What sum of money put at interest for 6 yr. 5 mo. 11 da. at 7% will earn \$3159.14?

18. Find the principal which will amount to \$2500 in 3 yr. 3 mo. 10 da. at $4\frac{1}{2}$%.

19. What principal will in 7 yr. 7 mo. 7 da. amount to \$700 at 7%?

20. The interest on a certain principal from Sept. 16th, 1884 to Oct. 20th, 1886 at 6% was \$22.37; what was the principal?

21. What sum of money put at interest July 23rd, 1885 at 6% will amount to \$1842 Mar. 11th, 1887?

22. What principal will produce \$125 interest from May 24th, 1883 to Nov. 5th, 1884 at 8%?

Promissory Notes.

132. A **promissory note,** commonly called a **note,** is a written or printed promise to pay a specified sum of money on demand or at a specified time. The sum whose payment is promised is called the **face** of the note; it should be written in words in the body of the note, and in figures either at the top or the bottom. The person signing the note is called the **maker**; the person to whom the sum of money is payable is the **payee**; the owner of the note is the **holder**; and a person who writes his name on the back of the note as security for its payment is an **indorser**.

A **negotiable note** is one that can be sold or transferred. To be negotiable it must be payable to the "bearer," or to

the "**order**" of the payee. A note should contain the words "value received," otherwise the holder may be required to prove that its value was received by the maker.

A note is nominally due at the expiration of the specified time; it **matures**, or becomes legally due, three days after the specified time has expired; these three days are called **days of grace**. The time when a note is due is often indicated by writing the date when nominally due and the date of maturity with a line between; for example, Oct. $^9/_{12}$, 1888.

If a note contains the words "with interest," it draws interest from date; otherwise it draws interest from the time of maturity until paid.

A **protest** is a written notice made by a notary public to the indorsers that the note has not been paid. A protest must be made out on the last day of grace, otherwise the indorsers are released from their obligation.

Each state makes its own laws in regard to all kinds of negotiable paper, and any note is governed by the laws of the state in which it is payable.

The following are common forms of promissory notes:

INDIVIDUAL NOTE.

$918 $\frac{32}{100}$. BOSTON, MASS., *Apr. 30th,* 188*9*.

For value received, *I* promise to pay to *James Brown,* ~~~~~ or order, ~~~ *Nine hundred eighteen* ~~~~ $\frac{32}{100}$ Dollars, in *Six months* from date.

Charles Howe.

NOTE PAYABLE AT A BANK.

$856 $\frac{25}{100}$. PITTSBURG, PA., *June 12th,* 188*9*.

Thirty days after date *we* promise to pay to the order of *John P. Alger,* *Eight hundred fifty-six* $\frac{25}{100}$ Dollars, at the **Allegheny National Bank.** Value received, without defalcation.

Wheeler & Jones.

NOTE. The law of Pennsylvania requires the words "without defalcation."

NON-NEGOTIABLE NOTE.

New York, Oct. 16th, 1888.

On demand, I promise to pay Frank H. Wood Fifty Dollars, for value received, with interest at six per cent per annum.

$50 $\frac{00}{100}$. *Henry Robinson.*

Partial Payments.

133. When **partial payments** are made on notes or other obligations bearing interest, allowance is made for these partial payments in computing the amount due at the time of settlement. The amounts of the payments and their dates are written on the back of the obligation; such payments are called **indorsements.** The Supreme Court of the United States has adopted a rule for partial payments, which is known as

The United States Rule.

Find the amount of the principal to the time when the payment, or the sum of the payments, equals or exceeds the interest due. From the amount subtract the payment, or the sum of the payments, and, with the remainder as a new principal, proceed as before to the time of settlement.

This rule is based on the following principles:

1st, payments must be applied first to the discharge of interest due, and the balance, if any, toward the discharge of the principal.

2nd, unpaid interest must not be added to the principal to draw interest.

3rd, only unpaid principal can draw interest.

I. A note for $750 is dated July 11th, 1884 and bears the following indorsements: Feb. 17th, 1885, $225; Dec. 24th, 1885, $25; May 19th, 1886, $375. What balance is due Dec. 1st, 1886, reckoning interest at 6%?

1885 — 2 — 17
1884 — 7 — 11
7 — 6
.036

1885 — 12 — 24
1885 — 2 — 17
10 — 7
$.051\frac{1}{6}$

1886 — 5 — 19
1885 — 2 — 17
1 — 3 — 2
$.075\frac{1}{3}$

1886 — 12 — 1
1886 — 5 — 19
6 — 12
.032

750.
.036
4500
2250
27.000
750.
777.
225.
552.
$.051\frac{1}{6}$
92
552
2760
28.244

552
$.075\frac{1}{3}$
184
2760
3864
41.584
552.
593.58
400.
193.58
.032
38716
58074
6.19456
193.58
$199.77

The amount of $750 from July 11th, 1884 to Feb. 17th, 1885 is $777, and subtracting the 1st payment from this amount, we obtain $552 for the 2nd principal. The interest of $552 from Feb. 17th, 1885 to Dec. 24th, 1885 is $28.24, which is more than the 2nd payment; hence we pass on to the next date. The amount of $552 from Feb. 17th, 1885 to May 19th, 1886 is $593.58, and subtracting the sum of the 2nd and 3rd payments from this amount, we obtain $193.58 for the 3rd principal. The amount of $193.58 from May 19th, 1886 to Dec. 1st, 1886 is $199.77.

NOTE 1. When the rate is any other than 6%, each separate interest must be found at the given rate.

NOTE 2. When the interest is required on a principal on which partial payments have been made, find the amount due, and from that amount subtract the difference between the principal and the sum of the payments.

EXAMPLES.

1. On a note for \$1400, given Apr. 12th, 1882, two payments were made: Aug. 30th, 1884, \$400; Aug. 30th, 1886, \$600. At 6% interest, what was due Dec. 30th, 1887?

2. A note for \$1000, dated Jan. 1st, 1883, and bearing interest at 6%, is indorsed with three payments of \$80 each, made on Jan. 1st of 1884, 1885, and 1886. What was due on the note at settlement Oct. 1st, 1886?

3. Find what is due Oct. 1st, 1888 on a note for \$1750 at 6% interest, dated Dec. 13th, 1884, with three payments indorsed, viz.: June 10th, 1886, \$360; Jan. 1st, 1887, \$40; Aug. 10th, 1887, \$500.

4. On a note for \$2000, dated July 15th, 1885, and bearing interest at 7%, there was paid \$100 May 5th, 1886, and \$200 Jan. 1st, 1887. Find what remained due Aug. 1st, 1887.

5. A note of \$600, dated Aug. 10th, 1885, had indorsements as follows: Feb. 4th, 1886, \$50; July 27th, 1886, \$10; Oct. 9th, 1886, \$75. How much was due Dec. 15th, 1886 at 5% interest?

6. A note for \$1580, dated Oct. 19th, 1882, is indorsed Sept. 6th, 1883, with \$640; Jan. 30th, 1884, with \$20; Oct. 9th, 1884, with \$380. What balance is due Feb. 3rd, 1885, interest at $4\frac{1}{2}$%?

7. A note for \$300, dated May 15th, 1878, and bearing interest at 5%, is indorsed as follows: Feb. 25th, 1881, \$40; Sept. 15th, 1882, \$25; May 4th, 1883, \$150. What balance is due Jan. 1st, 1884?

8. On a note for \$1000, dated Jan. 1st, 1878, due in one year, and bearing interest at the rate of 6% from the date of maturity, the following payments were made: Aug. 16th,

1879, \$300; Feb. 12th, 1880, \$200; Oct. 3rd, 1881, \$50; Jan. 17th, 1882, \$19; May 31st, 1883, \$22. What was due Jan. 1st, 1884?

9. What is the interest at $4\frac{1}{2}\%$ of \$360.45 from July 5th, 1883 to Nov. 4th, 1885, allowing a credit of \$75 paid Oct. 6th, 1884?

134. When the whole period of time is not longer than one year, business men commonly employ

The Merchants' Rule.

Find the amount of the principal for the whole time the note is on interest; find also the amount of each payment from the time it was made until settlement; from the amount of the principal subtract the amounts of the payments.

I. Find the balance due Mar. 1st, 1888 on a note for \$875, given Apr. 18th, 1887, on which the following payments had been made: July 7th, 1887, \$360; Oct. 28th, 1887, \$250.

1888 – 3 – 1	1888 – 3 – 1	1888 – 3 – 1	374.04
1887 – 4 – 18	1887 – 7 – 7	1887 – 10 – 28	255.13
10 – 13	7 – 24	4 – 3	629.17
$.052\frac{1}{6}$	.039	$.020\frac{1}{2}$	

875.	360.	250.	920.65
$.052\frac{1}{6}$	.039	$.020\frac{1}{2}$	629.17
$145\frac{5}{6}$	3240	125	\$291.48
1750	1080	5000	
4375	14.040	5.125	
$45.645\frac{5}{6}$	360.	250.	
875.	374.04	255.13	
920.65			

The amount of \$875 from Apr. 18th, 1887 to Mar. 1st, 1888 is \$920.65. The amount of \$360 from July 7th, 1887 to Mar. 1st, 1888 is \$374.04,

and the amount of \$250 from Oct. 28th, 1887 to Mar. 1st, 1888 is \$255.13. The sum of the amounts of the payments is \$629.17, and subtracting this from \$920.65, we find \$291.48 to be the balance due.

EXAMPLES.

1. On a note for \$1500, dated Jan. 1st, 1886, and bearing interest at $4\frac{1}{2}\%$, there was paid \$550 Apr. 1st, 1886, and \$725 Oct. 1st, 1886. Find what remained due Jan. 1st, 1887.

2. A lends B \$1000 Feb. 12th, 1885; B pays \$200 Mar. 27th, 1885, and \$50 Dec. 12th, 1885. Find what is due Jan. 18th, 1886 at 6% interest.

3. Find what is due Oct. 1st, 1888 on a note for \$2500 at 9% interest, dated Jan. 1st, 1888, on which payments of \$600 each have been made: Mar. 1st, May 1st, and July 1st, 1888.

4. Find the balance due Sept. 1st, 1888 on a note for \$600, given Sept. 1st, 1887, on which the following payments had been made: Feb. 15th, 1888, \$120; May 24th, 1888, \$350; July 20th, 1888, \$100.

5. A note for \$1372.50, dated Nov. 10th, 1887, and bearing interest at 7%, is indorsed as follows: Jan. 20th, 1888, \$321; Mar. 29th, 1888, \$490; June 14th, 1888, \$275. What balance is due Sept. 10th, 1888?

6. Payments were made on a debt of \$2470, due May 7th, 1886, as follows: June 24th, 1886, \$420; Aug. 3rd, 1886, \$345; Oct. 20th, 1886, \$500; Nov. 29th, 1886, \$790. What was due Jan. 1st, 1887 at 5% interest?

7. What is the interest at 5% of \$722.85 from Oct. 19th, 1886 to May 3rd, 1887, allowing a credit of \$500 paid Jan. 4th, 1887?

Compound Interest.

135. Compound interest is interest reckoned on both the principal and the unpaid interest added at regular intervals. The interest may be compounded, or added to the principal, annually, semi-annually, or for any other period of time according to agreement.

I. Find the compound interest of $800 for 2 yr. 8 mo. 12 da. at 7%.

```
     800
    1.07
    5600
    800
    856.
    1.07
    5992
   856
  915.92
    .042
  183184
 366368
6)38.46864
  6.41144
 44.88008
 915.92
 960.80
 800.
$160.80
```

The amount of $1 for 1 yr. at 7% is $1.07, and the amount of $800 is 800 times $1.07, which equals $856. Taking this as a new principal, we find the amount at compound interest for 2 yr. to be $915.92, which we take as the principal for the remaining 8 mo. 12 da. Thus the amount of $800 for 2 yr. 8 mo. 12 da. at compound interest is $960.80. The original principal subtracted from this amount gives $160.80 as the compound interest.

Note. When the interest is compounded semi-annually, the interest must be found for each half-year at one half the yearly rate, and similarly for any other period of time. Interest is compounded annually if nothing is stated to the contrary.

EXAMPLES.

1. What is the compound interest on $1000 for 3 yr. at 7%?

2. Find the amount of $100 at the end of 3 yr. at 4½% compound interest.

3. To how much will $1000 amount in 4 yr. at 20% compound interest?

4. How much would $350 amount to in 7 yr. at 6% compound interest?

5. What is the amount, at compound interest, of $500 for 2 yr. 6 mo. at 7%?

6. What is the compound interest of $25 for 3 yr. 5 mo. at 6%?

7. Find the amount of $1000 for 2 yr. 2 mo. 12 da. at 6% compound interest.

8. Find the compound interest of $200 for 2 yr. 6 mo. 18 da. at 4%.

9. What is the amount of $5216.75 from Jan. 21st, 1885 to July 3rd, 1888 at 8% compound interest?

10. What is the compound interest on £47 13s. 6d. for 3 yr. 4 mo. 15 da. at 3½%?

11. Find the compound interest of $720 for 2 yr. at 7%, interest being payable semi-annually.

12. What will be the amount of $103 for 2 yr. 6 mo. at 5%, the interest being compounded semi-annually?

13. What is the amount of $340 at 8% for 1 yr. 3 mo., the interest being compounded semi-annually?

14. What is the amount of $450 for 1 yr. 2 mo. 18 da. at 6%, interest compounding quarterly?

15. Find the compound interest of $122.50 from Sept. 1st, 1884 to Nov. 25th, 1885 at 4%, interest being payable quarterly.

To Find the Principal when the Compound Interest (or Amount), Rate Per Cent, and Time are Given.

136. I. Find the principal on which the compound interest for 2 yr. 6 mo. at 6% is $108.75.

1.06	.157308)108.750000($691.32
1.06	943848
636	1436520
106	1415772
1.1236	207480
1.03	157308
33708	501720
11236	471924
1.157308	297960

The compound interest of $1 for 2 yr. 6 mo. at 6% is $0.157308. To produce an interest of $108.75 will require as many dollars as $0.157308 is contained times in $108.75, which equals $691.32.

NOTE. When the amount is given in place of the interest, the divisor should be the amount of $1 for the given time at the given rate.

EXAMPLES.

1. What principal will in 2 yr. at 5% produce a compound interest of $350?

2. Find the principal on which the compound interest for 3 yr. at 4% is $468.24.

3. What sum of money at 6% compound interest will amount to $2703 in 1 yr. 4 mo.?

4. At 4% compound interest what sum of money will amount in 2 yr. to $594.88?

5. What sum of money, at 10% compound interest, will amount to $8651.50 in 3 yr.?

6. Find what principal will amount to $1000 in 3 yr. 6 mo. at $3\frac{1}{2}$% compound interest.

7. What principal will produce $250 interest in 1 yr. 8 mo. 24 da. at 6%, the interest being compounded semi-annually?

8. What principal will amount to $2000 in 1 yr. 4 mo. 15 da. at 4%, the interest being compounded quarterly?

ANNUAL INTEREST.

137. Annual interest is simple interest reckoned on the principal and also on each year's interest after it is due.

I. Find the annual interest of $1710 for 3 yr. 4 mo. 12 da. at 5%.

```
 3 yr. . . .18
 4 mo. . . .02
12 da. . . .002
           ----
           .202

     1710
     .202
     ----
     3420
   3420
 ---------
 6)345.420
 ---------
    57.57
   287.85
```

```
 2—4—12
 1—4—12
   4—12
 ------
 4—1— 6
   .246

    1710.
      .05
    -----
    85.50
     .246
    -----
    51300
   34200
  17100
 ----------
 6)21.03300
 ----------
    3.5055
   17.53
  287.85
  -------
 $305.38
```

The simple interest of $1710 for 3 yr. 4 mo. 12 da. is $287.85. The interest due at the end of the first year draws interest for 2 yr. 4 mo. 12 da.; the interest due at the end of the second year draws interest for 1 yr. 4 mo. 12 da.; the interest due at the end of the third year draws interest for 4 mo. 12 da.; the sum of the interests of the yearly unpaid interests is equivalent to the interest of one year's interest for the sum of these periods. The interest of $1710 for 1 yr. is $85.50, and the interest of $85.50 for 4 yr. 1 mo. 6 da. is $17.53; adding $287.85 to this amount, we find $305.38 to be the entire annual interest.

EXAMPLES.

1. What is the interest for 3 yr. on a debt of $1800 at 6% annual interest?

2. How much interest is due on a debt of $1500, at 6% annual interest, at the end of 3 yr. 6 mo.?

3. Find the annual interest of $1200 for 4 yr. 3 mo. 10 da. at 5%.

4. A note for $500, with annual interest at 6%, is due 4 yr. 6 mo. after date; if no interest has been paid, what will be due at maturity?

5. A note for $2250, with interest payable annually at 8%, was paid 3 yr. 3 mo. 18 da. after date, and no interest had been previously paid; what was the amount due?

6. A note was given May 8th, 1883 for $625, interest payable annually at 5%; if no payment is made, what will be due Mar. 8th, 1886?

7. Find the interest due Dec. 20th, 1888 on a note for $725, dated June 11th, 1884, with interest payable annually at 7%, when no interest has been paid.

8. Find the amount due Apr. 4th, 1888 on a note for $1150, dated Nov. 22nd, 1883, with interest payable annually at 4½%, when no interest has been paid.

TRUE DISCOUNT.

138. Discount is a deduction made for the payment of a debt before it is due.

The **present worth** of any sum of money due at a future time without interest, is that sum which put at interest for the given time, will amount to the given sum. The differ-

ence between the given sum and its present worth is called the **true discount.**

I. Find the present worth of $800, due in 1 yr. 7 mo. 24 da., at 5%.

1 yr. . . .06	1.0825)800.0000($739.03
7 mo. . . .035	75775
24 da. . . .004	42250
6).099	32475
.0165	97750
.0825	97425
	32500

The amount of $1 for 1 yr. 7 mo. 24 da. is $1.0825; hence the present worth of $1.0825 is $1. The present worth of $800 is as many times $1 as $1.0825 is contained times in $800, which equals $739.03. The process is the same as finding the principal, when the amount, rate per cent, and time are given, as shown in § 131.

NOTE. When the money is at compound interest, the amount of $1 should be found at compound interest.

EXAMPLES.

1. Find the present worth and discount of $3230, due in 4 yr. 10 mo. 12 da., at 6%.

2. Find the present worth and discount of $2000, due in 1 yr. 8 mo., at $4\frac{1}{2}$%.

3. Find the present worth and discount of $2500, due in 3 mo., at 8%.

4. Find the present worth and discount of $1926.94, due in 8 mo. 3 da., at 7%.

5. Find the present worth and discount of $1025, due in 36 da., at 10%.

6. What is the present worth of $3471.50, due 3 mo. 9 da. hence, at 7%?

7. What is the present worth of $1609.30, due in 10 mo. 24 da., when money is worth 5%?

8. Find the present worth of a note for \$313.31, due in 2 yr. 2 mo. 2 da., at $3\frac{1}{2}\%$.

9. What is the present worth of \$10000, due 3 yr. hence, at 5% compound interest?

10. What is the present worth of \$678.75, due 3 yr. 8 mo. hence, at 7% compound interest?

Bank Discount.

139. Bank discount is a deduction made by a bank for advancing money on a note before it is due, and it is the interest on the face of the note from the day of discount to the day of maturity; this period of time is called the **term of discount,** and the rate of interest is called the **rate of discount.**

The sum of money received for a note when it is discounted at a bank is called the **proceeds** or **avails,** and it equals the face of the note minus the bank discount.

NOTE. If no day of discount is given, a note is understood to be discounted on the day of its date; in such a case the term of discount equals the time specified in the note plus three days of grace.

When the time a note has to run is designated by months, the term of discount is determined by subtracting dates; when the time is designated by days, the term of discount is determined by exact days.

I. Find the bank discount on a note for \$450, due in 60 days, at 5%.

```
     450.
      63
    1350
    2700
6000)28350
   6)4.725
     .7875
   $3.94
```

The term of discount is 60 da. + 3 da., which is 63 da. The interest of \$450 for 63 da. equals \$3.94.

II. Find the proceeds of a note for \$1200, dated Apr. 10th, payable in 90 days, and discounted May 14th at 7%.

17	1200	1200.
30	59	13.77
12	10800	\$1186.23
59 da.	6000	
	6000)70800	
	6)11.80	
	1.967	
	13.77	

90 days after Apr. 10th is July 9th; hence the note becomes due July 9/12. The term of discount is from May 14th to July 12th, which equals 59 da. The interest of \$1200 for 59 da. at 7% is \$13.77. The proceeds is \$1200 − \$13.77, which equals \$1186.23.

Note. As the day of maturity was not needed in this problem, the term of discount could have been determined as follows: from Apr. 10th to May 14th is 20 da. + 14 da., or 34 da., and 93 da. − 34 da. = 59 da.

III. A note for \$2020, dated Oct. 31st, 1887, and payable in 6 months with interest at $4\frac{1}{2}$%, was discounted Mar. 12th, 1888 at 6%; find the proceeds.

2020	1888—5— 3	2066.21
.030½	1888—3—12	17.56
1010	1—21	\$2048.65
60600	.008½	
4)61.610		
15.4025	2066.21	
46.2075	.008½	
2020.	1033101½	
2066.21	1652968	
	17.562781½	

When a note "with interest" is discounted, the discount is computed on the amount due at maturity. The amount of \$2020 for 6 mo. 3 da. is \$2066.21.

6 mo. after Oct. 31st is called Apr. 30th; since April has no 31st day, the time expires on the last day; hence the note becomes due Apr. 30/May 3. The term of discount is 1 mo. 21 da., and the interest of \$2066.21 for this time is \$17.56. The proceeds is \$2066.21 − \$17.56, which equals \$2048.65.

EXAMPLES.

1. Find the bank discount on a note for \$125, due in 3 months, at 5%.

2. What is the discount on a note for \$475, due in 75 days, discounted at a bank at $4\frac{1}{2}\%$?

3. If you have a note for \$1000, payable in 60 days, discounted at a bank at 6%, what sum will you receive?

4. How much money should be received on a note of \$1000, payable in 4 months, discounted at a bank where the rate of discount is 6%?

5. A man buys \$800 worth of goods and gives his note for that sum, payable in 90 days. Find the sum realized on the note if it is immediately discounted at a bank at 6%.

6. If the rate of discount is 5%, how much can be obtained on a note for \$600, payable in 4 months, discounted at a bank?

7. If the rate of discount is $5\frac{1}{2}\%$, how much can be obtained on a note for \$1000, payable in 60 days, discounted at a bank?

8. Find the proceeds of a note for \$1225, due in 30 days, discounted at $5\frac{1}{2}\%$.

9. Find the proceeds of a note of \$620.25, discounted at a bank for 53 days.

10. Find the bank discount and proceeds of a note of \$1285, dated Mar. 28th, 1883, payable Jan. 5th, 1885, and discounted at 4%.

11. Find the proceeds of a four-months' note for \$1350, discounted 15 days after date at 7%.

12. Find the proceeds of a note for \$25, dated Aug. 17th, payable in 30 days, and discounted Sept. 1st at 5%.

13. Find the proceeds of a note for \$250, dated July 31st, payable in 4 months, and discounted Sept. 15th at $4\frac{1}{2}\%$.

14. A note for $500, dated Mar. 9th, at 3 months, is discounted Apr. 11th at 8%; what is received for the note?

15. Find the bank discount on a note for $400, dated Jan. 12th, 1887, due in 90 days, and discounted Feb. 1st at 6%.

16. Find the proceeds of a note for $384.22, at 60 days, dated July 17th, and discounted Aug. 2nd.

17. Find the proceeds of a note, dated Oct. 5th, 1886, for $428.50, payable in 6 months, and discounted Jan. 1st, 1887 at 5%.

18. What is the bank discount on a note for $392, payable in 90 days with interest at 6%, and discounted 15 days after date at 7%?

19. Find the proceeds of a note for $625 at 5% interest, due in 60 days, dated Aug. 1st, and discounted Sept. 21st at 5%.

20. A note for $150, dated June 14th, and payable in 4 months with interest at 5%, was discounted July 20th at 7%; find the proceeds.

21. A note for $1000, dated Jan. 17th, 1888, and payable in 90 days with interest at 7%, was discounted Mar. 1st at 6%; find the proceeds.

22. A note for $1875, dated Aug. 30th, 1887, and payable in 6 months with interest at 6%, was discounted Oct. 27th, 1887 at 8%; find the proceeds.

To Find the Face of a Note to Yield a given Proceeds.

140. I. Find the face of a note for 90 days which, when discounted at 4%, will yield $300.

$$\begin{array}{r} 3).0155 \\ \hline .0051\frac{2}{3} \\ \hline .0103\frac{1}{3} \end{array} \qquad \begin{array}{r} 1.0000 \\ .0103\frac{1}{3} \\ \hline .9896\frac{2}{3} \end{array}$$

$$\begin{array}{l} .9896\tfrac{2}{3})300. \\ \quad\;\; 3 \qquad\;\; 3 \\ \hline 2.969 \;\;)900.000(\$303.13 \\ \qquad\qquad 8907 \\ \qquad\qquad \overline{\;\;9300} \\ \qquad\qquad\;\; 8907 \\ \qquad\qquad\;\; \overline{\;\;3930} \\ \qquad\qquad\;\;\;\; 2969 \\ \qquad\qquad\;\;\;\; \overline{\;\;9610} \end{array}$$

The bank discount on \$1 for 90 days is the same as the interest for 93 days, which equals \$0.0103$\frac{1}{3}$, and the proceeds of \$1 equals \$0.9896$\frac{2}{3}$. To produce a proceeds of \$300 will require as many dollars as \$0.9896$\frac{2}{3}$ is contained times in \$300, which equals \$303.13.

EXAMPLES.

1. What must be the face of a note having 4 months to run that it may yield \$1959 when discounted?

2. What must be the face of a note which, when discounted at a bank for 60 days at 6%, shall give as its proceeds \$500?

3. For what sum must a note be drawn at 90 days to net \$2050 when discounted at 7%?

4. Find the face of a note at 2 months that would realize \$4500 when discounted at a bank, interest being 6%.

5. I wish to borrow \$560 at a bank; for what sum must I give my note for 90 days at 8%?

6. What must be the face of a note which, discounted at a bank for 30 days, would realize \$200?

7. If the rate of discount is 5%, for what amount must a note, payable in 4 months, be given to realize \$600?

8. Find the face of a note payable in 60 days, so that the proceeds shall be \$1200 when discounted at 5%.

9. For what amount must a note, payable in 120 days, be given to a bank discounting at 6% to obtain \$500?

10. For what sum must I give my note for 90 days at a bank, in order to receive \$1100, money being worth 7%?

EXCHANGE.

141. A **bill of exchange**, or **draft**, is a written or printed order from one person to another directing the payment of a specified sum of money to a third person. The person signing the draft is called the **drawer**; the person to whom the draft is addressed is the **drawee**; and the person to whom the money is payable is the **payee**.

A **sight draft** is a draft which is payable on presentation to the drawee. A **time draft** is one which is payable at a specified time after presentation, or after date.

When the drawee accepts a draft, he writes the word "Accepted" with the date and his signature; the draft is then called an **acceptance**, and the drawee, who now becomes an **acceptor**, is responsible for its payment.

The following are common forms of bills of exchange:

SIGHT DRAFT.

\$*150*$\frac{00}{100}$. BUFFALO, N.Y., *June 12th,* 188*9*.

At sight ~~~~~~~~ pay to

the order of ~~~~~~ *Walter Johnson,* ~~~~~~

~~~~~~ *One hundred fifty* ~~~~~~~~ Dollars,

value received, and charge to the account of

To *Thomas Powell,* *Andrew Hooper.*

Cincinnati, O.

~~~~~~

TIME DRAFT.

$\$2000\frac{00}{100}$. CHICAGO, ILL., *Nov. 1st,* 188*8*.

Thirty days ~~~~~~~~ after date pay to the order of ~~~~~~ *George K. Foster,* ~~~~~ *Two thousand* ~~~~~~~~~~~~~~~~~~~~ Dollars, value received, and charge the same to the account of

To *the Atlas National Bank,* *J. Moore.*
Boston, Mass.

FOREIGN BILL OF EXCHANGE.

Exchange for *£35.* NEW YORK, *Aug. 20th,* 188*9*.

On demand pay for this Bill of Exchange to the order of ~~~~~~ *Albert Hall,* ~~~~~~ *Thirty-five Pounds Sterling.*

To *the Alliance Bank,* *B. Fox & Co.*
London, England.

The system of making payments in distant places by transmitting drafts instead of money is called **exchange.** The business is commonly carried on through bankers, who have credit in distant places, and sell drafts to persons wishing to make payments in those places.

When a draft sells for its face value, exchange is said to be **at par**; when a draft sells for more than its face value, exchange is **above par,** or **at a premium**; when a draft sells for less than its face value, exchange is **below par,** or **at a discount.**

DOMESTIC OR INLAND EXCHANGE.

142. Exchange between places in the same country is called **domestic** or **inland exchange.**

I. Find the cost of a draft on Chicago for \$1320 when exchange is $1\frac{1}{8}\%$ discount.

1320	1320.
$.01\frac{1}{8}$	14.85
165	\$1305.15
1320	
14.85	

The discount on \$1 is $\$0.01\frac{1}{8}$, and on \$1320 it is \$14.85. Subtracting this from \$1320, we find the cost of the draft to be \$1305.15.

II. Find the cost of a draft on Omaha for \$1400, payable in 60 days, when exchange is $\frac{1}{2}\%$ premium, and interest 5%.

1400	1400.
.0105	12.25
7000	1387.75
1400	7.
6)14.70	\$1394.75
2.45	
12.25	

The bank discount on \$1400 for 60 da. at 5% is the same as the interest for 63 da., which equals \$12.25, and the proceeds is \$1387.75; this would be the cost of the draft if bought at par. At $\frac{1}{2}\%$ the premium on \$1400 is \$7; adding this to \$1387.75, we find the cost of the draft to be \$1394.75.

EXAMPLES.

1. What is the value of a sight draft on Buffalo for \$1800, when exchange is at a premium of 1%?

2. Find the cost of a sight draft on New York for $1300, when exchange is $1\frac{1}{4}\%$ premium.

3. Find the cost of a sight draft on Detroit for $840, when exchange is $\frac{3}{8}\%$ discount.

4. What will be the cost of a draft for $3000, payable in 30 days after sight, exchange being 1% discount, and interest 6%?

5. Find the cost of a draft for $700, payable in 60 days, when exchange is at par, and interest 7%.

6. What must be paid for a draft of $925, at 60 days, at 6%, exchange being $\frac{5}{8}\%$ premium?

7. What must be paid for a draft of $450 on New Orleans, at 90 days, exchange being $\frac{3}{4}\%$ discount, and interest $4\frac{1}{2}\%$?

8. What will be the cost of a draft for $750, payable in 60 days after sight, exchange being $\frac{1}{2}\%$ premium, and interest 7%?

9. Find the cost of a draft on Baltimore for $1237.50, payable in 30 days after sight, exchange being $\frac{1}{8}\%$ discount, and interest 5%.

To Find the Face of a Draft when the Cost is Given.

143. I. Find the face of a sight draft bought for $3559.50, when exchange is $1\frac{1}{8}\%$ discount.

1.
```
 .01125
 ------
 .98875) 3559.50000 ($3600
         296625
         ------
          593250
          593250
          ------
              00
```

At $1\frac{1}{8}\%$ discount a sight draft for $1 would cost $0.98875. $3559.50 will buy a draft for as many dollars as $0.98875 is contained times in $3559.50, which equals $3600.

II. What is the face of a draft, payable in 90 days, that can be bought for \$2000, exchange being $1\frac{1}{2}\%$ premium, and interest 4%?

$$
\begin{array}{r}
3\overline{)\,.0155} \\
.0051\frac{2}{3} \\
\hline
.0103\frac{1}{3}
\end{array}
\qquad
\begin{array}{r}
1. \\
.0103\frac{1}{3} \\
\hline
.9896\frac{2}{3} \\
.015 \\
\hline
1.0046\frac{2}{3}
\end{array}
$$

$$
\begin{array}{l}
1.0046\frac{2}{3})\,2000. \\
\quad\ \ 3 \qquad\quad 3 \\
\hline
3.014\ \ )\,6000.000\,(\$1990.71 \\
\qquad\qquad 3014 \\
\qquad\qquad 29860 \\
\qquad\qquad 27126 \\
\qquad\qquad\ \ 27340 \\
\qquad\qquad\ \ 27126 \\
\qquad\qquad\quad\ 21400 \\
\qquad\qquad\quad\ 21098 \\
\qquad\qquad\qquad\ 3020
\end{array}
$$

The bank discount on \$1 for 90 days at 4% is $\$0.0103\frac{1}{3}$, and the proceeds is $\$0.9896\frac{2}{3}$. At $1\frac{1}{2}\%$ premium the cost of a draft for \$1 is $\$1.0046\frac{2}{3}$. \$2000 will buy a draft for as many dollars as $\$1.0046\frac{2}{3}$ is contained times in \$2000, which is \$1990.71.

EXAMPLES.

1. What is the face of a sight draft that can be purchased for \$2351.70, when exchange is $\frac{1}{2}\%$ premium?

2. How large a sight draft can be bought for \$2500, exchange being $\frac{3}{8}\%$ discount?

3. What is the face of a sight draft bought for \$1650, when exchange is $3\frac{1}{2}\%$ discount?

4. How large a draft, payable in 30 days after sight, can be bought for \$4018, exchange being 1% premium, and interest 6%?

5. How large a draft on Cincinnati, at par, at 30 days, can be bought for \$1989, when money is worth 6%?

6. What is the face of a draft, payable in 60 days after date, that can be bought for \$2386.20, exchange being 1% premium, and interest 9%?

7. Find the face of a draft on New York, at 90 days sight, bought for \$450, exchange at $1\frac{3}{4}\%$ premium, and interest 5%

8. What is the face of a draft on St. Paul for 60 days which may be bought for $1000, exchange being $\frac{7}{8}\%$ discount, and interest 7%?

9. Find the face of a draft on Boston, at 90 days sight, bought for $75, exchange at $\frac{1}{2}\%$ premium, and interest 4%.

Foreign Exchange.

144. Exchange between places in different countries is called **foreign exchange**.

Exchange with Europe is carried on principally through large commercial cities, as London, Paris, Antwerp, Geneva, Hamburg, Frankfort, Bremen, and Berlin.

Sterling Exchange, as exchange with Great Britain and Ireland is called, is quoted at a certain number of dollars per pound sterling.

Exchange with France, Belgium, and Switzerland is quoted at a certain number of francs per dollar.

Exchange with Germany is quoted at a certain number of cents per four reichsmarks (marks).

I. Find the cost of a bill of exchange on London for £326 16 s., when sterling exchange is quoted at $4.83\frac{1}{2}$.

$$\begin{array}{r} 326.8 \\ \underline{4.83\frac{1}{2}} \\ 1634 \\ 9804 \\ 26144 \\ \underline{13072} \\ 1580.078 \end{array}$$

£326 16 s. equals £326.8. Since the value of £1 is $\$4.83\frac{1}{2}$, the value of £326.8 is 326.8 times $\$4.83\frac{1}{2}$, which equals $1580.08.

Ans. $1580.08.

II. Find the cost of a bill of exchange on Paris for 4730 francs, when Paris exchange is quoted at $5.15\frac{1}{2}$.

```
5.155)4730.000($917.56
       46395
        9050
        5155
        38950
        36085
         28650
         25775
          28750
```

Since 5.155 francs are worth \$1, 4730 francs are worth as many dollars as 5.155 francs are contained times in 4730 francs, which equals \$917.56.

III. Find the cost of a bill of exchange on Berlin for 2760 marks, when German exchange is quoted at $96\frac{1}{2}$.

$$\begin{array}{r} 2760 \\ .24\frac{1}{8} \\ \hline 345 \\ 11040 \\ 5520 \\ \hline \$665.85 \end{array}$$

Since $\$0.96\frac{1}{2}$ is the value of 4 marks, the value of 1 mark is $\frac{1}{4}$ of $\$0.96\frac{1}{2}$, or $\$0.24\frac{1}{8}$. The value of 2760 marks is 2760 times $\$0.24\frac{1}{8}$, which equals \$665.85.

The face of a bill of exchange can be found when the cost is given by performing the reverse operation to that used in finding the cost when the face is given.

EXAMPLES.

1. What must be paid in New York for a bill of exchange on London for £725 10 s., when sterling exchange is quoted at $4.87\frac{1}{4}$?

2. Find the cost of a bill of exchange on Dublin for £296 8 s. 6 d., when sterling exchange is quoted at $4.85\frac{1}{2}$.

3. What is the cost of a bill of exchange on Liverpool for £137 15s. 4 d., exchange at 4.86?

4. How large a bill of exchange on Edinburgh can be bought for \$2500, when sterling exchange is quoted at 4.87?

5. When exchange on London is quoted at 4.85, what will be the face of a draft that can be bought for \$3889.70?

6. What is the face of a bill of exchange on Liverpool for which $4800 was paid, exchange at 4.84½?

7. What is the cost of a bill of exchange on Paris for 975 francs, exchange at 5.16?

8. Find the cost of a bill of exchange on Geneva for 1822 francs, exchange at 5.17.

9. Find the cost of a bill of exchange on Antwerp for 2025.25 francs, exchange at 5.17¼.

10. What is the face of a bill of exchange on Paris bought for $2240.25, when Paris exchange is quoted at 5.15?

11. How large a bill of exchange on Geneva can be bought for $850, exchange at 5.16½?

12. When exchange on Paris is quoted at 5.21¼, what will be the face of a draft that can be bought for $2046.50?

13. How much must be paid in Boston for a bill of exchange on Hamburg for 2672 marks, exchange at 95?

14. Find the cost of a bill of exchange on Bremen for 1685.25 marks, exchange at 94¼.

15. Find the cost of a bill of exchange on Berlin for 1050 marks, exchange at 95⅝.

16. When exchange on Frankfort is quoted at 95½, what will be the face of a draft that can be bought for $2871.40?

17. Find the face of a bill of exchange on Hamburg costing $892.76, when German exchange is quoted at 95¼.

18. How large a bill of exchange on Berlin can be bought for $1500, exchange at 96¾?

Equation of Payments.

145. Equation of payments is the process of finding the time when several payments due at different times can all

be paid at once without loss to either debtor or creditor. This time is called the **equated time**.

I. A man owes \$1200, of which \$600 is due in 3 months, \$400 in 5 months, and \$200 in 6 months; find the equated time of payment.

$$\begin{array}{lr} & 600 \times 3 = 1800 \\ & 400 \times 5 = 2000 \\ & \underline{200} \times 6 = \underline{1200} \\ 1200 &)\overline{5000} \\ & 4\tfrac{1}{6} \text{ mo.} \\ & = 4 \text{ mo. } 5 \text{ da.} \end{array}$$

The use of \$600 for 3 mo. is equivalent to the use of \$1 for 1800 mo.; the use of \$400 for 5 mo. is equivalent to the use of \$1 for 2000 mo.; and the use of \$200 for 6 mo. is equivalent to the use of \$1 for 1200 mo. This amounts to the use of \$1 for 5000 mo., which is equivalent to the use of \$1200 for $\frac{1}{1200}$ of 5000 mo., which equals $4\frac{1}{6}$ mo., or 4 mo. 5 da.

II. A merchant bought the following bills of goods: Jan. 15th, \$600 on 2 months' credit; Feb. 1st, \$300 on 3 months' credit; Mar. 25th, \$550 on 30 days' credit; and Apr. 8th, \$400 on 60 days' credit. Find the equated time of payment.

$$\begin{array}{lll} \text{Mar. 15.} & 600 \times 0 = & 0 \\ \text{May 1.} & 300 \times 47 = & 14100 \\ \text{Apr. 24.} & 550 \times 40 = & 22000 \\ \text{June 7.} & \underline{400} \times 84 = & \underline{33600} \\ & 1850 &)69700(37.7 \\ & & \underline{5550} \\ & & 14200 \\ & & \underline{12950} \\ & & 12500 \end{array}$$

Ans. Apr. 22nd.

Write the dates on which the several payments are due with the amounts opposite. Take the earliest date, Mar. 15th, as a convenient date from which to reckon (sometimes called *focal date*). The periods of time, reckoning from Mar. 15th, are 0, 47, 40, and 84 days respectively. Computing as in the previous example, we find the equated time to be 38 days after Mar. 15th, which is Apr. 22nd.

III. A man owes \$2000 due in 8 months; he pays \$500 in 2 months and \$800 in 3 months; when in equity should he pay the balance?

$$\begin{array}{r} 500 \times 6 = 3000 \\ \underline{800 \times 5 = 4000} \\ 2000 - \overline{1300} = 700 \overline{)7000} \\ \overline{10} \text{ mo.} \end{array}$$

$500 paid in 2 mo. is paid 6 mo. before it is due, and its use is equivalent to the use of $1 for 3000 mo.; $800 paid in 3 mo. is paid 5 mo. before it is due, and its use is equivalent to the use of $1 for 4000 mo.; this amounts to the use of $1 for 7000 mo. To offset these payments made before maturity, the balance of $700 can be retained after maturity for $\frac{1}{700}$ of 7000 mo., which equals 10 mo.

EXAMPLES.

1. A man owes $300 due in 4 months, and $600 due in 7 months; find the equated time of payment.

2. What is the equated time for paying $20 due in 20 days, $60 due in 30 days, $40 due in 50 days, and $80 due in 75 days?

3. A man buys a house for $2500, and agrees to pay $500 down, and the rest in 4 equal annual instalments; when can he justly pay the whole at once?

4. A merchant owes $2400, of which $400 is payable in 6 months, $800 in 10 months, and $1200 in 16 months; what is the equated time of payment?

5. Find the equated time for the payment of $400 due in 30 days, $250 due in 60 days, and $200 due in 90 days.

6. Of a debt, $\frac{1}{3}$ is to be paid in 2 months, $\frac{1}{4}$ in 6 months, $\frac{1}{6}$ in 10 months, and the balance in a year. Find at what time in equity the whole should be paid if all the payments were converted into one.

7. A debt is to be paid $\frac{1}{3}$ down, $\frac{1}{4}$ in 6 months, $\frac{1}{6}$ in 8 months, and the balance in a year; if the payments are all converted into one, what is the equated time of payment?

8. Three bills are due as follows: Sept. 5th, $275; Oct. 1st, $180; and Nov. 20th, $350. Find the equated time of payment.

9. What is the equated time for the payment of $170 due Mar. 12th, $250 due Apr. 12th, $280 due May 17th, and $325 due June 12th?

10. I owe three notes bearing interest from date: the first, dated June 1st, 1886, is for $450; the second, dated Dec. 17th, 1886, is for $750; the third, dated Mar. 15th, 1887, is for $600. I wish to substitute for these a single note for $1800; what should be the date of it?

11. A merchant bought goods on 6 months' credit as follows: Mar. 20th, $420; May 3rd, $270; and June 12th, $340. When shall a note to settle for the whole be made payable?

12. Find the equated time of payment for the following bills of merchandise: Oct. 10th, 1887, $625 on 60 days' credit; Nov. 1st, 1887, $314 on 3 months' credit; and Jan. 4th, 1888, $266 on 30 days' credit.

13. A merchant bought the following bills of goods: Dec. 23rd, 1887, $428 on 90 days' credit; Jan. 17th, 1888, $206 on 2 months' credit; Feb. 3rd, 1888, $90 on 30 days' credit; and Feb. 8th, 1888, $214 on 60 days' credit. Find the equated time of payment.

14. A man bought a horse and carriage for $500 on 6 months' credit; if he pays $200 down, when should he pay the balance?

15. A man owes $1600 due in 9 months; he pays $300 in 4 months, $200 in 6 months, and $300 in 8 months; when is the balance due?

16. A man owes $600 due in 6 months, $900 due in 10 months, and $1200 due in 12 months; at the end of 8 months he pays $1800; when in equity should the remainder be paid?

17. On a debt of $5000 due in 8 months from Jan. 1st, the following payments were made: Apr. 1st, $500; June 1st, $600; and Aug. 1st, $1000. When is the balance due?

Average of Accounts.

146. Average of accounts is the process of finding the time when the balance of an account can be paid without loss to either debtor or creditor.

I. Find the equated time for paying the balance of the following account:

Dr. B. R. Harvey. *Cr.*

1888.			1888.		
Aug. 2	To Mdse., 30 da.	$400	Aug. 4	By Draft, 60 da.	$200
" 29	" "	350	" 31	" Cash.	300
Sept. 7	" " 2 mo.	250	Sept. 8	" "	400

Solution.

Sept. 1.	$400 \times 3 =$	1200	Oct. 6.	$200 \times 38 =$	7600
Aug. 29.	$350 \times 0 =$	0	Aug. 31.	$300 \times 2 =$	600
Nov. 7.	$250 \times 70 =$	17500	Sept. 8.	$400 \times 10 =$	4000
	1000	18700		900	12200
	900	12200			
	100	)6500			
		65 da.	*Ans.* Nov. 2nd.		

Following the method of equation of payments, taking Aug. 29th for the focal date, we find the amount of the debtor side of the account to be $1000, equivalent to the use of $1 for 18700 days. We find the amount of the creditor side to be $900, equivalent to the use of $1 for 12200 days. The balance on the debtor side is $100, equivalent to the use of $1 for 6500 days, and the equated time is 65 days after Aug. 29th, or Nov. 2nd.

Note. In determining the maturity of a note or draft, 3 days of grace must be added to the specified time.

When the balance of the account and the difference between the sums of the products fall on the same side, the result is of the same nature as a result in equation of payments, and the equated time is later than the focal date. When the balance of the account and the difference between the sums of the products fall on opposite sides, it is readily seen that an earlier focal date could be taken which would give the same sum of products on each side, and thus this date is the equated time; hence the equated time is earlier than the focal date regularly taken.

The method may be stated as follows:

Write each item with its date of maturity on the respective sides of the account, and take as the focal date the earliest date of maturity.

Multiply each item by the number of days intervening between the focal date and the date of maturity, and find the sums of these products on each side of the account. Divide the difference between the sums of the products by the balance of the account, and the quotient is the number of days between the focal date and the equated time.

When the balance of the account and the difference between the sums of the products fall on the same side, count forward from the focal date; when they fall on opposite sides, count backward.

EXAMPLES.

1. Find the equated time for paying the balance of the following account:

Dr. M. P. BARTLETT. *Cr.*

1888.			1888.		
Apr. 20	To Mdse., 30 da.	\$520	May 15	By Cash.	\$600
May 10	" "	135			

2. Find the time when a note for the balance of the following account should begin to draw interest:

Dr. R. J. MINER. *Cr.*

1888.			1888.		
Mar. 1	To Cash.	$1500	May 14	By Mdse.	$2050
June 8	" Mdse.	235	July 10	" Real Estate.	145

3. Find the equated time for the payment of the balance of the following account:

Dr. J. H. ADAMS. *Cr.*

1888.			1888.		
May 10	To Mdse., 30 da.	$420	May 4	By Draft, 30 da.	$750
June 15	" "	380	June 12	" Cash.	400
" 20	" "	450			

4. Find the equated time for the settlement of the following account:

Dr. B. P. HARPER. *Cr.*

1888.			1888.		
Jan. 10	To Mdse.	$672	Jan. 28	By Cash.	$475
Feb. 7	" " 30 da.	428	Apr. 10	" Mdse.	462
" 24	" " 2 mo.	550	May 18	" Cash.	250

5. Find the face of a note that will balance the following account, and the date at which it should begin to draw interest:

Dr. A. F. BRACKETT. *Cr.*

1887.			1887.		
Sept. 14	To Mdse., 30 da.	$1950	Nov. 19	By Cash.	$750
Oct. 16	" " 3 mo.	532	Dec. 1	" Draft, 60 da.	1000
" 20	" " 2 mo.	1178	1888. Feb. 4	" Cash.	600

CHAPTER XI.

STOCKS.

147. A **corporation** is an association of individuals authorized by law to transact business as a single person. The capital invested in the business is called **stock,** and it is divided into equal parts called **shares.** The owners of the shares are called **stockholders,** each of whom holds a document called a **certificate of stock,** which is issued by the corporation and specifies the number of shares owned.

The usual value of a share is $100, although it varies in different corporations. In this book it will be regarded as $100, unless otherwise stated.

The original or face value of a share is called the **par value,** and the value at which it sells is called the **market value.** When shares sell for their face value, they are said to be **at par**; when they sell for more than their face value, they are **above par,** or **at a premium**; when they sell for less than their face value, they are **below par,** or **at a discount.** The market value is quoted at a certain per cent of the par value. For example, when stock is at par, it is quoted at 100; when it is 8% above par, it is quoted at 108; when it is 15% below par, it is quoted at 85.

Stocks are generally bought and sold through the agency of brokers, who receive a commission, called **brokerage,** reckoned on the par value of the stock. The usual rate of brokerage is $\frac{1}{8}$%, but other rates may be charged.

A **dividend** is a sum paid to stockholders from the profits of the business. An **assessment** is a sum sometimes required of stockholders to meet losses or pay expenses. Dividends

and assessments are generally reckoned at a certain per cent of the par value. Dividends are usually declared annually, semi-annually, or quarterly, and the rate per cent is called the **rate of dividend.**

Bonds are interest-bearing notes issued by governments or corporations; they are bought and sold in the same manner as shares of corporations. Bonds are commonly designated according to the rates of interest which they bear. For example, Virginia 6's are bonds issued by the state of Virginia bearing 6% interest.

The methods of percentage apply to stocks. The par value of the stock is the base, and the premium, discount, dividend, or assessment is a percentage of the par value.

I. Find the cost of 32 shares of railroad stock at $8\frac{1}{2}$% discount.

$$\begin{array}{r} 91\frac{1}{2} \\ \underline{32} \\ 16 \\ 182 \\ \underline{273} \\ \$2928 \end{array}$$

At $8\frac{1}{2}$% discount the cost of one share is $\$91\frac{1}{2}$, and the cost of 32 shares is 32 times $91\frac{1}{2}$, or \$2928.

II. How much, including brokerage at $\frac{1}{8}$%, must be paid for \$15000 U. S. 4's at $111\frac{1}{2}$?

$$\begin{array}{r} 15000 \\ \underline{1.11\frac{5}{8}} \\ 7500 \\ 1875 \\ 15000 \\ 15000 \\ \underline{15000} \\ \$16743.75 \end{array}$$

If the brokerage is $\frac{1}{8}$%, $111\frac{1}{2} + \frac{1}{8}$, or $111\frac{5}{8}$, represents the price paid. $111\frac{5}{8}$% of \$15000 is \$16743.75.

III. How much bank stock at $131\frac{1}{2}$ must be sold in order to receive \$4725, brokerage $\frac{1}{4}$%?

```
1.3125)4725.0000($3600
        39375
        78750
        78750
           00
```

If the brokerage is $\frac{1}{4}$%, $131\frac{1}{2} - \frac{1}{4}$, or $131\frac{1}{4}$, represents the price received. \$4725 is $131\frac{1}{4}$% of the amount obtained by dividing \$4725 by 1.3125, which is \$3600.

IV. Find the quoted price of stock when 15 shares cost \$1886.25.

```
15)1886.25
   125.75 = 125¾.
```

If 15 shares cost \$1886.25, one share costs $\frac{1}{15}$ of \$1886.25, which is \$125.75. Hence the quoted price is $125\frac{3}{4}$.

V. A man owns \$5000 of the stock of a railroad which declares a dividend of 4%; what is the amount of his dividend?

```
  5000
   .04
$200.00
```

4% of \$5000 is \$200.

VI. How much 8% stock must be bought to yield an income of \$3000?

```
.08)3000.00
    $37500
```

\$3000 is 8% of \$37500.

NOTE. In this book the brokerage is included in the price of a stock, unless otherwise stated; hence no account should be taken of brokerage when it is not mentioned in the problem.

EXAMPLES.

1. How much must be paid for \$8500 Iowa 6's at $112\frac{1}{2}$?

2. What is the cost at $63\frac{1}{2}$ of stock having a par value of \$2800?

3. A man bought 28 shares of railroad stock at 18% discount; what did they cost him?

4. Find the cost of 36 shares of bank stock at $121\frac{1}{4}$, brokerage $\frac{1}{8}$%.

5. How much, including brokerage at $\frac{1}{4}\%$, must be paid for $3500 Tennessee 6's at $88\frac{1}{2}$?

6. How much will be received from the sale of $11200 U. S. $3\frac{1}{2}$'s at $107\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

7. A speculator bought 45 shares of stock at $4\frac{1}{4}\%$ discount, and sold it at $2\frac{1}{2}\%$ premium; what was his gain?

8. A broker bought 84 shares of railroad stock at 19% discount. He sold 35 shares at $27\frac{1}{2}\%$ discount, and the balance at 8% discount. Did he gain or lose, and how much?

9. How many shares of stock at 78% premium can be bought for $9790?

10. How much stock can be bought for $14178, when the quoted price is $208\frac{1}{2}$?

11. What amount of Union Pacific bonds at $104\frac{1}{2}$ can be bought for $7837.50?

12. Find the number of shares of bank stock at 105 that can be bought for $25260, including brokerage at $\frac{1}{4}\%$.

13. A broker receives $3762.50 to invest in stocks at $75 per share and cover his brokerage at $\frac{1}{4}\%$. How many shares should he purchase?

14. How much canal stock must be sold at $136\frac{5}{8}$ in order to receive $6552, brokerage $\frac{1}{8}\%$?

15. A broker received $10.50 for selling stock at $122\frac{1}{2}$; how many shares did he sell, brokerage $\frac{1}{8}\%$?

16. I sent $40100 to a broker for the purchase of bank stock at par. If the brokerage is $\frac{1}{4}\%$, what does he pay for the stock, and what is his brokerage?

17. A man exchanged 72 shares of bank stock at 85 for railroad stock at 136; how many shares of railroad stock did he receive?

18. A speculator bought stock at $1\frac{1}{2}\%$ discount, and gained \$495 by selling the same at 6% premium; how many shares did he purchase?

19. Bought bonds at 115, and sold at 110, losing \$300. How many bonds of \$1000 each did I buy?

20. Find the quoted price of stock when 35 shares cost \$2931.25.

21. What must be the quoted price in order that \$6800 stock may be bought for \$2941?

22. How should U. S. $4\frac{1}{2}$'s be quoted when \$10093.75 is paid for bonds having a par value of \$8500?

23. When the cost of \$4500 telegraph stock, including brokerage at $\frac{1}{4}\%$, is \$7380, what is the quoted price?

24. Find the quoted price of railroad stock when the cost of 250 shares, including brokerage at $\frac{1}{8}\%$, is \$30312.50.

25. Find the quoted price of bank stock when \$10175 is received for 110 shares, brokerage $\frac{1}{8}\%$.

26. A man bought stock at 115, and sold the same at $128\frac{1}{2}$; what per cent of the investment did he gain?

27. If I buy railroad stock at 20% discount, and sell at 10% premium, what per cent do I gain?

28. A railroad declares a dividend of $3\frac{1}{2}\%$; how much will a man owning 48 shares receive?

29. The capital of a manufacturing company is \$300000, and it declares a semi-annual dividend of 4%; find the entire amount of the dividend.

30. An insurance company calls an assessment of $2\frac{3}{4}\%$ to meet losses; how much is the assessment on \$7200 stock?

31. A man owns 150 shares of mining stock, and the company declares a dividend of 6% payable in stock; how many shares will he then own?

32. Find the total income from \$4000 9% stocks and \$7800 7% stocks.

33. How much 7% stock must a man own in order to receive an income of \$4200?

34. If a person receives \$360 when a $4\frac{1}{2}$% dividend is declared, how many shares, \$50 each, does he own?

35. The net earnings of a corporation are \$2625, from which a dividend of $6\frac{1}{4}$% is declared; find the capital.

36. Find the number of shares owned by a person after receiving 12 shares when a stock dividend of 15% is declared.

37. Find the rate of dividend when a man owning 52 shares receives \$182.

38. A company with a capital of \$325000 calls an assessment of \$4875; what is the rate?

39. A company, whose capital is \$275000, has \$15125 from its earnings to divide. What per cent dividend can it declare?

40. The capital of a company is \$175000; the gross receipts are \$35930, and the expenses are \$19205; find the rate of dividend that can be declared after reserving a surplus of \$2725.

148. In making investments, it is necessary to consider both the market value of a stock and the rate of dividend.

I. What income will be realized from investing \$10650 in 8% stock at 142?

1.42)10650.00(7500
  994
  710
  710
   00

7500
.08
\$600.00

If \$10650 is invested at 142, the par value of the stock is \$7500; 8% of \$7500 is \$600.

II. What sum must be invested in 6% bonds at $92\frac{1}{2}$ to yield an income of \$1500?

$$\begin{array}{r}
.06\overline{)1500.00} \\
25000 \\
\underline{.92\frac{1}{2}} \\
12500 \\
50000 \\
\underline{225000} \\
\$23125.00
\end{array}$$

To yield an income of \$1500, the par value of the stock must be \$25000, and this amount of stock at $92\frac{1}{2}$ will cost \$23125.

III. When 9% stock is quoted at 192, what rate of interest does the investment pay?

$\frac{9}{192} = \frac{3}{64}$.

$$\begin{array}{l}
64)3.00(.04\frac{11}{16} = 4\frac{11}{16}\% \\
\quad \underline{256} \\
\quad\ \frac{44}{64} = \frac{11}{16}
\end{array}$$

The cost of a share is \$192, and the income is \$9, which is $\frac{9}{192}$, or $4\frac{11}{16}\%$, of the cost.

IV. What is the quoted price of a 6% stock which pays $4\frac{4}{9}\%$ interest on the investment?

$$\begin{array}{r}
.04\frac{4}{9})6.00 \\
\underline{9 \qquad 9} \\
.40\overline{)54.00} \\
135
\end{array}$$

The income is \$6, which is $4\frac{4}{9}\%$ of the market value; hence the quoted price is as much as $0.04\frac{4}{9}$ is contained times in 6, which is 135.

EXAMPLES.

1. What annual income would a man receive from \$9810 invested in railroad stock at 109, and paying 5% dividend?

2. What income will \$10120 yield if invested in 4% bonds bought at 115?

3. What income would a man receive from \$9525 invested in Mexican Central 4's at $63\frac{1}{2}$?

4. How much will be realized yearly from an investment of \$7620 in a 5% stock bought at 95, brokerage $\frac{1}{4}\%$?

5. A man invests $11459 in telephone stock at $204\frac{1}{2}$, paying $\frac{1}{8}\%$ brokerage. What will he receive when a dividend of 5% is declared?

6. How much must be invested in 8% stock at $170\frac{3}{4}$ to afford an income of $2000?

7. What sum must I invest in 6% bonds, selling at $2\frac{1}{2}\%$ premium, to secure an annual income of $840?

8. How much must be invested in a stock at $213\frac{1}{2}$, which pays 5% semi-annual dividends, to realize an annual income of $420?

9. What sum must be invested in U. S. 4's at $121\frac{1}{4}$, brokerage $\frac{1}{8}\%$, to secure an annual income of $700?

10. When Wisconsin Central 5's are selling at $85\frac{1}{2}$, how much must be invested to produce an income of $750, brokerage $\frac{1}{4}\%$?

11. If a 6% stock is at 120, what rate per cent will an investor receive on his money?

12. Bank stock, which sells at 170, pays an annual dividend of $12\frac{1}{2}\%$; what rate of interest does a buyer receive?

13. Received 6% dividend on stock bought at 25% below par; what rate of interest did the investment pay?

14. Stock bought at 20% below par paid 7%; what was the rate on the investment?

15. What per cent of income does stock paying 8% dividends yield, if bought at $168\frac{1}{2}$?

16. Which is the better investment, a 4% stock at 120, or a 5% stock at $166\frac{2}{3}$?

17. Which is the more profitable stock to invest in, 3% at $83\frac{1}{2}$, or $3\frac{1}{2}\%$ at 97?

18. Which is the more profitable, $8400 invested in 5 per cents at 105, or in 7 per cents at 150?

19. Which will yield the better income, a 4% stock at 73, or a 7% stock at 126¾, brokerage ⅛% in each case?

20. At what price must I purchase 8% stock that the investment shall pay 5%?

21. What must be paid for 7% bonds that the investment may yield 6%?

22. A bank declares a semi-annual dividend of 4%; what could I afford to pay for its shares if I wish to get 6% a year for my money?

23. At what price must a stock paying 6% dividends be bought to pay the same income as an 8% stock at par?

24. If money is worth 3%, what is the premium on government 3½% bonds?

25. If I invest $1500 in 3% stock at 75, what is my income, and what rate per cent do I get on my investment?

26. If a man invests $1338 in bank stock at 167¼, what is the rate of dividend when he receives $120?

27. A 5% stock pays a dividend of $510; if it is sold for $11985, what premium is paid?

28. What must be the price of a 5% stock in order to yield the same rate of income as a 4% stock at 87?

29. When stock is quoted at 120, what rate of dividend must be paid in order to yield the same rate of income as a 6% stock at 144?

30. A man having a certain sum of money to invest has an opportunity of purchasing 7% stock at 95, but delays until it has risen to 110. What per cent is his income less than if he purchased at the first price?

31. A man sells \$10000 $3\frac{1}{2}\%$ bonds at $109\frac{1}{4}$, and reinvests the proceeds in 3% bonds at 92; is his income increased or diminished, and by what amount?

32. If a man sells \$4000 6% bonds at $113\frac{3}{4}$, and invests the proceeds in $4\frac{1}{2}\%$ bonds at 91, is his income increased or diminished, and by what amount?

33. A man sold \$6000 of 6% stock at $144\frac{1}{2}$, and invested the proceeds in 8% stock at 170. How much 8% stock did he buy, and what was the change in his income?

34. How much $3\frac{1}{2}\%$ stock must I sell at 84, to enable me to buy \$7700 4% stock, the value of the stock being proportional to the dividends they pay?

35. If I exchange 48 shares of a 9% stock at 176 for U. S. 4's at $116\frac{1}{2}$, how much must I add to my investment to secure the same income?

36. If I sell \$5000 Alabama 6's at 132 and buy sufficient U. S. $4\frac{1}{2}$'s at 108 to secure an income of \$225, how much shall I have left, brokerage $\frac{1}{8}\%$ for each transaction?

37. A man has a certain sum of money to invest. He finds that by buying 5% stock at 90 his income will be \$10 more than if he bought 8% stock at 150. How much money has he to invest?

38. A man sold \$4500 of 9% stock at $172\frac{1}{2}$, and invested the proceeds in 4% stock, thereby increasing his income by \$55. Find the price of the 4% stock.

39. By selling his 6% stock at 147, and investing the proceeds in a 5% stock at $96\frac{1}{4}$, a man increases his income by \$54. How much 6% stock did he sell?

40. A person sells a certain amount of 5% stock for 86, and invests in 6% stock at 103, and by so doing changes his income by \$1. Is the change an increase or decrease? How much stock does he sell?

CHAPTER XII.

INVOLUTION AND EVOLUTION.

149. A **power** of a number is the number itself, or the product obtained by taking the number several times as a factor.

A **root** of a number is one of the equal factors of that number.

A power or root receives its name from the number of equal factors. For example,

$3^1 = 3$. 3 is the first power of 3.

$3^2 = 9$. 9 is the second power, or square, of 3; and 3 is the second root, or square root, of 9.

$3^3 = 27$. 27 is the third power, or cube, of 3; and 3 is the third root, or cube root, of 27.

$3^4 = 81$. 81 is the fourth power of 3; and 3 is the fourth root of 81.

The **radical sign,** $\sqrt{\ }$, indicates a root. The name of the root is indicated by a small figure placed in the opening of the sign, called the **index** of the root. In expressing square root, the radical sign is generally used alone. For example, $\sqrt{49}$, or $\sqrt[2]{49}$, denotes the square root of 49; $\sqrt[4]{256}$ denotes the fourth root of 256.

NOTE. A root may also be indicated by a fractional exponent. For example, $25^{\frac{1}{2}}$ denotes the square root of 25; $64^{\frac{2}{3}}$ denotes the cube root of the square of 64.

INVOLUTION.

150. The process of finding a power of a number is called **involution.**

I. Find the fourth power of 6.

$$6^4 = 6 \times 6 \times 6 \times 6 = 1296.$$

The fourth power of 6 is the product of four factors, each equal to 6, which equals 1296.

EXAMPLES.

1. What is the square of 11? of 0.11?
2. Find the square of 0.9; of three millionths.
3. What is the third power of 0.1? of 100?
4. What is the third power of 3? of 0.3? of 0.03? of 30?
5. Find the cube of 10.1; of 1.01.
6. What is the cube of $\frac{7}{3}$? of 0.006?
7. What is the fourth power of 2? of 0.2? of 0.02?
8. Find the fifth power of 5; of 50; of 0.5.

NOTE. The student will find it advantageous to commit to memory the squares of all the numbers from 1 to 25 inclusive, and the cubes of all the numbers from 1 to 10 inclusive.

EVOLUTION.

151. The process of finding a root of a number is called **evolution**. When the exact root of a number can be found, the number is called a **perfect power**; all other numbers are **imperfect powers**. The roots of perfect powers can readily be found by factoring.

I. Find the cube root of 9261.

$$\begin{array}{r} 3\underline{)9261} \\ 3\underline{)3087} \\ 3\underline{)1029} \\ 7\underline{)343} \\ 7\underline{)49} \\ 7 \end{array}$$

$$3 \times 7 = 21.$$

The prime factors of 9261 are $3^3 \times 7^3$; hence the cube root of 9261 is 3×7, or 21.

EXAMPLES.

1. Find the square root of 3136.
2. Find the square root of 5184.
3. Find the square root of 11025.
4. Find the cube root of 32768.
5. Find the cube root of 91125.
6. Find the cube root of 456533.
7. Find the fourth root of 331776.
8. Find the fourth root of 1185921.
9. Find the fifth root of 1889568.
10. Find the sixth root of 2985984.

Square Root.

152. To obtain a general method for finding the square root of numbers, we must investigate the relations between simple numbers and their squares.

The first step in extracting the square root of a number is to determine the number of figures in the root. $1^2 = 1$, $10^2 = 100$, $100^2 = 10000$, $1000^2 = 1000000$, and so on. Hence the square of any number between 1 and 10 is a number between 1 and 100, the square of any number between 10 and 100 is a number between 100 and 10000, the square of any number between 100 and 1000 is a number between 10000 and 1000000, and so on. Thus we see that the square of a number contains twice as many figures as the number, or twice as many less one. If, therefore, a number be separated into periods of two figures each by placing a dot over every alternate figure, beginning with the units' figure, the number of figures in the root equals the number of periods.

NOTE 1. The left-hand period has but one figure when the number consists of an odd number of figures.

NOTE 2. The principle applies also to decimals, because the square of a decimal contains twice as many decimal places as the decimal itself.

153. The component parts of the square of a number of two figures may be learned from the following multiplication:

$$\begin{array}{rrl}
 & 47 = & 40+7 \\
 & 47 = & 40+7 \\
\hline
 & 329 = & 40\times 7+7^2 \\
 & 1880 = 40^2 + & 40\times 7 \\
\hline
47^2 = & 2209 = & 40^2+2\times(40\times 7)+7^2
\end{array}$$

In general, the square of any number composed of tens and units is equal to *the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

I. Find the square root of 5329.

$$\begin{array}{rl}
 & \dot{5}3\dot{2}9(70+3=73 \\
 & 4900 \\
\hline
140+3=143) & 429 \\
 & 429 \\
\hline
\end{array}$$

Or more briefly

$$\begin{array}{rl}
 & \dot{5}3\dot{2}9(73 \\
 & 49 \\
\hline
143) & 429 \\
 & 429 \\
\hline
\end{array}$$

Since the number consists of two periods, the square root will consist of two figures. The square of the tens' figure of the root must be the largest square in 53 hundreds, which is 49 hundreds; hence the tens' figure of the root is 7. Subtracting 4900, the square of 70, from 5329, the remainder is 429. Since the square of the tens has been subtracted, 429 equals twice the product of the tens by the units plus the square of the units; this is the same as twice the tens plus the units multiplied by the units. Thus the two factors of 429 are twice the tens plus the units and the units. We can find the units' figure by dividing 429 by the other factor; however, this factor is known only in part, so we take twice the tens, the part known, as a *trial divisor*, and we find that 140 is contained in 429 three times. The units' figure, therefore, is 3, and the *complete divisor* is 140 + 3, or 143. Multiplying 143 by 3, we obtain

429, and there is no further remainder. Hence the square root of 5329 is 73.

In the shorter arrangement of work, 14 may be considered as the trial divisor, and the units' figure is found by dividing 42 by 14, which gives the same result as dividing 429 by 140.

If the number consists of more than two periods, after finding the first two figures of the root, we can consider them as tens in reference to the next figure, and then proceed as before. At each stage of the work the trial divisor is obtained by doubling that part of the root already found.

II. Find the square root of 6796449.

$$\begin{array}{rl}
 & \dot{6}7\dot{9}6\dot{4}4\dot{9}(2607 \\
 & 4 \\
46) & \overline{279} \\
 & 276 \\
5207) & \overline{36449} \\
 & 36449 \\
\end{array}$$

After finding 26 in the root, the trial divisor is 52; 52 is not contained in 36, and the next figure of the root is 0. Then the trial divisor is 520, which can be used at once by bringing down another period.

III. Find the square root of 2.5 to four decimal places.

$$\begin{array}{rl}
 & \dot{2}.5\dot{0}(1.5811 \\
 & 1 \\
25) & \overline{150} \\
 & 125 \\
308) & \overline{2500} \\
 & 2464 \\
3161) & \overline{3600} \\
 & 3161 \\
31621) & \overline{43900} \\
\end{array}$$

A zero must be annexed to 5 to complete the first period after the decimal point. Other periods of two zeros each can be brought down as they are needed.

The square root of a fraction in its lowest terms may be obtained by taking the square root of both terms when they are perfect squares. For example, the square root of $\frac{25}{36}$ is $\frac{5}{6}$; the square root of $7\frac{1}{9}$ is the same as the square root of $\frac{64}{9}$, which equals $\frac{8}{3}$, or $2\frac{2}{3}$. When either term is not a per-

fect square, reduce the fraction to a decimal and then extract the root.

When the denominator of a fraction is the square root of a number, the work may be simplified by first multiplying both terms of the fraction by the denominator. For example, $\frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$.

EXAMPLES.

Find the square root (to five decimal places when the number is not a perfect square) of

1.	676.	13.	7.333264.	25.	$\frac{36}{121}$.
2.	1681.	14.	39.037504.	26.	$\frac{49}{529}$.
3.	624100.	15.	0.9.	27.	$\frac{1}{3}$.
4.	46656.	16.	0.001.	28.	$\frac{1}{10}$.
5.	6.7081.	17.	0.196.	29.	$\frac{5}{7}$.
6.	49.2804.	18.	530.	30.	$\frac{31393}{798}$.
7.	747.4756.	19.	3369.	31.	$30\frac{1}{4}$.
8.	0.005625.	20.	79000.	32.	$22\frac{9}{16}$.
9.	1361610000.	21.	0.002539.	33.	$5\frac{2}{3}$.
10.	4.190209.	22.	0.01952.	34.	$24\frac{1}{4}$.
11.	3444736.	23.	102.002.	35.	$42\frac{2}{5}$.
12.	0.05331481.	24.	0.001601.	36.	$201\frac{1}{4}$.

37. Find the value of $\sqrt{\frac{1.23}{0.625}}$ to four decimal places.

38. Find the value of $\sqrt{(1.06)^5}$ to five decimal places.

39. Find the value of $\sqrt{\frac{1}{21}}$ to three decimal places.

40. Find the value of $\frac{1}{\sqrt{3}}$ to three decimal places.

41. Calculate the value of $\sqrt{3+2\sqrt{2}}$ to two decimal places.

42. Multiply 3.15 by 0.075, and extract the square root of the product to three decimal places.

43. Multiply 903.14 by 0.063, and extract the square root of the product to three decimal places.

44. Divide 3.63 by 2.353, and find the square root of the quotient to three decimal places.

45. Extract the square root of $0.875 \div 2.63$ to three decimal places.

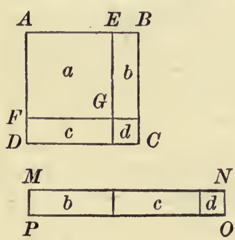
46. Multiply $\sqrt{2}$ by $\sqrt{0.123}$, and carry the result to three decimal places.

47. The area of a square is 655.36 sq. ft.; what is the length of its side?

48. If a square field contains 10.24^{Ha}, find the length of its side in meters.

154. Square root can also be explained by the aid of diagrams.

The area of a square surface is found by squaring the length of one side; hence the length of one side may be found by extracting the square root of the number denoting the area.



$$\begin{array}{r|l} \dot{6}7\dot{6} & (26 \\ 4 & \\ 46)\ 276 & \\ 276 & \end{array}$$

Let *ABCD* represent a square containing 676 square units; we wish to determine the length of one side.

Since the number denoting the area consists of two periods, the square root will consist of two figures. The square of the tens' figure of the root must be the largest square in 6 hundreds, which is 4 hundreds; hence the tens' figure of the root is 2, and the length of a side of the square is 20 plus the units' figure of the root. Let *AE* be 20 units in length; then *a* represents a square whose area is 400. Subtracting this from 676, we find the area of the irregular figure *EBCDFG* to be 276. This figure con-

sists of the two rectangles b and c and the square d; arranging them as in $MNOP$, we have a rectangle whose width is the units' figure of the root, and whose length is 40 plus the units' figure of the root. When the area and length of a rectangle are known, the width can be found by dividing the area by the length; in this case there is a small part of the length unknown, so we take 40 as a trial divisor, and by dividing 276 by 40, we obtain 6 for the units' figure. The entire length, then, is 46, and the breadth is 6. The product of 46 and 6 is 276, and there is no further remainder. Hence 26 is the square root of 676.

Cube Root.

155. A general method for finding the cube root of numbers can be obtained by pursuing lines of investigation similar to those made use of in determining the method for square root.

Since $1^3 = 1$, $10^3 = 1000$, $100^3 = 1000000$, and so on, the cube of any number between 1 and 10 is a number between 1 and 1000, the cube of any number between 10 and 100 is a number between 1000 and 1000000, and so on. Thus we see that the cube of a number contains three times as many figures as the number, or three times as many less one or two. If, therefore, a number be separated into periods of three figures each by placing a dot over every third figure, beginning with the units' figure, the number of figures in the root equals the number of periods.

Note 1. The left-hand period may contain but one or two figures.

Note 2. The principle applies also to decimals, because the cube of a decimal contains three times as many decimal places as the decimal itself.

156. The component parts of the cube of a number of two figures may be learned from the following multiplication:

$$\begin{array}{rrl}
47^2 = & 2209 = & 40^2 + 2 \times (40 \times 7) + 7^2 \\
 & 47 = & 40 + 7 \\
\hline
 & 15463 = & 40^2 \times 7 + 2 \times (40 \times 7^2) + 7^3 \\
 & 8836 = & 40^3 + 2 \times (40^2 \times 7) + 40 \times 7^2 \\
\hline
47^3 = & 103823 = & 40^3 + 3 \times (40^2 \times 7) + 3 \times (40 \times 7^2) + 7^3
\end{array}$$

In general, the cube of any number composed of tens and units is equal to *the cube of the tens, plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units.*

I. Find the cube root of 175616.

$$\begin{array}{r|l}
 & 17\dot{5}61\dot{6}(50+6=56 \\
 & \underline{125000} \\
3\times 50^2 = 7500 & 50616 \\
3\times 50\times 6 = 900 & \\
6^2 = \underline{36} & \\
\underline{8436} & \underline{50616}
\end{array}$$

Or more briefly

$$\begin{array}{r|l}
 & 17\dot{5}61\dot{6}(56 \\
 & \underline{125} \\
7500 & 50616 \\
900 & \\
\underline{36} & \\
\underline{8436} & \underline{50616}
\end{array}$$

Since the number consists of two periods, the cube root will consist of two figures. The cube of the tens' figure of the root must be the largest cube in 175 thousands, which is 125 thousands; hence the tens' figure of the root is 5. Subtracting 125000, the cube of 50, from 175616, the remainder is 50616. Since the cube of the tens has been subtracted, 50616 equals three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units; this is the same as three times the square of the tens plus three times the product of the tens by the units plus the square of the units multiplied by the units. Thus the two factors of 50616 are three times the square of the tens plus three times the product of the tens by the units plus the square of the units and the units. We can find the units' figure by dividing 50616 by the other factor; however, this factor is known only in part, so we take three times the square of the tens, the part known, for the trial divisor, and we find that 7500 is contained in 50616 six times. The units' figure, therefore, is 6, and the complete divisor is $7500 + 3 \times 50 \times 6 + 6^2$, or 8436. Multiplying 8436 by 6, we obtain 50616, and there is no further remainder. Hence the cube root of 175616 is 56.

If the number consists of more than two periods, after finding the first two figures of the root, we can consider

them as tens in reference to the next figure, and then proceed as before.

II. Find the cube root of 22069.810125.

$$2\dot{2}06\dot{9}.81\dot{0}12\dot{5}\,(28.05$$

	8
1200	14069
480	
64	
1744	13952
23520000	117810125
42000	
25	
23562025	117810125

After finding 28 in the root, the trial divisor is 3×280^2, or 235200. This is not contained in 117810, so the next figure of the root is 0. Then the trial divisor is 23520000, which can be used at once by bringing down another period.

The quotient obtained by dividing the dividend by the trial divisor often proves to be too large; in such a case try a smaller figure in the root.

When there is a remainder after using all the periods, further figures in the root can be obtained by continuing the process, bringing down periods of three zeros each.

The cube root of a fraction in its lowest terms may be obtained by taking the cube root of both terms when they are perfect cubes. When either term is not a perfect cube, reduce the fraction to a decimal and then extract the root.

EXAMPLES.

Find the cube root (to four decimal places when the number is not a perfect cube) of

1. 4913.	5. 96702.579.	9. 0.000000148877.
2. 24389.	6. 8365.427.	10. 225866529.
3. 250047.	7. 0.000032768.	11. 1027243.729.
4. 636056000.	8. 0.001295029.	12. 12000.8121619.

13. 0.27.
14. 10.
15. 1.025.
16. 3.7.
17. 0.0093.
18. 18.65.
19. 0.61.
20. 51.
21. 1729.
22. 9358.
23. $\frac{64}{125}$.
24. $\frac{729}{1728}$.
25. $\frac{5}{7}$.
26. $\frac{17}{33}$.
27. $15\frac{5}{8}$.
28. $12\frac{1}{2}$.
29. $46\frac{2}{3}$.
30. $81\frac{5}{11}$.

31. Find the value of $\sqrt[3]{\frac{4.29}{0.125}}$ to three decimal places.

32. Find the value of $\sqrt[3]{\frac{\frac{3}{4}}{1\frac{9}{16}}+\frac{2\frac{1}{2}}{16}}$ to three decimal places.

33. Find the value of $\sqrt[3]{(1.05)^4}$ to three decimal places.

34. Find the value of $\sqrt[3]{5+2\sqrt[3]{5}}$ to two decimal places.

35. Multiply 2.49 by 22.32, and extract the cube root of the product to two decimal places.

36. Divide 3.15 by 0.075, and extract the cube root of the quotient to two decimal places.

37. Divide 6 by 0.89, and extract the cube root of the quotient to two decimal places.

38. Multiply 108 billionths by two thousand, and extract the cube root of the product.

39. Multiply $\sqrt[3]{4}$ by $\sqrt[3]{0.456}$, and carry the result to two decimal places.

40. Find the difference between the sum of the cube roots of 32768 and 0.000512 and the cube root of their sum.

41. A cubical vessel contains 1331^{l}; what is the length of its edge in meters?

42. A cubical block contains 12695.24 cu. in.; find the length of one side.

157. Cube root can also be explained by the aid of diagrams.

The solid contents of a cube are found by cubing the length of an edge; hence the length of an edge may be found by extracting the cube root of the number denoting the solid contents.

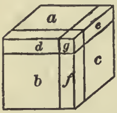


FIG. 1.

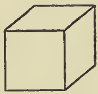


FIG. 2.

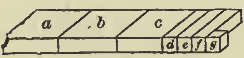


FIG. 3.

$$\begin{array}{r|l}
 & \dot{1}38\dot{2}4(24 \\
 & \underline{8} \\
1200 & 5824 \\
240 & \\
\underline{16} & \\
1456 & \underline{5824}
\end{array}$$

Let Fig. 1 represent a cube containing 13824 cubic units; we wish to determine the length of an edge.

Since the number denoting the cubic contents consists of two periods, the cube root will consist of two figures. The cube of the tens' figure of the root must be the largest cube in 13 thousands, which is 8 thousands; hence the tens' figure of the root is 2, and the length of an edge of the cube is 20 plus the units' figure of the root. Let Fig. 2 represent a cube whose edge is 20 units; then its volume is 8000. Subtracting this from 13824, we have left an irregular solid whose volume is 5824. This irregular solid consists of the three rectangular solids a, b, and c, the three smaller rectangular solids d, e, and f, and the cube g. Arranging them as in Fig. 3, we have a series of solids which have for their common thickness the units' figure of the root. The bases of a, b, and c are each 20^2, or 400; the bases of d, e, and f are each the product of 20 and the units' figure of the root; and the base of g is the square of the units' figure of the root. When the volume and base of a rectangular solid are known, the thickness can be found by dividing the volume by the base; the common thickness of the solids in Fig. 3 can be found by dividing their entire volume by the sum of their bases. a, b, and c are the only solids whose bases are known, so we take the sum of these bases, 3×20^2, or 1200, for the trial divisor, and by dividing 5824 by 1200, we have 4 for the units' figure of the root. The sum of the bases, then, is $3 \times 20^2 + 3 \times 20 \times 4 + 4^2$, or 1456. The product of 1456 and 4 is 5824, and there is no further remainder. Hence 24 is the cube root of 13824.

HIGHER ROOTS.

158. By means of the processes of square and cube root, we can find any root whose index contains only the factors 2 and 3. For example, the fourth root of a number may be found by taking the square root of the square root; the sixth root of a number may be found by taking the square root of the cube root or the cube root of the square root.

EXAMPLES.

1. Find the fourth root of 1874161.
2. Find the fourth root of 8.25 to three decimal places.
3. Find the sixth root of 1291467969.
4. Find the sixth root of 0.184 to three decimal places.
5. Find the eighth root of 2562890625.
6. Find the eighth root of 5 to three decimal places.
7. Find the ninth root of 134217728.
8. Find the ninth root of $3\frac{1}{8}$ to two decimal places.
9. Find the twelfth root of 13841287201.
10. Find the twelfth root of 0.75 to two decimal places.

CHAPTER XIII.

SERIES.

159. A **series** is a succession of numbers, each of which is derived from one or more of the preceding by a fixed law. The numbers which compose a series are called its **terms**; the first and last terms are called the **extremes**, and the other terms are called the **means**. An **ascending series** is one in which the terms increase regularly from the first term; a **descending series** is one in which the terms decrease regularly from the first term.

Arithmetical Progression.

160. When a series increases or decreases by a common difference, it is called an **arithmetical series** or **arithmetical progression.** For example, 2, 5, 8, 11, 14, 17 is an ascending arithmetical progression, in which the common difference is 3.

In every arithmetical progression there are five elements to be considered, — *the first term*, *the last term*, *the common difference, the number of terms*, *and the sum of the terms.* These five elements bear such a relation to each other that when any three are given, the other two can be found. This gives rise to twenty distinct cases, a few of the more important of which will here be illustrated.

I. In an ascending arithmetical series the first term is 7, and the common difference is 4; find the 10th term.

$7 + 9 \times 4 = 7 + 36 = 43.$ The first term is 7; the second term equals 7 plus the common difference; the third term equals the second term plus the common difference, or

7 plus twice the common difference; the fourth term equals the third term plus the common difference, or 7 plus three times the common difference. In like manner, the tenth term equals 7 plus nine times the common difference.

II. Find the first term of an ascending arithmetical series, the last term of which is 47, the common difference 6, and the number of terms 8.

$$47 - 7 \times 6 = 47 - 42 = 5.$$

The first term must be such a number that if 7×6 be added to it, the result will be 47; hence, if 7×6 be subtracted from 47, the remainder is the first term.

III. The extremes of an arithmetical progression are 8 and 63, and the number of terms is 12; what is the common difference?

$$\frac{63-8}{12-1} = \frac{55}{11} = 5.$$

The last term is determined by adding the common difference to the first term as many times as there are terms less one; hence $63-8$, the difference between the extremes, equals the common difference taken $12-1$ times, and the common difference equals $63-8$ divided by $12-1$.

IV. The extremes of an arithmetical progression are 4 and 103, and the common difference is 9; what is the number of terms?

$$\frac{103-4}{9} + 1 = 11 + 1 = 12.$$

The difference between the extremes equals the common difference taken as many times as there are terms less one; hence $\frac{103-4}{9}$ equals one less than the number of terms, and $\frac{103-4}{9} + 1$ equals the number of terms.

V. The first term of an arithmetical progression is 4, the last term 19, and the number of terms 6; find the sum of the terms.

$$\frac{6 \times (4+19)}{2} = \frac{6 \times 23}{2} = 69.$$

Using the method shown in example III., the common difference is $\frac{19-4}{6-1}$, which equals $\frac{15}{5}$, or 3. The sum of the series can then be written

$$4 + 7 + 10 + 13 + 16 + 19,$$

or $19 + 16 + 13 + 10 + 7 + 4$. By addition we find twice the sum to be $23 + 23 + 23 + 23 + 23 + 23$, which equals $6 \times (4 + 19)$; hence the sum of the terms equals

$$\frac{6 \times (4 + 19)}{2}.$$

Let the first term be represented by a, the last term by l, the common difference by d, the number of terms by n, and the sum of the terms by s; then the principles illustrated in the foregoing examples may be briefly expressed as follows:

$$l = a + (n - 1) \times d.$$

$$a = l - (n - 1) \times d.$$

$$d = \frac{l - a}{n - 1}.$$

$$n = \frac{l - a}{d} + 1.$$

$$s = \frac{n}{2} \times (a + l).$$

NOTE. The principles as stated apply to ascending series. They can be stated so as to apply to descending series by interchanging a and l.

To solve a problem in arithmetical progression, it is merely necessary to substitute the given values in the proper formula, and then simplify the expression thus obtained.

EXAMPLES.

1. The first term of an ascending arithmetical series is 6, and the common difference is 5; find the 20th term.

2. In an ascending arithmetical series the first term is 8, and the common difference is $\frac{2}{3}$; what is the 30th term?

3. The first term of a descending arithmetical series is 120, the common difference 6, and the number of terms 15; what is the last term?

4. The first term of a descending arithmetical series is 54, and the common difference is 4; find the 12th term.

5. The 12th term of an ascending arithmetical series is 60, and the common difference is 2; what is the first term?

6. The last term of a descending arithmetical series is 3, the common difference 4, and the number of terms 11; what is the first term?

7. The extremes of an arithmetical progression are 9 and 49, and the number of terms is 9; what is the common difference?

8. The extremes of an arithmetical progression are $1\frac{1}{2}$ and 24, and the common difference is $2\frac{1}{2}$; find the number of terms.

9. The extremes of an arithmetical progression are 0 and 150, and the number of terms is 16; find the sum of all the terms.

10. Find the sum of the first 12 terms of the series 3, 7, 11, etc.

11. Find the sum of the first 10 terms of the series 24, $22\frac{1}{2}$, 21, etc.

12. Find the sum of the odd numbers from 1 to 49 inclusive.

13. How many strokes does the hammer of a clock strike in 12 hours?

14. How far can a man walk in 10 days, going 12 miles the first day, and increasing the rate 3 miles a day?

15. A man travelled 13 days, travelling each day $\frac{5}{6}$ of a mile more than the preceding day. If he went 18 miles the last day, how many miles did he travel the first day?

16. A man going a journey, travelled the first day 5 miles, the last day 32 miles, and each day 3 miles more than the preceding day. How many days did he travel?

17. A man travels 11 days, travelling 5 miles the first day, and increasing the distance equally each day, so that the last day's journey is 20 miles; find the daily increase.

18. A man has 8 children, whose several ages differ alike; the youngest is 2 years old, and the oldest 30. What is the common difference of their ages?

19. A laborer worked for 40 cents the first day, and on each succeeding day his wages were increased 5 cents; on the last day he received $2.50. How many days did he work?

20. A stone falls 16.08 ft. during the first second, 48.24 ft. during the next second, 80.4 ft. during the third second, and so on; how far will it fall during the ninth second? How far will it fall in nine seconds?

21. Find the sum of an arithmetical series whose first two terms are 6 and 13, and whose last term is 62.

22. Find the amount of $300 for 15 years at 5% simple interest.

23. In how many years will $150 amount to $330 at 6% simple interest?

24. The amount of $500 for 18 years is $860; what is the yearly interest?

Geometrical Progression.

161. When a series increases or decreases by a common ratio, it is called a **geometrical series** or **geometrical progression**. For example, 2, 6, 18, 54, 162 is an ascending geometrical progression, in which the ratio is 3.

An **infinite series** is a descending series of an infinite number of terms. For example, 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, etc.; the last term is infinitely small and is regarded as zero.

In every geometrical progression there are five elements to be considered, — *the first term, the last term, the ratio, the number of terms, and the sum of the terms.* As in arithmetical progression, the five elements bear such a relation to each other that when any three are given, the other two can be found.

I. The first term of a geometrical series is 2, and the ratio is 3; what is the 7th term?

$2 \times 3^{7-1} = 2 \times 3^6 = 2 \times 729 = 1458.$ The first term is 2; the second term equals 2 multiplied by the ratio; the third term equals the second term multiplied by the ratio, or 2 multiplied by the square of the ratio; the fourth term equals the third term multiplied by the ratio, or 2 multiplied by the cube of the ratio. In like manner, the seventh term equals 2 multiplied by the sixth power of the ratio.

II. The last term of a geometrical series is 640, the ratio 2, and the number of terms 8; what is the first term?

$\frac{640}{2^{8-1}} = \frac{640}{2^7} = \frac{640}{128} = 5.$ The first term must be such a number that if it be multiplied by 2^7, the result will be 640; hence, if 640 be divided by 2^7, the quotient is the first term.

III. The first term of a geometrical series is 7, the last term 567, and the number of terms 5; what is the ratio?

$\sqrt[5-1]{\frac{567}{7}} = \sqrt[4]{81} = 3.$ 567, the fifth term, equals 7 multiplied by the 4th power of the ratio; hence, if 567 be divided by 7, the quotient, 81, is the fourth power of the ratio, and the ratio is the 4th root of 81, or 3.

IV. The extremes of a geometrical progression are 3 and 768, and the ratio is 4; find the sum of the terms.

$$\frac{768 \times 4 - 3}{4 - 1} = \frac{3072 - 3}{3} = \frac{3069}{3} = 1023.$$

The sum of the series may be written

$3 + 12 + 48 + 192 + 768$. Four times the sum equals $12 + 48 + 192 + 768 + 3072$. Subtracting the upper line from the lower line, we have $3072 - 3$, which equals three times the sum of the series; hence the sum of the series equals $\frac{3072-3}{3}$, or 1023.

NOTE. The last term, 768, equals the first term multiplied by 4^{5-1}; hence 768×4, or 3072, equals the first term multiplied by 4^5, and the value of the sum might be written $\frac{3 \times 4^5 - 3}{4-1}$, which equals 1023.

V. Find the sum of the first 5 terms of the series 768, 192, 48, etc.

$$\frac{768 - 3 \times \frac{1}{4}}{1 - \frac{1}{4}} = \frac{768 - \frac{3}{4}}{\frac{3}{4}} = \frac{767\frac{1}{4}}{\frac{3}{4}} = \frac{3069}{3} = 1023.$$

The sum of the series may be written

$768 + 192 + 48 + 12 + 3$. One fourth of the sum equals $192 + 48 + 12 + 3 + \frac{3}{4}$. Subtracting the lower line from the upper line, we have $768 - \frac{3}{4}$, which equals three fourths of the sum of the series; hence the sum of the series equals $\frac{768 - \frac{3}{4}}{\frac{3}{4}}$, or 1023.

NOTE. The value of the sum might be written $\frac{768 - 768 \times (\frac{1}{4})^5}{1 - \frac{1}{4}}$, which equals 1023.

VI. Find the sum of the infinite series 768, 192, 48, etc.

$$\frac{768}{1 - \frac{1}{4}} = \frac{768}{\frac{3}{4}} = \frac{3072}{3} = 1024.$$

The last term is regarded as 0; hence the expression for the sum as found in the preceding example becomes $\frac{768 - 0 \times \frac{1}{4}}{1 - \frac{1}{4}}$, which equals $\frac{768}{\frac{3}{4}}$, or 1024.

Let the first term be represented by a, the last term by l, the ratio by r, the number of terms by n, and the sum of the terms by s; then the principles illustrated in the foregoing examples may be briefly expressed as follows:

$$l = a \times r^{n-1}.$$

$$a = \frac{l}{r^{n-1}}.$$

$$r = \sqrt[n-1]{\frac{l}{a}}.$$

$$s = \frac{l \times r - a}{r - 1} \text{ or } \frac{a \times r^n - a}{r - 1}, \text{ for ascending series.}$$

$$s = \frac{a - l \times r}{1 - r} \text{ or } \frac{a - a \times r^n}{1 - r}, \text{ for descending series.}$$

$$s = \frac{a}{1 - r}, \text{ for infinite series.}$$

To solve a problem in geometrical progression, it is merely necessary to substitute the given values in the proper formula, and then simplify the expression thus obtained.

EXAMPLES.

1. The first term of a geometrical series is 8, and the ratio is 4; find the 8th term.

2. The first term of a geometrical series is 27, and the ratio is $\frac{1}{3}$; find the 7th term.

3. The 6th term of a geometrical series is 3888, and the ratio is 6; find the first term.

4. The last term of a geometrical series is $60\frac{3}{4}$, the ratio $\frac{3}{4}$, and the number of terms 7; what is the first term?

5. The first term of a geometrical series is 10, and the 6th term is 2430; what is the ratio?

6. The first term of a geometrical series is $\frac{1}{150}$, the last term $104\frac{1}{6}$, and the number of terms 7; find the ratio.

7. Find the sum of the first 9 terms of the series whose first term is 6 and ratio 4.

8. Find the sum of the first 5 terms of the series whose first term is 100 and ratio $\frac{1}{4}$.

9. Find the sum of an infinite series whose first term is 3 and ratio $\frac{1}{3}$.

10. Find the sum of the first 8 terms of the series 8, 4, 2, etc. Find the sum of the same series to infinity.

11. The second term of a geometrical progression is 36; find the sum of 4 terms when the ratio is $1\frac{1}{3}$; also when the ratio is $1\frac{1}{2}$.

12. A merchant doubles his capital every 5 years; if he begins with $2000, how much has he at the end of 25 years?

13. If a ball be put in motion by a force which would move it 10 feet the first second, 8 feet the second, 6.4 feet the third, and so on, how far would it move?

14. What sum of money can be paid by 10 instalments, the first of which is $1, the second $2, the third $4, and so on in a geometrical progression?

15. A man worked 8 days on condition that he should receive 1 cent the first day, 5 cents the second day, and so on, the wages of each day being 5 times the wages of the previous day; how much did he receive?

16. A man travels 4 miles the first day, 8 miles the second day, 16 miles the third day, and so on. How far does he travel the 7th day? How far does he travel in 7 days?

Compound Interest.

162. Problems in compound interest can be solved by the principles of geometrical progression.

Let P represent the principal, r the interest of $1 for 1 year, n the number of years, and A the amount of the given principal for n years. In computing compound interest P is multiplied by $1+r$ as many times as there are years. Thus P is the first term of a geometrical series, of which A is the last term, and $1+r$ the ratio; the number of terms is

one more than the number of years, or $n+1$. Hence the first three formulæ for geometrical progression when applied to compound interest become

$$A = P \times (1+r)^n.$$

$$P = \frac{A}{(1+r)^n}.$$

$$1+r = \sqrt[n]{\frac{A}{P}}, \text{ or } r = \sqrt[n]{\frac{A}{P}} - 1.$$

I. Find the amount of $250 for 3 years at 5% compound interest.

$$A = 250 \times (1.0\dot{5})^3.$$

$$\begin{array}{r} 1.05 \\ \underline{1.05} \\ 525 \\ \underline{105} \\ 1.1025 \\ \underline{1.05} \\ 55125 \\ \underline{11025} \\ 1.157625 \end{array} \qquad \begin{array}{r} 1.157625 \\ \underline{250} \\ 57881250 \\ \underline{2315250} \\ 289.406250 \end{array}$$

Ans. $289.41.

To solve the problem it is merely necessary to substitute the values for P, r, and n in the formula for A, and then simplify.

II. At what rate per cent must $500 be put out at compound interest to amount to $571.20 in 3 years?

$$r = \sqrt[3]{\frac{571.20}{500}} - 1.$$

$$500)\underline{571.20}$$

$$\begin{array}{r|l} & \dot{1}.14\dot{2}40\dot{0}(1.045 \\ & \underline{1} \\ 30000 & 142400 \\ 1200 & \\ \underline{16} & \\ \underline{31216} & \underline{124864} \\ 3\dot{2}44800) & 17536000 \end{array}$$

Ans. 4.5%.

Substitute values of A, P, and n in the formula for r, and then simplify.

EXAMPLES.

1. Find the amount of $600 for 5 years at 6% compound interest.

2. Find the amount of $1500 for 6 years at 7% compound interest.

3. Find the amount of $75 for 8 years at 4% compound interest.

4. What principal at 5% compound interest will amount to $53.98 in 4 years?

5. What principal at 6% compound interest will amount to $1000 in 6 years?

6. What sum must be invested at 4% compound interest to amount to $600 in 8 years?

7. At what rate of compound interest will $2500 amount in 3 years to $4320?

8. At what rate of compound interest will $500 amount in 4 years to $631.24?

9. At what rate of compound interest will a sum of money double itself in 6 years?

10. At what rate of compound interest will a sum of money treble itself in 12 years?

Annuities.

163. An **annuity** is a sum of money, payable yearly, to continue for a certain number of years, for life, or forever. The term is also applied to a sum of money payable at any regular intervals of time.

The **amount** or **final value** of an annuity is the sum of all the payments plus the interest on each payment from the time it becomes due until the annuity ceases.

The **present worth** of an annuity is such a sum of money as will, if put at interest, amount to the final value.

ANNUITIES AT SIMPLE INTEREST.

164. Problems in annuities at simple interest can be solved by the principles of arithmetical progression.

I. What is the amount of an annuity of $300 for 5 years at 6% simple interest?

$$\begin{aligned} l &= a + (n-1) \times d \\ &= 300 + (5-1) \times 18 \\ &= 300 + 72 = 372. \end{aligned}$$

$$\begin{aligned} s &= \frac{n}{2} \times (a + l) \\ &= \frac{5}{2} \times (300 + 372) \\ &= \frac{5}{\cancel{2}} \times \overset{336}{\cancel{672}} = \$1680. \end{aligned}$$

The payment at the end of the fifth year is $300, the payment at the end of the fourth year amounts to $300 plus the interest for 1 year, the payment at the end of the third year amounts to $300 plus the interest for 2 years, and so on. These sums form an arithmetical progression, of which $300 is the first term, the interest of $300 for 1 year, or $18, is the common difference, and 5 is the number of terms. By principles of arithmetical progression, we find the sum of the terms to be $1680.

To find the present worth of the annuity of example I., find the present worth of $1680 for 5 years, as shown in § 138.

II. Find the annuity whose amount for 6 years at 5% simple interest is $1350.

$$\begin{aligned} l &= a + (n-1) \times d \\ &= 1 + (6-1) \times .05 = 1.25. \end{aligned}$$

$$\begin{aligned} s &= \frac{n}{2}(a + l) \\ &= \frac{6}{2}(1 + 1.25) = 6.75. \end{aligned}$$

```
6.75)1350.00($200
     1350
     ----
       00
```

If the annuity were $1, we would have $a = 1$, $l = 1 + (6-1) \times .05 = 1.25$, and $s = \frac{6}{2}(1 + 1.25) = 6.75$. It takes an annuity of as many dollars to amount to $1350 as $6.75 is contained times in $1350, which equals $200.

EXAMPLES.

1. What is the amount of an annuity of \$500 for 8 years at 6% simple interest?

2. What is the amount of an annuity of \$1000 for 10 years at 5% simple interest?

3. What is the amount of an annuity of \$200 for 5 years at $4\frac{1}{2}$% simple interest?

4. A clerk's salary of \$1000 a year, payable quarterly, remained unpaid for three years; find the amount then due, reckoning interest at 6%.

5. What is the present worth of an annuity of \$600 for 6 years at 6% simple interest?

6. What is the present worth of an annuity of \$150 for 12 years at 4% simple interest?

7. Find the annuity whose amount for 6 years at 6% simple interest is \$3450.

8. Find the annuity whose amount for 10 years at 5% simple interest is \$3675.

9. Find the annuity whose amount for 5 years at 4% simple interest is \$1500.

Annuities at Compound Interest.

165. Problems in annuities at compound interest can be solved by the principles of geometrical progression.

I. What is the amount of an annuity of \$200 for 4 years at 6% compound interest?

$$s = \frac{a \times r^n - a}{r - 1} = \frac{200 \times (1.06)^4 - 200}{1.06 - 1}.$$

1.06	1.26247696
1.06	200
636	252.49539200
106	200
1.1236	.06)52.495392
1.06	\$874.92
67416	
11236	
1.191016	
1.06	
7146096	
1191016	
1.26247696	

The payment at the end of the fourth year is \$200, the payment at the end of the third year amounts to \$200 plus the interest for 1 year, the payment at the end of the second year amounts to \$200 plus the compound interest for 2 years, and so on. These sums form a geometrical progression, of which \$200 is the first term, 1.06 is the ratio, and 4 is the number of terms. By the principles of geometrical progression, we find the sum of the terms to be \$874.92.

To find the present worth of the annuity of example I., find the sum of money which put on interest for 4 years at 6% compound interest will amount to \$874.92, as shown in § 162.

II. Find the annuity whose amount for 3 years at 5% compound interest is \$504.40.

$$s = \frac{a \times r^n - a}{r - 1} = \frac{1 \times (1.05)^3 - 1}{1.05 - 1}.$$

1.05	3.1525)504.4000(\$160
1.05	31525
525	189150
105	189150
1.1025	0
1.05	
55125	
11025	
1.157625	
1.	
.05).157625	
3.1525	

If the annuity were \$1, we would have $a = 1$, and $s = \frac{1 \times (1.05)^3 - 1}{1.05 - 1} = 3.1525.$ It takes an annuity of as many dollars to amount to \$504.40 as \$3.1525 is contained times in \$504.40, which equals \$160.

EXAMPLES.

1. What is the amount of an annuity of $50 for 5 years at 6% compound interest?

2. What is the amount of an annuity of $200 for 6 years at 7% compound interest?

3. What is the amount of an annuity of $1000 for 8 years at 4% compound interest?

4. If a man deposits $100 a year in a savings bank that pays 3% compound interest, how much will he have in the bank at the end of 10 years?

5. What is the present worth of an annuity of $60 for 4 years at 6% compound interest?

6. What is the present worth of an annuity of $200 for 7 years at 5% compound interest?

7. Find the annuity whose amount for 3 years at 6% compound interest is $95.51.

8. Find the annuity whose amount for 5 years at 6% compound interest is $2818.55.

9. Find the annuity whose amount for 6 years at 5% compound interest is $1000.

CHAPTER XIV.

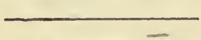
MENSURATION.

166. **Mensuration** is the process of finding the lengths of lines, the areas of surfaces, or the volumes of solids. The principles of mensuration that apply to rectangles and rectangular solids are given in sections 80 and 81. The present chapter contains such principles as are useful to students of Arithmetic, but the proofs of these principles must be learned from Geometry.

Definitions.

167. A **point** is that which has only position.

A **line** is that which has length without breadth or thickness. A **straight line** is a line which has the same direction throughout its whole extent. A **curved line** is a line which changes its direction at every point.



Straight Line.

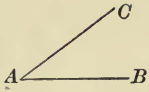
Curved Line.

A **surface** is that which has length and breadth without thickness. A **plane surface** is a surface such that if any two of its points be joined by a straight line, that line lies wholly in the surface. A **curved surface** is a surface no portion of which is plane.

A **solid** is that which has length, breadth, and thickness.

Parallel lines are lines in the same plane which have the same direction. They are equally distant and can never meet.

Parallel Lines.

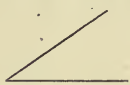


An **angle** is the difference in direction between two lines which meet at a point, called the **vertex**; the lines are called the **sides** of the angle. BAC is an angle whose vertex is the point A, and whose sides are AB and AC.

RIGHT ANGLE.

A **right angle** is an angle such that if one of its sides be produced through the vertex, the two angles thus formed are equal. The two sides of a right angle are said to be **perpendicular** to each other.

OBTUSE ANGLE.



ACUTE ANGLE.

All angles not right angles are called **oblique angles.** An angle greater than a right angle is called an **obtuse angle,** and an angle less than a right angle is called an **acute angle.**

A **plane figure** is a portion of a plane surface bounded by straight or curved lines.

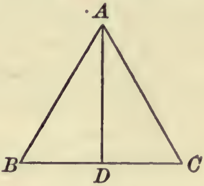
A **polygon** is a plane figure bounded by straight lines; the lines are called the **sides** of the polygon. The **perimeter** of a polygon is the sum of its sides. The **area** of a polygon is the surface included within the perimeter. A **diagonal** of a polygon is a line joining the vertices of two angles not adjacent.

An **equilateral polygon** has all its sides equal. An **equiangular polygon** has all its angles equal. A **regular polygon** is both equilateral and equiangular.

Polygons are named according to the number of sides. A polygon of three sides is a **triangle,** four sides a **quadrilateral,** five sides a **pentagon,** six sides a **hexagon,** seven sides a **heptagon,** eight sides an **octagon,** nine sides a **nonagon,** ten sides a **decagon,** and so on.

TRIANGLES.

168. A polygon of three sides is called a **triangle.** The **base** of a triangle is the side on which it is supposed to stand. The **vertical angle** is the angle opposite the base, and its vertex is called the **vertex** of the triangle. The **altitude** is the perpendicular distance from the vertex to the base. The base and altitude are called the **dimensions** of the triangle. In the triangle



ABC, *BC* is the base, the angle *BAC* is the vertical angle, and *AD* is the altitude.

An **equilateral triangle** has all its sides equal. (Fig. 1.)

An **isosceles triangle** has two of its sides equal. (Fig. 2.)

A **scalene triangle** has no two sides equal. (Fig. 3.)

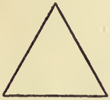


FIG. 1.

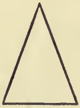


FIG. 2.

FIG. 3.

FIG. 4.

A **right triangle** is a triangle having one right angle. The side opposite the right angle is called the **hypotenuse** or **hypothenuse,** and the side perpendicular to the base is called the **perpendicular.** Fig. 4 is a right triangle, in which *AB* is the hypotenuse, *CB* the base, and *AC* the perpendicular.

An **obtuse triangle** is a triangle having one obtuse angle. (Fig. 3.)

An **acute triangle** is a triangle having three acute angles. (Fig. 2.)

An **equiangular triangle** has all its angles equal. (Fig. 1.)

To find the area of a triangle when the base and altitude are given, *take one half the product of the base by the altitude.*

To find the area of a triangle when three sides are given, *subtract each side separately from half the sum of the three sides; then multiply the continued product of these three remainders by half the sum of the sides, and extract the square root of the product.*

To find one dimension of a triangle when the area and the other dimension are given, *divide twice the area by the given dimension.*

Let A represent the area of a triangle, h the altitude, b the base, a and c the other two sides, and s half the sum of the sides; then

$$A = \tfrac{1}{2} \times b \times h.$$

$$A = \sqrt{s \times (s-a) \times (s-b) \times (s-c)}.$$

$$b = \frac{2 \times A}{h}.$$

$$h = \frac{2 \times A}{b}.$$

EXAMPLES.

1. A triangle has a base of 40 ft., and an altitude of 15 ft.; how many square feet does it contain?

2. Find the area of a triangle, the length of whose base is 25 ft., and the height 12 ft. 4 in.

3. Find the number of acres in a triangular field whose base is 20.28 ch. and altitude 14.5 ch.

4. How many acres are there in a triangular lot whose base is 432 ft. and altitude 320 ft.?

5. Find the number of hektars in a triangular field whose base is 196.8^{m} and altitude 85^{m}.

6. Find the area of a triangle whose sides are respectively 4 ft., 6 ft., and 8 ft.

7. Find the number of hektars in a triangular field whose sides are respectively 62.4^{m}, 84.2^{m}, and 106.8^{m}.

8. At 60 cents a square yard, find the cost of paving a triangular court whose sides are respectively 80 ft., 75 ft., and 60 ft.

9. Find the altitude of a triangle whose area is $137\frac{1}{2}$ sq. ft. and base 20 ft.

10. Find the altitude of a triangle whose area is 3.25^{Ha} and base 502^{m}.

11. Find the base of a triangle whose area is 20 A. and altitude 80 rd.

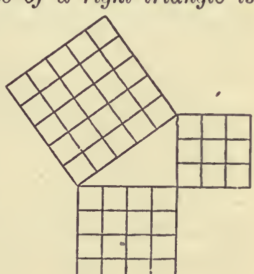
12. Find the base of a triangle whose area is 12.6^{a} and altitude 30^{m}.

13. Find the area and altitude of an equilateral triangle whose sides are each 12 ft. long.

14. Find the perpendicular distances from the vertices to the opposite sides of a triangle, when the sides are respectively 12^{cm}, 15^{cm}, and 20^{cm}.

Right Triangles.

169. *The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.* This principle is illustrated in the annexed diagram.



To find the hypotenuse of a right triangle when the other two sides are given, *extract the square root of the sum of the squares of the other two sides.*

To find one side of a right triangle when the hypotenuse and other side are given, *extract the square root of the difference between the squares of the hypotenuse and the other side.*

NOTE. The length and breadth of a rectangle form two sides of a right triangle of which the diagonal of the rectangle is the hypotenuse.

To find the diagonal of a rectangular parallelopiped, *extract the square root of the sum of the squares of the three dimensions.*

EXAMPLES.

1. Find the hypotenuse of a right triangle whose base is 30 ft. and perpendicular 16 ft.

2. The hypotenuse of a right triangle is $16\frac{1}{4}$ ft., and the base is 15 ft.; what is the perpendicular?

3. The hypotenuse of a right triangle is 3.25^m, and the perpendicular is 3^m; find the base.

4. A flag-pole 140 ft. high casts a shadow 105 ft. in length; what is the distance from the top of the pole to the end of the shadow?

5. What is the length of a ladder that will just reach to the top of a house 12^m high, when its foot is placed 8.4^m from the house?

6. Find the height of the eaves of a house that can be reached by a ladder 40 ft. long, when the foot of the ladder stands 24 ft. from the house.

7. A pole was broken 26 ft. from the bottom, and fell so that the end struck 19 ft. 6 in. from the foot; find the length of the pole.

8. Find the width of a street, from a point in which a ladder 36 ft. long will reach a window 28 ft. high on one side, and one $25\frac{1}{2}$ ft. high on the other.

9. A steamer goes due north at the rate of 15 miles an hour, and another due west 18 miles an hour. How far apart will they be in 6 hours?

10. A rectangular field is 96 rd. long and 72 rd. wide; find the length of the diagonal.

11. Find the longest straight line that can be drawn on a floor 4.5^m long and 3.2^m wide.

12. The side of a square field is 40 rd.; find the distance between two diagonally opposite corners.

13. The diagonal of a square equals 16 ft.; find the length of a side.

14. What is the length of the diagonal of a room 20 ft. long, 16 ft. wide, and 12 ft. high?

15. Find the length of the diagonal of a box 4 ft. 8 in. long, 2 ft. 4 in. wide, and 7 in. deep.

16. Find the diagonal of a cubical block whose edge is $3\frac{1}{4}$ inches.

17. The diagonal of a cube equals 10^{cm}; find the length of an edge.

Quadrilaterals.

170. A polygon of four sides is called a **quadrilateral.**

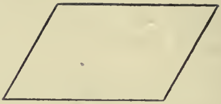
A **parallelogram** is a quadrilateral whose opposite sides are parallel. A **rectangle** is a parallelogram whose angles are right angles; a **square** is a rectangle whose sides are all equal. A **rhomboid** is a parallelogram whose angles are oblique angles; a **rhombus** is a rhomboid whose sides are all equal.

A **trapezoid** is a quadrilateral which has two sides parallel.

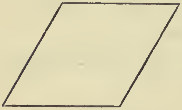
A **trapezium** is a quadrilateral which has no two sides parallel.

RECTANGLE.

SQUARE.



RHOMBOID.



RHOMBUS.

TRAPEZOID.

TRAPEZIUM.

The side upon which a parallelogram is supposed to stand and the opposite are called the lower and upper **bases.** The parallel sides of a trapezoid are called the **bases.** The perpendicular distance between the bases of a parallelogram or trapezoid is the **altitude.**

To find the area of any parallelogram, *multiply the base by the altitude.*

To find the area of a trapezoid, *multiply half the sum of the parallel sides by the altitude.*

To find the area of a trapezium, *multiply the diagonal by half the sum of the perpendiculars to it from the vertices of opposite angles.*

The area of any polygon may be found by dividing it into triangles and obtaining the sum of their areas.

EXAMPLES.

1. Find the number of square yards in a parallelogram whose base is 25 ft. and altitude $22\frac{1}{2}$ ft.

2. Find the number of hektars in a parallelogram whose base is 640^{Dm} and altitude 180^{Dm}.

3. Find the area of a rhomboid whose altitude is 132^{m} and base 154.4^{m}.

4. Find the area of a rhombus, of which one of the equal sides is 358 ft., and the perpendicular distance between it and the opposite side is 194 ft.

5. The parallel sides of a trapezoid are 30^m and 25.2^m, and the altitude is 18.2^m; find the area.

6. Two sides of a field, which are parallel, are respectively 262 yd. and 486 yd., and the perpendicular distance between them is 440 yd. How many acres does it contain?

7. Find the area of a trapezium whose diagonal is 21 ft. and the perpendiculars to this diagonal are 9 ft. and 8 ft.

8. What is the area of a trapezium, the length of a diagonal of which is 25 ft., and of the perpendiculars from the opposite vertices to the diagonal 5 ft. and $17\frac{1}{2}$ ft.?

Circles.

171. For definitions, see § 60.

The ratio of the circumference to the diameter is the same for all circles, and it is customary to represent this ratio by the Greek letter π (*pi*).

The numerical value of π cannot be obtained exactly, but the value $\pi = 3.1416$ is correct to four decimal places.

Note. $\pi = 3\frac{1}{7}$ is sufficiently accurate for many purposes, and it is used to a great extent. In this book $\pi = 3.1416$ is the value used.

To find the circumference of a circle when the diameter is given, *multiply the diameter by* 3.1416.

To find the diameter of a circle when the circumference is given, *divide the circumference by* 3.1416.

To find the area of a circle when the radius is given, *multiply the square of the radius by* 3.1416.

To find the radius of a circle when the area is given, *divide the area by* 3.1416, *and extract the square root of the quotient.*

Let A represent the area of a circle, C the circumference, D the diameter, and R the radius; then

$$C = \pi \times D \text{ or } 2 \times \pi \times R.$$

$$D = \frac{C}{\pi}.$$

$$R = \frac{C}{2 \times \pi}.$$

$$A = \pi \times R^2 \text{ or } \tfrac{1}{4} \times \pi \times D^2.$$

$$R = \sqrt{\frac{A}{\pi}}.$$

NOTE. When a circle is circumscribed about a square, the diagonal of the square is the diameter of the circle. When a circle is inscribed in a square, the diameter of the circle is equal to a side of the square.

EXAMPLES.

1. Find the circumference of a circle whose diameter is 22 ft.

2. Find the circumference of a circle whose diameter is 50^{m}.

3. Find the diameter of a circle whose circumference is 284^{cm}.

4. Find the radius of a circle whose circumference is 82 ft. 4 in.

5. Find the diameter of a tree whose circumference is 25 ft. 10 in.

6. Find the number of acres in a circular field whose radius is 32 rd.

7. Find the number of hektars in a circular field whose radius is 325^{m}.

8. Find the area of a circle whose diameter is 2 ft. 7 in.

9. Find the radius of a circle whose area is $163\frac{1}{4}$ sq. rd.

10. Find the radius of a circle whose area is $1560^{\text{sq cm}}$.

11. Find the diameter of a circle whose area is 38 sq. ft.

12. Find the circumference of a circle whose area is 25^{a}.

13. Find the number of acres in a circular park whose circumference is $3\frac{1}{2}$ miles.

14. Find the diameter of a wheel which turns 23 times in going 103.5^{m}.

15. How many times will a wheel whose radius is 0.762^{m} revolve in running 1.6043^{Km}?

16. How many turns per minute does a pulley 1.3^{m} in diameter make when the belt travels 50^{Km} per hour?

17. A horse, tied to a stake, can graze to the distance of 35 ft. from the stake; find the number of square yards of surface on which he can graze.

18. In a board 6 ft. long and 16 in. wide are two round holes, one of which is 10 in. across, and the other 12 in. across. Find the area remaining.

19. Find the width of the ring between two concentric circles whose circumferences are respectively 225 ft. and 300 ft.

20. What is the area of a circular ring formed by two concentric circles whose diameters are respectively 6 ft. 4 in. and 4 ft. 6 in.?

21. Find the area of a circle inscribed in a square containing 225 sq. ft.

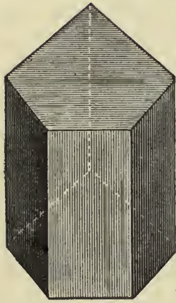
22. Find the area of a circle circumscribed about a square containing $144^{\text{sq m}}$.

23. Find the side of the largest square that can be laid out in a circular enclosure whose diameter is 10 rd.

24. Find the side of a square inscribed in a circle whose area is 78.54$^{\text{sq m}}$.

Prisms and Cylinders.

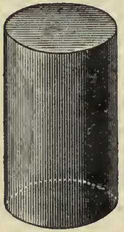
172. A **prism** is a solid whose ends, or **bases**, are equal and parallel polygons, and whose sides are parallelograms. A prism is *triangular*, *quadrangular*, *pentagonal*, etc., according as its ends are triangles, quadrilaterals, pentagons, etc. A **right prism** is a prism whose sides are perpendicular to the bases.



Pentagonal Prism.

A **cylinder** is a solid whose ends, or **bases**, are circles, and whose lateral surface is a uniformly curved surface. The **axis** of a cylinder is a straight line joining the centres of the two bases. A **right cylinder** is a cylinder whose axis is perpendicular to the bases.

The perpendicular distance between the bases of a prism or cylinder is called the **altitude**.



Cylinder.

To find the lateral surface of a right prism or right cylinder, *multiply the perimeter of a base by the altitude.*

To find the volume of a prism or cylinder, *multiply the area of a base by the altitude.*

Let S represent the lateral surface of a right prism or right cylinder, V the volume of any prism or cylinder, B the area of a base, P the perimeter of a base, and H the altitude; then

$$S = P \times H.$$
$$V = B \times H.$$

Let R represent the radius of a base of a cylinder, and D the diameter; then the following formulæ are true for cylinders:

$$S = 2 \times \pi \times R \times H \text{ or } \pi \times D \times H.$$
$$V = \pi \times R^2 \times H \text{ or } \tfrac{1}{4} \times \pi \times D^2 \times H.$$

EXAMPLES.

1. How many square feet are there in the lateral surface of a right prism whose altitude is 3 ft., and whose base is a regular hexagon, each side of which is 6 in. long?

2. The radius of the base of a cylinder is 8 in., and the altitude is $2\frac{1}{2}$ ft.; how many square feet are there in the lateral surface? in the whole surface?

3. The sides of the base of a triangular prism are respectively 12, 15, and 24 feet, and the altitude is 20 ft.; find the cubic contents.

4. Find the volume of a prism whose base contains $7\frac{1}{5}$ sq. ft., and the square of whose height equals five times the number of square feet in the base.

5. Find the capacity in gallons of a cylindrical cistern, measuring 16 ft. across and 15 ft. deep.

6. Find the number of liters contained in a cup, measuring 20^{cm} across and 31.831^{cm} deep.

7. Find the number of cubic feet in a log $28\frac{1}{2}$ ft. long and 6 ft. 2 in. round.

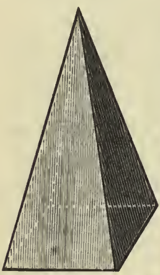
8. How many kiloliters must be drawn from a cylindrical tank, the diameter of the base being 10^{m}, in order to lower the surface 7^{dm}?

9. A cylindrical vessel 1^m high is made of sheet iron 2^{cm} thick, and holds 100^l. What is its outer diameter?

10. The diameter of a cylindrical vessel filled with water is 6 in. An immersed stone displaces $1\frac{1}{2}$ in. in depth of the water. How many cubic inches are there in the stone?

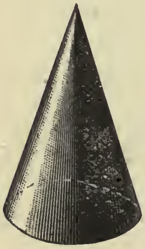
Pyramids and Cones.

173. A **pyramid** is a solid whose base is a polygon, and whose sides are triangles meeting in a common point, called the **vertex**. A pyramid is *triangular, quadrangular, pentagonal,* etc., according as its base is a triangle, quadrilateral, pentagon, etc. A **right pyramid** is a pyramid whose base is a regular polygon, and in which the perpendicular from the vertex passes through the centre of the base.



Quadrangular Pyramid.

A **cone** is a solid whose base is a circle, and whose lateral surface tapers uniformly to a point, called the **vertex**. The **axis** of a cone is a straight line drawn from the vertex to the centre of the base. A **right cone** is a cone whose axis is perpendicular to the base.



Cone.

The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

The **slant height** of a right pyramid or right cone is the shortest distance from the vertex to the perimeter of the base.

A **frustum** of a pyramid or cone is the part of the pyramid or cone that remains after cutting off the upper part by a plane parallel to the base.

The **altitude** of a frustum is the perpendicular distance

between the two bases, and the **slant height** is the shortest distance between the perimeters of the bases.

To find the lateral surface of a right pyramid or right cone, *multiply the perimeter of the base by one half of the slant height.*

To find the lateral surface of the frustum of a right pyramid or right cone, *multiply one half the sum of the perimeters of the bases by the slant height.*

To find the volume of a pyramid or cone, *multiply the area of the base by one third of the altitude.*

To find the volume of a frustum of a pyramid or cone, *multiply the sum of the areas of the two bases and the square root of their product by one third of the altitude.*

Let S represent the lateral surface of a right pyramid or right cone, V the volume of any pyramid or cone, B the area of the base, P the perimeter of the base, H the altitude, and L the slant height; then

$$S = \tfrac{1}{2} \times P \times L.$$
$$V = \tfrac{1}{3} \times B \times H.$$

Let R represent the radius of the base of a cone, and D the diameter; then the following formulæ are true for cones:

$$S = \pi \times R \times L \text{ or } \tfrac{1}{2} \times \pi \times D \times L.$$
$$V = \tfrac{1}{3} \times \pi \times R^2 \times H \text{ or } \tfrac{1}{12} \times \pi \times D^2 \times H.$$

The formulæ for a frustum, representing the areas of the bases by B and B', and the perimeters of the bases by P and P', are as follows:

$$S = \tfrac{1}{2} \times (P + P') \times L.$$
$$V = \tfrac{1}{3} \times (B + B' + \sqrt{B \times B'}) \times H.$$

Representing the radii of the bases of a frustum of a cone by R and R', and the diameters by D and D', the formulæ for a frustum of a cone are

$$S = \pi \times (R + R') \times L \text{ or } \tfrac{1}{2} \times \pi \times (D + D') \times L.$$

$$V = \tfrac{1}{3} \times \pi \times (R^2 + R'^2 + R \times R') \times H$$

$$\text{or } \tfrac{1}{12} \times \pi \times (D^2 + D'^2 + D \times D') \times H.$$

EXAMPLES.

1. How many square feet are there in the lateral surface of a right pyramid whose slant height is 6 ft., and whose base is a regular octagon, each side of which is 4 ft. long?

2. The radius of the base of a right cone is 16 in., and the slant height is 4 ft.; how many square feet are there in the lateral surface? in the whole surface?

3. The slant height of a frustum of a right pyramid is 5^m, and the perimeters of the two bases are 12^m and 8^m respectively; find the lateral area of the frustum.

4. The slant height of a frustum of a right cone is 8 ft., and the radii of the bases are 8 ft. and 5 ft. respectively; how many square feet are there in the lateral surface? in the whole surface?

5. The altitude of a pyramid is 8^m, and its base is a rectangle 3^m by 2^m; find the volume.

6. The altitude of a cone is 18 ft., and the radius of its base is 6 ft.; find the volume.

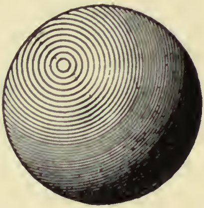
7. The base of a right triangular pyramid is an equilateral triangle, each side of which is 6 ft., and the altitude is 9 ft.; find the cubic contents.

8. Find the volume of a pyramid whose base is 5^m square, and whose height equals the diagonal of the base.

9. Find the capacity in liters of a pail 25^{cm} deep, measuring 28^{cm} across the top and 18^{cm} across the bottom.

SPHERES.

174. A **sphere** or **globe** is a solid bounded by a curved surface, every point of which is equally distant from a point within called the **centre**. A straight line passing through the centre and having its extremities in the surface is called a **diameter**; a straight line drawn from the centre to the surface is a **radius**. A section of a sphere made by a plane passing through the centre is called a **great circle**, and the **circumference** of a sphere is the same as a circumference of a great circle.



SPHERE.

To find the surface of a sphere, *multiply the circumference by the diameter.*

To find the volume of a sphere, *multiply the surface by one third of the radius.*

Let S represent the surface of a sphere, V the volume, R the radius, and D the diameter; then

$$S = 4 \times \pi \times R^2 \text{ or } \pi \times D^2.$$

$$V = \tfrac{4}{3} \times \pi \times R^3 \text{ or } \tfrac{1}{6} \times \pi \times D^3.$$

EXAMPLES.

1. Find the number of square feet in the surface of a sphere whose radius is 8 ft.

2. Find the number of cubic feet in the volume of a sphere whose radius is 6 ft.

3. How many cubic centimeters are there in a cannon ball whose diameter is 18^{cm}?

4. How many square inches of leather will cover a ball 8 in. in circumference?

5. A ball contains 2144.6656 cu. in.; what is the diameter?

6. The earth's surface contains about $509294630^{\text{sq Km}}$; find the radius.

7. A hemispherical vessel measures $2\frac{1}{2}$ ft. across the top; how many gallons does it hold?

8. A spherical shell of copper has an outer radius of 2^{m} and is 5^{dm} thick. What is the weight of this shell in kilograms when it is filled with mercury, the specific gravity of the copper being 8.8 and mercury 13.6?

Similar Surfaces and Solids.

175. Surfaces or solids which have the same form are said to be **similar**.

Like dimensions of similar surfaces or similar solids are proportional.

The areas of similar surfaces are to each other as the squares of their corresponding dimensions.

The volumes of similar solids are to each other as the cubes of their corresponding dimensions.

I. A triangle whose base is 12 ft. has an area of 54 sq. ft.; find the base of a similar triangle whose area is 96 sq. ft.

$$54 : 96 :: 12^2 : x^2.$$

$$x^2 = \frac{\overset{16}{\cancel{96}} \times \overset{4}{\cancel{12}} \times \overset{4}{\cancel{12}}}{\underset{\underset{\cancel{3}}{\cancel{9}}}{\cancel{54}}} = 256.$$

Ans. 16 ft.

Let x represent the base of the second triangle. Since the areas are to each other as the squares of their corresponding dimensions, $54 : 96 :: 12^2 : x^2$. From this proportion we find that $x^2 = 256$. Hence the base of the triangle is the square root of 256, or 16 ft.

II. A cylinder which is 9 ft. high contains 504 cu. ft.; find the volume of a similar cylinder 6 ft. high.

$$504 : x :: 9^3 : 6^3.$$

$$x = \frac{\overset{\overset{56}{\cancel{168}}}{\cancel{504}} \times \overset{2}{\cancel{6}} \times \overset{2}{\cancel{6}} \times \overset{2}{\cancel{6}}}{\underset{\cancel{3}}{\cancel{9}} \times \underset{\cancel{3}}{\cancel{9}} \times \underset{3}{\cancel{9}}} = \frac{448}{3} = 149\tfrac{1}{3} \text{ cu. ft.}$$

Let x represent the volume of the second cylinder. Since the volumes are to each other as the cubes of their corresponding dimensions,

$$504 : x :: 9^3 : 6^3.$$

From this proportion we find that $x = 149\frac{1}{3}$.

EXAMPLES.

1. The bases of two similar triangles are 5 ft. and 8 ft. respectively, and the altitude of the former is 9 ft.; find the altitude of the latter.

2. The hypotenuse of a right triangle is 25^m; find the hypotenuse of a similar triangle which contains twice the area.

3. The area of a trapezoid is 108 sq. ft., and its altitude is 6 ft.; find the altitude of a similar trapezoid whose area is 192 sq. ft.

4. How many circles, each 4 in. in diameter, will equal in area a circle whose diameter is 2 ft.?

5. Two farms of exactly similar form contain respectively 16 and 25 acres. One side of the former is 60 rd. in length; find the corresponding side of the latter.

6. If a cistern can be filled in 30 min. by a pipe 1 in. in diameter, in what time can it be filled by a pipe 3 in. in diameter?

7. If a pyramid 6 ft. high contains 45 cu. ft., what is the height of a similar pyramid that contains 100 cu. ft.?

8. How many spheres, each 6 in. in diameter, will equal in volume a sphere whose diameter is 2 ft.?

9. If a man digs a small square cellar, measuring 6 ft. each way, in one day, how long would it take him to dig a similar one measuring 10 ft. each way?

10. If a stack of hay 5 ft. high weighs 100 lb., find the weight of a similar stack 24 ft. high.

11. If a rope 1 in. in diameter weighs $2\frac{1}{2}$ lb., what is the diameter of a rope of the same length which weighs 50 lb.?

12. How far from the base must a cone whose altitude is 8 ft. be cut off so that the frustum shall be equivalent to one half of the cone?

MISCELLANEOUS EXAMPLES.

1. Divide 3380321 by MDCCXCIX, and express the quotient by the Roman system of notation.

2. Find, by casting out the nines, whether the following is correct: $349751 \times 28637 = 10015819397$.

3. Multiply 4.32 by 0.00012.

4. Divide 0.002268 by 10.8.

5. Divide the product of 12, 20, and 30 by the product of 15, 24, and 18, by cancellation.

6. Find the factors and the greatest common divisor of 1498, 1582, and 2331.

7. Arrange in order of magnitude $\frac{64}{45}$, $\frac{38}{27}$, and $\frac{17}{10}$.

8. Reduce $\frac{38}{247}$, $\frac{77}{119}$, and $\frac{120}{442}$ to their least common denominator.

9. Divide $\frac{2}{3}$ of 47 by $\frac{1}{19}$ of 51.

10. Find the value of $\frac{7}{9} - \frac{1}{6} + 4\frac{3}{4} + \frac{29}{18} + \frac{7}{12}$; reduce the result to its lowest terms, and also to a decimal form.

11. At $1.75 a rod, what will it cost to fence a piece of ground 63.5 rd. long and 27.75 rd. wide?

12. From a piece of cloth containing $84\frac{3}{8}$ yd. there were sold $4\frac{3}{7}$ yd., $26\frac{1}{2}$ yd., and $\frac{1}{2}$ of $7\frac{3}{4}$ yd.; how much remains?

13. Name all the prime numbers in the series of numbers between 1 and 30 inclusive; resolve all the composite numbers into their prime factors; and name all the perfect squares, cubes, and other powers in the same series.

14. How much will be paid for 3760 lb. of coal at \$15 a ton?

15. Find the product of 157.757 and 15.3254 to two places of decimals.

16. Divide 1728 by 0.00144, and multiply the result by 0.000012.

17. Divide \$125 among 4 boys and 3 girls, and give each boy $\frac{2}{3}$ as much as each girl.

18. Bought 360 gallons of wine at \$2.60 a gallon; paid for carriage \$17.20, and for duties \$86.50. If $\frac{3}{20}$ of it be lost by leakage, at what price must the remainder be sold to gain \$50 on the whole transaction?

19. Find the product of three, three hundredths, thirty-three thousandths, three thousand millionths, and two twenty-fifths.

20. Divide ten thousand six hundred twenty-five billionths by seventeen thousandths, and extract the square root of the quotient.

21. Find the value of the following fraction to three decimal places: $\frac{1.0045 \times 0.0875}{0.0016}$.

22. Find the sum of five, five tenths, thirty-seven thousandths, one thousand millionths, XIX, MDCCCLXXXI, and $0.\dot{1}\dot{8}$.

23. Reduce $\dfrac{\sqrt{.36} + \dfrac{1.92}{1.28}}{2.1} \div 3$ to its simplest decimal form.

24. Reduce 3.36 inches to a decimal fraction of a rod.

25. Reduce a pressure of 22.5 lb. Avoirdupois per square foot to ounces per square inch.

26. If either 5 oxen or 7 horses will eat up the grass of a field in 87 days, in what time will 2 oxen and 3 horses eat up the same?

27. How many liters of water may be contained in a reservoir 10^{m} long, 6^{m} wide, and 4^{m} high? What will be the weight in kilograms?

28. A bin is $2\frac{2}{9}^{m}$ high, and contains $16\frac{1}{5}^{Kl}$. The base of the bin is a square; how many centimeters are there in one of its sides?

29. If 4 men can mow 15 acres in 5 days of 14 hours, in how many days of 13 hours can 7 men mow $19\frac{1}{2}$ acres?

30. I buy 300 bu. of grain consisting of wheat, rye, and oats, in the proportion of 3, 4, and 5. How many bushels of each do I buy?

31. The sum of two numbers is 100, and $\frac{1}{2}$ of one of them is $\frac{3}{4}$ of the other; find the numbers.

32. How much water must be mixed with 31 gal. of another liquid which cost $45.25, that the mixture may be sold at $1.25 per gallon, and 25% be gained?

33. What is the interest on $647.65 for 2 yr. 5 mo. 10 da. at 5%?

34. What sum will produce $12.50 interest in 20 days at 4%?

35. Find the selling price of goods by which there is a loss of 2% and an actual loss of $54.50.

36. If 12 barrels of corn will pay for 10 cords of wood, and 48 cords of wood will pay for 8 tons of hay, and 5 tons of hay will pay for 16 kegs of nails, how many barrels of corn will pay for 12 kegs of nails?

37. Find the reciprocal of 155 carried to five decimal places.

38. Convert into a decimal $\dfrac{\frac{1}{2} \div 3\frac{1}{4}}{0.075} \times 0.00025$.

39. What are the prime factors of 1716? How many integral divisors has this number, and what are they? What is the smallest integer by which this number can be multiplied, so that the product shall be a square?

40. Reduce to its simplest form the expression $\dfrac{3}{7}$ of $\dfrac{4\frac{5}{9}}{12\frac{1}{3}}$ of $\dfrac{3\frac{4}{11}}{11\frac{5}{7}} \div 1\frac{1}{11}$.

41. Reduce $5\frac{3}{8}$ and $10\frac{4}{5}$ to the decimal form, and divide the first by the second.

42. Find the greatest common divisor of $26\frac{1}{4}$, $28\frac{7}{8}$, and $29\frac{1}{6}$.

43. Find the least common multiple for the numbers $\frac{3}{4}$, 2.1, 5.25, and $\frac{7}{8}$.

44. Change 0.013 to an equivalent fraction whose denominator is 135.

45. $\dfrac{7\frac{3}{4}}{9\frac{7}{8}}$ of $\frac{5}{6}$ of $\frac{8}{9}$ ft. equals what decimal of a rod?

46. A block of stone (sp. gr. 2.5) is 1^{m} long, 8^{dm} wide, and 45^{cm} thick. How many kilograms does it weigh?

47. Find the number of liters in a vat 2^{m} by 75^{cm} by 50^{cm}. Also find the weight in kilograms of the sulphuric acid (sp. gr. 1.84) required to fill it.

48. If a meter is 39.37 inches, how many feet are there in a dekameter? How many square centimeters in a square kilometer?

49. If 4 masons build 27 yd. of wall in 5 days working 9 hr. a day, in how many days will 32 masons build 81 yd. of a similar wall if they work 10 hr. a day?

50. Separate $772\frac{2}{3}$ into three numbers, which shall be in the same proportion as $2\frac{1}{2}$, $\frac{7}{10}$, and $\frac{6}{10}$.

51. Compute the square of the sum of the cubes of the first twelve prime numbers, and check all the work by casting out the nines.

52. Multiply 34.056 by 0.065043, obtaining the product to four decimal places.

53. Divide 0.0144 by 4800; multiply the quotient by 6.004, and extract the square root of the product.

54. Simplify $\frac{5}{7}$ of $\frac{3\frac{1}{4} \div \frac{7}{8}}{1\frac{1}{2} \times 3\frac{1}{3}}$, and divide the result by 0.0018.

55. Find the least common multiple of 75, 105, 150, and 175.

56. Reduce to a common denominator and add $\frac{2}{3} \times \frac{4}{5} \times \frac{5}{8}$, $\frac{7}{15}$, $\frac{3}{4}$, and $\frac{9}{10}$.

57. If $\frac{4}{5}$ of a cord of wood cost $\$3.33\frac{1}{3}$, what would $\frac{7}{8}$ of a cord cost?

58. 75 miles equals how many kilometers? 75 pounds Avoirdupois equals how many kilograms? 75 quarts equals how many liters?

59. A man's height is 174^{cm}. What is his height in feet and inches?

60. Find the value of 17^{l} of sulphuric acid (sp. gr. 1.84) at 5 cents a kilogram.

61. By selling a horse for \$64.75, I lost $7\frac{1}{2}\%$; what per cent would I have gained by selling him for \$73.50?

62. Sold steel at \$25.44 a ton with a profit of 6% and a total profit of \$103.32. What quantity was sold?

63. If I buy stocks, par value \$187.50, at 15% below par and sell them at $19\frac{1}{2}\%$ above par, what is the gain per cent on my investment?

64. If I buy coal at \$4.12 per ton on six months' credit, for what must I sell it immediately to gain 10%?

65. Find the amount of \$342.42 from Feb. 5th, 1879 to Mar. 15th, 1881, with interest at 7%, and reduce it to pounds sterling.

66. Find the interest, discount, and bank discount on \$25 for 60 days at 7%.

67. Find the interest, discount, and bank discount on \$17.50, due in 30 days, at $4\frac{1}{2}$%.

68. A merchant bought flour for \$1000 cash and sold the same immediately for \$1200 on 6 mo. credit, for which he received a note. If he should get the note discounted at a bank at 5%, what would be the gain on the flour?

69. A and B can do a piece of work in 4 hours, A and C in $3\frac{3}{5}$ hours, and B and C in $5\frac{1}{7}$ hours. In what time can A do it alone?

70. Divide 52 into such parts that $\frac{1}{5}$ of one part shall equal $\frac{2}{3}$ of the other.

71. A cubical cistern holds 1331^{Kg} of water; what is the length of an inner edge?

72. Arrange in order of magnitude $\frac{22}{25}$, $\frac{49}{55}$, and 0.89.

73. Show that the square root of 0.3 lies between $\frac{11}{20}$ and $\frac{9}{17}$.

74. I have a rectangular lot of land, 64 rd. long and 36 rd. wide, and a square lot of the same area; how many more feet of fencing will be needed for the former lot than for the latter?

75. If 144 pounds Avoirdupois be equivalent to 175 pounds Troy, what is the ratio of the pennyweight Troy to the dram Avoirdupois?

76. Reduce to equivalent fractions having a common denominator $\frac{2}{3}$ of $\frac{2}{4}$, $2\frac{3}{8}$, $5\frac{2}{5}$, and $\frac{2}{5}$ of $\frac{1}{3}$ of $3\frac{1}{2}$.

77. The number 209.069673692836 is composed of three factors, of which two are 20083.6 and 0.260075; find the third factor.

78. Simplify $\dfrac{1\frac{1}{3} \times 3\frac{1}{7} \text{ of } 2\frac{6}{11}}{\frac{8}{9} + 1\frac{1}{8} - 1\frac{2}{3}}$.

79. Simplify $\dfrac{(3.2 + 0.004 - 1.111) \times 0.25}{(4 \div 0.2) - 17.907}$.

80. Simplify $\dfrac{1}{2 + \dfrac{3}{4 + \dfrac{5}{6}}}$.

81. Write in Arabic numerals the value of the expression $[\text{MDCCCLXXXIII} \div 16.6] \times [(2.5 - 1.25) \div 0.\dot{0}\dot{3}]$.

82. Reduce $\frac{345}{800}$ to its lowest terms; reduce the result to a decimal, and extract the square root to three figures.

83. What is the length in meters and decimeters of a side of a square which contains 0.1335^{a}?

84. Find the length in dekameters of the side of a square, the area of which equals the area of a rectangle which is $1^{\text{Km}}\ 8^{\text{m}}$ long and $4\frac{12}{25}^{\text{Hm}}$ wide.

85. A square field contains 1016064 sq. ft. What is the length of a side expressed in meters?

86. Find the side to millimeters of a cubical box that contains $1\frac{1}{2}^{\text{Hl}}$.

87. The volume of a sphere is 0.056 cu. yd. What is the length in inches of the side of a cube containing the same volume?

88. Find the edge of a cubical can which will hold 27.57^{Kg} of sulphuric acid, whose specific gravity is 1.8.

89. Find a fourth proportional to 0.37, 8.9, and 4.3, and extract the cube root of it to two decimal places.

90. Find the fourth term in $\sqrt[3]{4.913} : 0.0016 :: 48000 :$.

91. If 60 cannon firing 5 rounds in 8 min. kill 350 men in 75 min., how many cannon firing 7 rounds in 9 min. will kill 980 men in 25 min.?

92. A man travelled 2 days at the rate of 15 miles per day, 4 days at the rate of 20 miles per day, and 5 days at the rate of 30 miles per day; what was his average rate of travel per day?

93. A farmer divides among his 3 sons 246 A. 1 R. 32 P., sharing it among them as the numbers 3, 4, and 5; what were the shares?

94. How many kilograms are there in a cubic foot of water?

95. A rectangular box is 4^{m} long, 30^{dm} wide, and 20^{cm} deep.

(i) **Find its capacity in liters.**
(ii) **What weight of water will it contain?**
(iii) **What weight of mercury (sp. gr. 13.6) will it contain?**

96. An empty bottle weighs 380^{g}; when filled with water it weighs 0.985^{Kg}. How many liters does the bottle hold?

97. Find the present worth of a note for \$1320, due in 3 yr. 4 mo., money being worth 6%. Find also what could be obtained for the same note at bank discount.

98. Find the interest, discount, and bank discount on \$65.33 for 90 days at $4\frac{1}{2}\%$.

99. What is the difference between the true and bank discount of \$250, due 10 mo. hence, at 7%?

100. How long must a note of \$243 at $3\frac{1}{2}\%$ run that its interest may equal the interest on a note of \$125 for 7 mo. at 5%?

101. How many times does the least common multiple of 5, 25, 40, and 75 contain the square of their greatest common divisor?

102. Reduce $\frac{3}{910}$ and $\frac{7}{2470}$ to their least common denominator; add the results, and express the sum decimally to four places.

103. Divide $(\frac{7}{80} - \frac{8}{125})$ by ($\frac{3}{11}$ of 0.00616), carrying the quotient to five places of decimals.

104. Simplify $\dfrac{1 + \frac{8}{3} + (\frac{4}{3})^2}{(\frac{4}{3})^2 - 1}$.

105. $\dfrac{4\frac{1}{4} - 3\frac{3}{8}}{4\frac{1}{4} + 3\frac{3}{8}} + (3 - 2\frac{1}{3}) \div (4 - 3\frac{1}{4})$ equals what? Extract the square root of the result to two decimal places.

106. Find the value of $\left(\dfrac{4\frac{6}{7}}{8\frac{9}{10}} \text{ of } \dfrac{3}{7\frac{1}{8}}\right)$ divided by $\frac{6}{11}$, and extract the square root of the quotient to two decimal places.

107. Simplify the expression $\sqrt[3]{\dfrac{39\frac{1}{16}}{2\frac{1}{2}} \times \dfrac{27}{64}}$.

108. How many rods of fence will it take to enclose a 20 acre lot in the form of a square?

109. If a man can walk 16 rods in $\frac{5}{8}$ of a minute, in what time can he walk 0.00164 of a mile?

110. The length of a rectangular field containing 30 acres is 3 times its width. Find the length of the field in feet.

111. A cubic inch of gold is hammered out until it covers 6 acres; how many leaves of gold of this thickness would it take to make one inch?

112. A cube contains 79507 cu. in. How many square inches does its surface contain?

113. How many rods of fence will be required to enclose 640 acres of land in a square form?

114. A man lost $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{2}{5}$ of his money, and then had \$2600 left; what sum had he originally, and how much per cent had he lost?

115. Reduce $\frac{1}{2}$ of 6% of $1.05 \div \frac{2}{25}$ of $\frac{7}{10}$ to the simplest form.

116. I earned \$10 by collecting bills on which a discount of 10% was allowed for cash. My commission was 5%; how much did I collect?

117. If $4\frac{1}{2}$% Government bonds sell at 116, what sum of money invested in them will yield an interest of \$1 per day?

118. Bought a bill of goods on 6 months' credit for \$500. What would be the gain per cent on my bargain if I sold the same at once for \$525 cash, interest being reckoned at 6% per annum?

119. Find the difference between the true and bank discounts on a note for \$1000, due 3 mo. hence, money being worth 6%.

120. Find the interest, discount, and bank discount on \$327.19 for 90 days at $7\frac{1}{2}$%.

121. Bought \$1500 worth of goods, half on 6 months' and half on 9 months' credit. What sum at 7% interest, paid down, would discharge the bill?

122. Find the principal that will amount to \$962 in 4 yr. 6 mo. at $4\frac{1}{2}$%.

123. Find the compound interest on \$300 for 2 yr. at 4%, interest being compounded semi-annually.

124. Find the annual interest of \$200 for 3 yr. 1 mo. at 6%.

125. Find the greatest common divisor of 113.355 and 3.141592.

126. Divide $\frac{0.6}{15}+\frac{7}{9}$ by $\frac{3.71}{630}$.

127. Simplify $\frac{0.005}{25}+\frac{0.6}{0.002}$, expressing the result in decimal form.

128. $[5\frac{2}{3}\times\frac{17}{12}\times\frac{3}{4}+1.0176]\div[3\frac{3}{4}+300.003]$ equals what?

129. Reduce to a decimal form $\frac{3\frac{1}{5}}{0.025}$, and from it subtract 0.01 of $\frac{3}{4}$.

130. Reduce to a vulgar fraction the decimal $0.0001\dot{2}3\dot{4}$. Test your answer by reversing the process.

131. Find the square root, to three places of decimals, of

$$\frac{10\frac{1}{2}+\frac{15.75}{1.5}}{\frac{1}{3}\text{ of }2\frac{1}{6}}.$$

132. Find the sum of $3\frac{1}{4}$, $6\frac{5}{7}$, $8\frac{9}{15}$, and $65\frac{8}{9}$, reduce the fractional part to a decimal, and extract the cube root of the result.

133. A and B, 44 miles apart, travel towards each other. A travels $\frac{3}{11}$ of the whole distance, while B travels $\frac{4}{7}$ of the remainder. How far are they then apart?

134. Two engines, 40 miles apart, approach each other at the rate of 25 and 35 miles an hour. Find the time and place of their meeting.

135. A river 10^{m} deep and $\frac{1}{2}^{Km}$ wide flows 2^{Km} an hour; find the number of kiloliters of water that falls into the sea in a minute; also its weight in kilograms.

136. If a sheet of paper weighs 8^{Dg} per square meter, find the weight in grams of a piece $1\frac{1}{2}^{m}$ long and 25^{cm} wide.

137. What is the difference in volume between two blocks of granite, one 1^{m} long, 6^{dm} wide, and 0.5^{m} thick, the other 300^{cm} long, 40^{cm} wide, and 0.2^{m} thick?

138. Find the weight in kilograms, and in pounds, of a rectangular block of marble (sp. gr. 2.83) 3.7^{m} long, 7^{dm} wide, and 30^{cm} thick.

139. If I buy macaroni at 30 cents a kilo, pay $12 a metric ton for transportation, and sell at 14 cents a pound, what per cent do I gain or lose?

140. A square field contains 0.08346 of an acre. Find the length of one side of the field in meters, the hektar being equal to 2.4711 acres.

141. The stere contains 1.308 cu. yd. How many meters are there in the side of a cube containing 0.056 cu. yd.?

142. If 35 men can build a wall 50 ft. long, 2 ft. thick, and 10 ft. high in 8 days, how long will it take 50 men to build a wall 250 ft. long, 3 ft. thick, and 7 ft. high? If the first wall costs $910, what will the second one cost?

143. How many men would be required to cultivate a field of $2\frac{5}{8}$ acres in $5\frac{1}{2}$ days of 10 hours each, if each man completed 77 square yards in 9 hours?

144. An estate is divided among three persons, A, B, and C, so that A has $\frac{5}{8}$ of the whole, and B has twice as much as C. It is found that B has 27 acres more than C. How large is the estate?

145. Copper weighs 550 pounds, and tin 462 pounds to the cubic foot. What will be the weight of a cubic foot of a mixture 6 parts copper to 5 parts tin?

146. A man bought 16 horses and 19 cows for $1865. He paid upon the average $\frac{7}{15}$ as much for a cow as he did for a horse. What was the average price per head he paid for the horses?

147. By selling a lot of land for $783 I lost 13%. What would it have brought if I had sold it at a loss of $8\frac{1}{3}$%?

148. At what rate per cent is the deduction made when 19 s. $10\frac{1}{2}$ d. is taken from an account of £39 15 s. in consideration of immediate payment?

149. At what per cent premium must a 4% perpetual bond be bought in order that it may pay only $3\frac{1}{2}\%$ on the investment?

150. Find the bank discount of a note for $25000 for 2 yr. 6 mo. at $3\frac{1}{2}\%$.

151. Find the principal that will amount to $724.92 in 2 yr. 3 mo. at $3\frac{1}{2}\%$.

152. Find the simple interest, the annual interest, and the compound interest of $1200 for 2 yr. 6 mo. 18 da. at 4%.

153. What is the interest, discount, and bank discount on $127.42 for 65 days at 5%?

154. Find the difference between the amount of $1000 for 3 yr. at 6% compounded yearly, and at 3% compounded half yearly.

155. Find the square root of five million five thousand and five tenths to two decimal places.

156. Compute the value of $3 + \sqrt{3} + \sqrt[3]{29}$ to three places of decimals.

157. Multiply the square root of 0.173056 by the cube root of $\frac{15625}{32768}$.

158. What is the difference between the square root and the cube root of 1771561?

159. On a map whose scale is $\frac{1}{10}$ of an inch to a mile, what would be the area covered by a tract of land containing 720 square miles?

160. A rectangular tank, with a square base, 3 ft. deep, contains 675 cu. ft. Find the length of a side of the base.

161. A man paints two sides of a wall 7 ft. high in 31 hr. 6 min. 40 sec. If he can paint 4 sq. yd. in an hour, how long is the wall?

162. A certain square field contains 38.75 acres. Compute the length of one side of the field in meters. (Given $1^{\text{sq m}} = 1550$ sq. in.)

163. The specific gravity of iron is 7.2; find the volume in cubic decimeters, and the weight in kilograms, of a block of iron whose dimensions are 5, 8, and 11 inches.

164. A railroad train makes a mile in 57 seconds. What is its rate per hour, and what per cent of the hour is occupied in its making a single mile?

165. If 5 horses eat as much as 6 oxen, and 12 oxen eat 12 tons of hay in 40 days, how much hay will 7 horses and 15 oxen eat in 65 days?

166. Express the value of $\dfrac{\frac{1}{2} - \left(\frac{1}{2\frac{2}{3}} \text{ of } \frac{3}{7\frac{1}{2}} \text{ of } \frac{5}{6}\right)}{32}$ exactly as a decimal.

167. Which is the larger, $\frac{29}{51}$ or $\sqrt{\frac{1}{3}}$?

168. Reduce $\dfrac{5\frac{1}{2} + \dfrac{7\frac{1}{4}}{0.5} - 0.725}{\dfrac{4 + 3.45}{2\frac{1}{2}}}$ to an equivalent decimal.

169. Simplify $1\frac{1}{2}$ of $2\frac{4}{5} + 6\frac{7}{8} \div 2\frac{3}{4} + \left(5\frac{1}{2} + \dfrac{0.24 + 0.5\dot{3}}{2.2 - 0.6\dot{4}}\right)$.

170. From the sum of $3\frac{3}{4}$ and $4\frac{5}{6}$ subtract $6\frac{6}{7}$, multiply the difference by $\frac{3}{5}$ of $\frac{27}{40}$ of 88, and find what fraction the product is of 999.

171. A cistern 6.84^{m} long and 2.36^{m} wide contains 34^{Hl} of wine. What is the depth of the liquid?

172. How many cubic meters are there in a cord?

173. How many liters of water are there in a full rectangular tank 12 ft. long, 6 ft. wide, and 4 ft. deep?

174. A tank which holds 100 gal. can be filled by one pipe in 25 min., and emptied by another pipe in 45 min.; if both are opened together, how long will it take to fill, and how much water will have been lost?

175. If 8 men can build a brick wall 125 ft. long, 2 ft. wide, and 4 ft. high, in 4 days, working 10 hr. each day, how many days will it take 12 men to build a wall 465 ft. long, 3 ft. wide, and 6 ft. high, working 8 hr. each day?

176. Two men undertake to do a piece of work for $6. One could do it alone in 5 days, and the other in 8 days. With the assistance of a boy, they finish it in $2\frac{2}{3}$ days. How should the money be divided?

177. A gallon contains 231 cu. in., and a bushel 2150.4 cu. in.; how will a liquid quart compare with a dry quart?

178. What is the present worth of $1000, due 6 mo. hence, money being worth 6%?

179. Find the interest, discount, and bank discount on $416.03 for 60 days at $7\frac{1}{2}$%.

180. What is the difference between the true discount and that taken by banks on $1500, due one year hence without grace? The rate of discount in both cases is 5%.

181. What is the difference between the simple and compound interest on $700 for 2 yr. 6 mo. at 7%, interest compounded annually?

182. A note for $500 at 60 days without interest is bought for $450. What is the profit if money is worth 1% a month?

183. How many rods of fence will it take to enclose a square field containing exactly one acre?

184. The area of a circle is 5 sq. rd. What is the length in inches of one side of a square which contains the same area?

185. Find the depth in meters of a cubical cistern which has a capacity of 30000^{l}. Give the result to three decimal places.

186. A is 156 miles ahead of B. A travels 30 and B 42 miles a day. In how many days will B overtake A?

187. If 4 men or 6 boys can do a piece of work in $27\frac{1}{2}$ days, in how many days will 5 men and 9 boys do it?

188. If 8 horses consume $3\frac{1}{2}$ tons of hay in 30 days, how long will $4\frac{9}{10}$ tons last 10 horses and 15 cows, each cow consuming $\frac{3}{4}$ as much as a horse?

189. A carriage, at the rate of $8\frac{1}{2}$ miles an hour, completes $\frac{2}{5}$ of a certain distance in $3\frac{1}{3}$ days; in how many days will it complete $\frac{4}{7}$ of the same distance, going at the rate of 10 miles an hour?

190. There are two casks, one containing 15 gal. of water, and the other 35 gal. of spirits; how many gallons must be transferred from each to the other in order that the mixtures in each may be of the same strength?

191. Find the value to three decimal places of $\sqrt{(0.146)^2+(0.063)^2}$.

192. Find the value of $\dfrac{3}{\sqrt{19}-4}$, correct to four places of decimals.

193. Simplify $\dfrac{(2.01+2.25\times 0.004)\div(1.0337-31.09\times 0.03)}{4.5\div 900}$.

194. Simplify $\sqrt[3]{\dfrac{\dfrac{1\frac{8}{9}}{11\frac{1}{3}}-2\frac{5}{6}\text{ of }1\frac{11}{34}+4\frac{1}{4}}{\frac{2}{7}+1\frac{8}{9}\div\frac{17}{18}-\frac{1}{21}}}$.

195. Find the sum of $\frac{0.5 \times 0.006}{\frac{9}{5} \times \frac{4}{5} \times (\frac{1}{4})^2}$ and $\frac{\frac{1}{5} \text{ of } \frac{15}{16} \text{ of } (\frac{2}{3})^3}{1.6 + 0.625}$.

196. Simplify the following expressions: $\sqrt{2\frac{1}{4}}$, $\sqrt[3]{\frac{128}{250}}$, $\sqrt[6]{5\frac{451}{729}}$, and $(2\frac{1}{5})^3$.

197. A and B together have \$136, and $\frac{2}{3}$ of A's money is equal to $\frac{3}{4}$ of B's. How much has each?

198. A certain piece of work can be done by 8 men or 16 boys in 10 days. In how many days can the work be done by 8 men and 16 boys?

199. If 8 horses in 30 days eat $3\frac{1}{2}$ tons of hay, how long will $4\frac{9}{10}$ tons last 10 horses, 15 cows, and 10 sheep, each cow eating $\frac{3}{4}$ as much as a horse, and each sheep $\frac{1}{3}$ as much as a cow?

200. A pail will hold $5\frac{1}{2}^{l}$. The area of its base is $330^{\text{sq cm}}$. Required its height in inches.

201. One meter equals 39.4 inches. How many cubic inches are there in one liter?

202. Leap year is omitted once in every century except in those centuries whose number is divisible by four. What is the average length of a year?

203. What is the value in pounds sterling of half an acre of land at $9\frac{1}{2}$ pence per square foot?

204. Separate 280 into two such numbers that $\frac{3}{7}$ of one is equal to the other.

205. A milkman bought 40 gal. of new milk at 16 cents a gallon and 60 gal. of skimmed milk at 8 cents a gallon, which he mixed with 12 gal. of water, and sold the whole at 24 cents a gallon. What was his profit?

206. What sum placed at simple interest for 3 yr. 10 mo. at 7% will amount to the same as \$1500 placed at compound interest for the same time at $7\frac{1}{2}$%?

207. I buy goods to the amount of $4978.70, payable in 4 mo. with interest at 5%, and give my note without interest. What must be the face of the note?

208. Compute the value of $\sqrt{3} - 1 + \sqrt{6}$ to four decimal places.

209. Extract the square root of 2.26 to three places of decimals. Show how you can derive from the square root of this number that of 0.0226.

210. A rectangle is 1.25^{Km} long and 3.5^{Dm} wide; find the side of the equivalent square in dekameters.

211. What is the length of a cubical bin which will contain 4500 cu. ft.?

212. Find as circulating decimals the square of $0.\dot{4}$ and the square root of $0.69\dot{4}$.

213. A rectangular block of stone, square at the base and 8 ft. high, contains 162 cu. ft. What is the length of one side of the base?

214. The weight of a cubical block of stone, 2 ft. on each edge, is 1352 lb. What is the weight of a cubical block whose edge is 4 ft.?

215. A cubical vessel contains 150 lb. of pure water. Find the length of an inner edge of the vessel in decimeters.

216. How many books, each $10\frac{1}{2}$ in. long, $4\frac{1}{2}$ in. wide, and $1\frac{3}{8}$ in. thick, can be packed in a box 5 ft. 3 in. long, 3 ft. wide, and 2 ft. 9 in. thick?

217. Supposing that the driving wheels of a locomotive are 16 ft. in circumference, what number of revolutions must they make per minute so that the locomotive may attain a speed of 60 mi. per hour?

218. Find what decimal part the square root of $\frac{99}{128}$ is of the square root of $5\frac{1}{2}$.

219. A sum of £250 17 s. 6 d. is transmitted through Paris to New York; find the value of the sum in United States money (£1 = 24.79 francs; 9.2 francs = $1.75).

220. What sum of money is the same part of £14 7 s. $9\frac{3}{4}$ d. that 4 oz. 7 pwt. 5 gr. is of 8 oz. 10 pwt. 15 gr.?

221. The wages of A and B together for 22 days amount to the same sum as the wages of A alone for $38\frac{4}{7}$ days; for how many days will this sum pay the wages of B alone?

222. If Greenwich time be 5 hr. 8 min. 12 sec. later than Washington, what is the difference in time between Washington and a point 87° 35′ west of Greenwich?

223. How much carpeting $\frac{3}{4}$ yd. wide will cover the top and sides of a box 3 ft. 6 in. long, 2 ft. 3 in. wide, and 9 in. high?

224. What would it cost to paper the walls of a room 30 ft. 8 in. long, 20 ft. 4 in. wide, and 11 ft. high, the paper being 2 ft. 3 in. wide, and costing 63 cts. per roll of 12 yd.?

225. Two bells commence tolling together, one at the rate of 5 times in 24 sec., the other at the rate of 4 times in 23 sec.; in what time will they again toll together?

226. A man bought 200^m of cloth in France at $16\frac{1}{4}$ francs per meter; he paid $12\frac{1}{2}$ cents a yard for duty and freight, and sold it in Boston at $4.62½ a yard. What was the gain?

227. A block in the form of a perfect cube contains 12516 cu. in. How many square yards of paper are required to cover it?

228. Sold a hundred bushels of wheat, which cost $150, at 50 cents a peck, taking in payment a 6 months' note which was discounted immediately at the bank at 6%. What was the profit?

229. A note for $1000, with interest at 7% payable annually, has run 3 years, but no interest has been paid. What is now the amount of the note at simple interest? at annual interest? at compound interest?

230. What is the compound interest of $1 for 143 yr., allowing it to double once in 11 yr. 11 mo.?

231. A grocer makes a mixture of which 21.5 lb. contain $\frac{1}{2}$ lb. of rye, 12 lb. of wheat, 5 lb. of oats, and 4 lb. of barley. How much of each ingredient will be contained in 100 lb. of the mixture?

232. If the pay of a man, a woman, and a boy be in the ratio 3, 2, 1; and 24 men, 20 women, and 16 boys receive £20 8 s. a week, what will 27 men, 40 women, and 15 boys receive in 365 days?

233. Find how many yards of carpet a yard wide must be bought to cover a floor 23 ft. 6 in. by 17 ft. 5 in., supposing that the strips run lengthwise, and that the figure of the carpet is 8 ft. long, and is laid to match. Find also how much of the carpet must be turned under or cut off at the ends and sides.

234. In the Centigrade and Fahrenheit thermometers the freezing points are 0° and 32° respectively, and the boiling points 100° and 212° respectively. When the Centigrade stands at 37°, what will the Fahrenheit read?

235. A train travels 82 mi. 7 fur. 26 rd. 4 yd. in 3 hr. 48 min. $51\frac{1}{2}$ sec.; what is the rate per hour?

236. A person by selling an article, which cost him $60 per 100 pounds, at $67\frac{1}{2}$ cents per pound, makes 5% more than he would by selling the whole for $$267.67\frac{1}{2}$; how many pounds were there?

237. The amount of a certain principal at a certain rate of interest for 6 mo. is $949.76, and for 1 yr. at the same rate is $1003.52. Required the rate per cent and principal.

238. Lead is 11.4 times, and zinc 7.2 times, as heavy as water. If 3 lb. of lead and 2 lb. of zinc be melted together, compare the weight of the alloy with that of water.

239. There is a rectangular lot of ground 64.8 rd. long and 36.05 rd. wide, and a square lot of the same area; which will require the more feet of fencing, and how much?

240. A cubic foot of iron weighs 450 lb.; what will be the weight of a rectangular closed box made of iron $\frac{1}{16}$ of an inch thick, the extreme dimensions of the box being 7 ft. 5 in., 8 ft. 3 in., and 4 ft. 3 in.?

241. In 100.93^{g} of chemically pure saltpetre there are 39.04^{g} of potassium, 14.01^{g} of nitrogen, and 47.88^{g} of oxygen; determine the per cent of each of these elements in the compound, and how many grams of each there are in a kilogram of the latter.

242. A commission merchant sells 28000 lb. of cotton at $12\frac{1}{2}$ cents per pound; after deducting $35.36 for freight and cartage, $10.50 for storage, and his commission, he remits $3252.89 as net proceeds of the sale. At what rate did he charge commission?

243. I have bought a farm for $6500; $2000 of this is to be paid down, $500 in one year, and the remainder in two years. If a note for the whole amount were preferred, when would it become due?

244. On the first of January, 1884, A, B, and C enter into a partnership. A and B each furnish $4000, and C $8000. At the end of a year B withdraws $1500, while 6 months later C adds $2000. At the end of 2 years they find their profits are $1580. How shall the profits be divided between them? What per cent do they realize on their capital?

245. If the diameter of the earth is 7926 mi., what height in inches on a globe 2 ft. in diameter will represent a mountain 15000 ft. in height?

246. The least common multiple of four numbers is 283500. Prove that if three of the numbers are 140, 42, and 60, the fourth must contain 225 as a factor.

247. A house costs \$5000, and rents for \$25 a month, with \$25 to pay annually for repairs and \$50 for taxes; what is the difference in the income from this and from the same money invested in 6% stock at 96?

248. A cistern contains 23104^{Kg} of water. What is its volume in cubic meters? If it has a square base, and its depth is 25^{m}, what is the length of an edge of its base?

249. How many five-cent coins may be made from a bar of silver 0.3^{m} long, 0.6^{dm} wide, and 5^{cm} thick, if each coin weighs 5^{g}, and silver is 10.5 times as heavy as water?

250. Resolve 21600 into its prime factors; and use them to find the greatest square number, and also the greatest cube, that will divide 21600 without remainder.

251. Divide $\dfrac{\frac{629}{17} \text{ of } \sqrt{9\frac{43}{49}}}{\dfrac{215}{5\frac{3}{8}}}$ by $\sqrt[3]{67419143}$.

252. Find the value to three decimal places of the expression $\sqrt[3]{\dfrac{3\frac{7}{9} \times 1\frac{1}{7} + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - 7\frac{7}{8} \div 28\frac{7}{20} + \frac{1}{3}}}$.

253. The length of a rectangular field is $\frac{4}{3}$ of the breadth, and the area is 9 acres. Find the diagonal in rods, feet, and inches.

254. If A can row at the rate of $12\frac{1}{2}$ miles per hour, and B at the rate of $11\frac{3}{4}$ miles per hour, what start should A give B in a race of 500 yards in order to beat him by one yard?

255. A clock gains $3\frac{1}{4}$ min. in 23 hr. 59 min. 45 sec.; at noon it is 2 min. slow; when will it indicate correct time?

256. 5 cu. ft. of gold weigh 98.2 times as much as a cubic foot of water, and 2 cu. ft. of copper weigh 18 times as much as a cubic foot of water; how many cubic inches of copper will weigh as much as $\frac{7}{8}$ of a cubic inch of gold?

257. A wins 9 games out of 15 when playing against B, and 16 out of 25 when playing against C. How many games out of 118 should C win when playing against B?

258. A cube is formed of a certain number of pounds Avoirdupois of a substance, and the same number of pounds Troy of the same substance. What ratio will a side of the cube bear to a side of a cube formed of the same number of pounds as before, but all Avoirdupois? (175 lb. Troy = 144 lb. Avoirdupois).

259. A Frenchman sells a draft on Paris for 10000 francs in New York at 5.15 francs for $1, and with the proceeds buys a bill of exchange on London at $8\frac{1}{2}\%$ premium; what is the amount of the bill in English currency?

260. A man paid $\frac{2}{5}$ of his money for stock, $\frac{1}{3}$ of what remained for goods, and $\frac{1}{8}$ of what then remained for tools; he then found that $26 was $\frac{1}{25}$ of one half of what was left. Find what part of the whole was left, and how much money he had at first.

261. Find in acres the area of a rectangular field of which the longer side is to the shorter as 15 : 8, and which a person walking at the rate of $3\frac{3}{11}$ miles per hour takes 5 min. 45 sec. to walk around.

262. A rectangular piece of ground is 13 ch. 44 li. by 8 ch. 40 li. How many square feet would be occupied on paper by a plan of the land drawn upon a scale of $1\frac{1}{2}$ inches to a chain?

263. A and B run a race, their rates of running being as 17 to 18. A runs $2\frac{1}{3}$ mi. in 16 min. 48 sec., and B runs the entire distance in 34 min. What was the entire distance?

264. Mr. A. buys a house for \$10000 and rents it for \$65 a month, paying \$150 per annum for taxes and repairs. He also buys 181 shares of railroad stock (par \$50) for \$55 each, and receives a dividend of 7%. What is his income and rate of interest from each investment? What is his total investment, income, and rate of interest?

265. Find to two decimal places the sixth root of the least common multiple of 899 and 1073.

266. A sum of money was placed at interest at 6% per annum; at the end of the first year the interest was added to the principal, and at the end of the second year the amount was \$842.70; what was the original sum?

267. Six men, working 9 hours a day, can do a piece of work in 15 days. In how many days will a party of men, working 10 hours a day, do the work, the number of men being equal to the number of days?

268. A square of 25^{m} on a side is inscribed in a circular walk 5^{m} wide. This walk is covered with asphalt 10^{cm} thick. What is the weight of the asphalt in metric tons if its specific gravity is 10?

269. A owes B \$1080 due July 5th, 1885, \$250 due Sept. 15th, 1885, \$700 due Dec. 10th, 1885, and \$300 due Mar. 20th, 1886; B owes A \$500 due Aug. 25th, 1885, \$400 due Jan. 1st, 1886, and \$350 due June 20th, 1886. Find the time when the balance due B may be paid without loss to either party. Find also the equitable value of that balance if payment were made Sept. 15th, 1886, the rate of interest being 6%.

270. A loaded wagon weighs 2 T. 3 cwt. 48 lb.; the wagon itself weighs 18 cwt. 75 lb. The load consists of 215 packages, each of the same weight. Find the weight of each, and reduce it to kilograms.

271. A box with a lid measures externally 16 in. each way, and the wood of which it is made is 1 in. thick; what would be the weight of the box when filled with paper, a cubic foot of paper weighing 792 oz. and a cubic foot of wood 840 oz.?

272. A box in the form of a cube is partially filled with water. Half a dozen balls, each 4 in. in diameter, are thrown in, and the water rises $\frac{1}{2}$ in. in consequence. Find the length of an edge of the box.

273. English shillings are coined from a metal which contains 37 parts of silver to 3 parts of alloy; one pound of this metal is coined into 66 shillings. The United States dollar weighs 412.5 grains, and consists of 9 parts silver to 1 of alloy. What fraction of the United States dollar will contain the same amount of silver as one English shilling?

274. Two bodies let fall at different instants from the same point are found, $\sqrt{\frac{5000}{981}}$ seconds after the latter of them started, to have fallen, the one 25^m, the other 100^m. These distances being to one another as the squares of the times during which the bodies have been falling, how many seconds must the one body have started before the other?

275. Using the prefix "*mega-*" for *a million times*, and "*micro-*" for *a millionth part of*, show how many megameters (roughly) make up the earth's circumference, and how many cubic micrometers of water weigh a microgram.